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Population Synthesis for Mira Variables

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Abstract By means of a population synthesis code, we investigate the Mira variables. Their birth rate (over $0.65\,\mathrm{yr}^{-1}$) and their number ($\sim 130\,000$) in the Galaxy are estimated. For all possible Mira variables, ranges of their initial masses, pulsating periods, mass losses and lifetimes are given. We check our model with the observed Mira variables near the Sun and our results prove to be valid.

Key words: stars: late-type – stars: variables – stars: fundamental parameters

1 INTRODUCTION

As stars ascend the asymptotic giant branch (AGB): it appears that they begin to pulsate when their effective temperature drops below a certain level (usually taken to be 3800 K (Percy 1997)) and the luminosity increases to a certain value. The amplitude tends to increase with decreasing temperature and appreciable periodicity which starts in a high radial overtone will gradually drop into lower overtones, then the stars approach the Mira stage. Mira variables are late M-type giant stars, pulsating with periods of 80 to 1000 days with visual amplitudes more than 2.5 magnitudes. They are the coolest and most luminous AGB stars, having a carbon core, and surrounded by helium-rich layers, then in turn surrounded by a hydrogen-rich envelope. At the AGB stage, two significant processes occur: (1) in the interior, due to instabilities in the thermonuclear reactions in the thin shell of helium, which flashes every few thousand years or so, causing cyclic changes in the temperature, luminosity, and pulsation period (Vassiliadis & Wood 1993); (2) in the outer layers, mass loss occurs as a result of the low escape velocity, the radiation field of the star and dust formation. The mass loss is enhanced by the pulsation of the star (Bowen & Willson 1991) and the star can lose a significant fraction of its mass within a million years. However, the study of the Mira variables has always been a very difficult task (Wood 1990). Theoretical analysis of the properties of the envelope has been made very uncertain by the problem of convective energy transport, while the study of the atmospheres needs a clear knowledge of the opacity resulting from the huge number of lines of molecules such as CN, CO, TiO, H_2O and so on. Although the models of atmospheres for Mira variables have been constructed by Bessell et al. (1989), we will investigate Mira variables from an evolutionary stand.

In this paper we attempt to find all possible Mira variables and to estimate their birth rates and lifetimes with an evolutionary population synthesis code. We shall evaluate the number of

Mira variables in the Galaxy, and obtain their distributions over potential observable ranges of masses, pulsating periods, mass-loss rates and so on. In Section 2 we present our assumptions and describe some details of the algorithm. In Section 3 we discuss our main results. A brief conclusion will be given in Section 4.

2 MODEL

We use the rapid single star evolution code of Hurley et al. (2000) for the single star tracks. Below we describe our algorithm and give some details that are important for the understanding of our model.

2.1 Model Input and Monte Carlo Simulations

For the population synthesis of a single stellar population the main input parameters are: (1) the initial mass function; (2) the lower and upper mass cut-offs M_l and M_u to the initial mass function; (3) the metallicity Z of the stars; (4) and the relative age of the single stellar population.

A simple approximation to the IMF of Miller & Scalo (1979) is used. The primary mass is generated with the formula of Eggleton, Fitchett & Tout (1989) (also Han et al. 1995; Han 1998),

$$M_1 = \frac{0.19X}{(1-X)^{0.75} + 0.032(1-X)^{0.25}} \ , \tag{1}$$

where X is a random variable uniformly distributed in the range [0, 1]. If M_1 is larger than $8.0\,M_\odot$, the number of stars is very small and the lifetimes on the stage of AGB are too short. The lower mass cut-off is taken $0.1\,M_\odot$. So M_1 is the stellar mass from 0.1–8 $M_\odot$. We take the solar metallicity ($Z = 0.02$) throughout this work. The maximum age of the single stellar population is 15 Gyr.

2.2 The Position of Mira Variables in the HR-diagram

The large amplitude pulsations exhibited by the stars when they become Miras are generally thought to drive the observed massive mass-loss of such stars in the upper part of the AGB. A good review is in Willson (2000). These stars appear in a limited region in the HR-diagram which has been discussed in Wood & Zarro (1981), Groenewegen & de Jong (1994) and Gautschy (1999). A most focused attempt to determine the region in the $L - T_{\rm eff}$-plane where stars become Mira variables is the work of Gautschy (1999), on which we will base our calculations. The effective temperature determined by Gautschy is a kind of equilibrium temperature which corresponds to the equilibrium states of the atmosphere. These temperature are therefore appropriate and can directly be related to the effective temperatures calculated for the envelopes of the single stellar population. We use the results of Gautschy (1999) for fundamental mode pulsators, since Mira variables are nearly certainly fundamental mode pulsators (Wood et al. 1999).

For the Galaxy Gautschy (1999) found the blue edge of the Mira instability strip to be located at log $T_{\rm eff} = 3.49$. The red edge is not well defined because of the low number of data points, possibly it is located below log $T_{\rm eff} = 3.43$. This would be in accord with the theoretical results of Xiong et al. (1998). The lower border of luminosity is not well defined, but in Gautschy (1999) it is about at log $L/L_\odot = 3.20$.

In our model we only care about the blue edge and lower border of the Mira instability strip which are taken respectively log $T_{\rm eff} = 3.49$ and log $L/L_\odot = 3.25$ (Ferrerotti & Gail 2005). We consider that the stage of Mira variables to be over when the stars finish the AGB evolution. So the lifetime of Mira variables in our model will be longer than that of observed.

2.3 The Mass-Loss Rates and Pulsating Periods

No complete theory of mass loss from AGB stars exists at present. In this paper, we use the empirical formula relating the mass-loss rate to the pulsating period (Vassiliadis & Wood 1993), in which the authors assumed that Mira variables and dust-enshrouded variable AGB stars are pulsating in the fundamental mode. The pulsating period is calculated using the period-mass-radius relation:

$$\log\ P(\mathrm{days}) = -2.07 + 1.94 \log R/R_\odot - 0.90 \log\ M/M_\odot, \tag{2}$$

with R and M being the stellar radius and mass respectively. If M <2.5M$_\odot$, the mass-loss rate ($M_\odot$ yr^{-1}) is

$$\log\ \dot{M} = -11.40 + 1.23 \times 10^{-2} P. \tag{3}$$

For $M \geq 2.5 M_\odot$,

$$\log\ \dot{M} = -11.40 + 1.23 \times 10^{-2} (P - 100(M/M_\odot - 2.5)). \tag{4}$$

When $P \sim 500$ days, there will be super wind. The mass-loss rate will be

$$\dot{M} = 2.1 \times 10^{-8} \frac{L/L_\odot}{v_\infty}\ (\mathrm{km\ s^{-1}}), \tag{5}$$

and

$$v_\infty = -13.50 + 5.60 \times 10^{-2} P\ \mathrm{km\ s^{-1}}, \tag{6}$$

where v_∞ is the terminal velocity of the wind.

3 RESULTS AND DISCUSSION

We calculate 1×10^5 single stars and pick out all possible Mira variables. Our results and analysis are as follows.

3.1 The Distribution of Initial Masses, Birth Rate and Lifetime

In our models, there are 11342 Mira variables in 1×10^5 stars. That is, a 1/10 of stars goes through the Mira instability strip. From Fig. 1 the initial masses lie at $1.01\, M_\odot \sim 4.0\, M_\odot$, with the majority smaller than $1.7\, M_\odot$. Stars with initial masses less than $1.0 M_\odot$ cannot evolve to the Mira instability strip because they have evolved to post-AGB and turned into white dwarfs before their effective temperatures do not drop to the blue edge of the Mira instability strip. For stars with too small initial mass, they cannot evolve to AGB in Hubble time. In this paper, we assume that the Galaxy makes $1\,\mathrm{yr}^{-1}$ star with mass larger than $0.8\, M\odot$ (Kenyon 1994; Yungelson et al. 1995). Then the birth rate of Mira variables is $\sim 0.65\,\mathrm{yr}^{-1}$. Figure 2 shows the lifetimes of the Mira variables of different initial masses. We estimate the average lifetime of Mira variables to be $\sim 2 \times 10^5$ yr. The observations suggest a mean Mira lifetime of $\sim 6 \times 10^4$ (Wood 1990). The most important reason for the discrepancy is our assumption in Section 2.2. Then the number of Mira variables in the Galaxy is about $\sim 1.3 \times 10^5$.

3.2 The Distribution of Pulsating Periods and Mass Loss of Mira Variables

Two main characteristics are the pulsating period and mass loss. In Fig. 3, we give the distribution of the minimal pulsating periods of Mira variables. Most of the minimal pulsating periods are smaller than 375 days. Figure 4 shows the range of pulsating periods for all the Mira variables of different initial masses. For the most parts the pulsating periods are less than

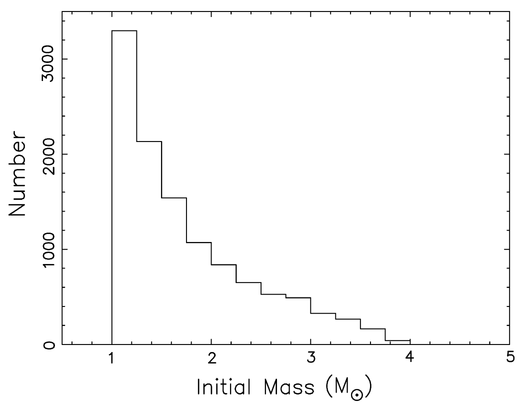

Fig. 1 Distribution of initial masses of Mira variables.

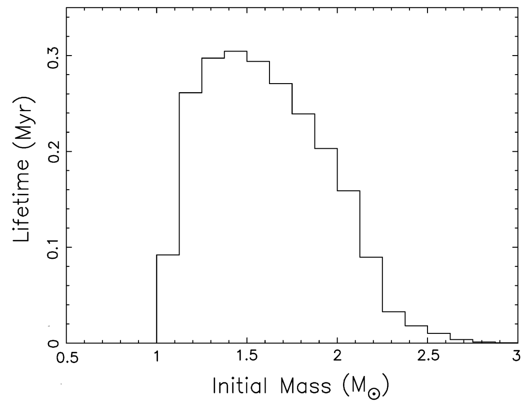

Fig. 2 Average lifetimes of Mira variables plotted against initial stellar masses.

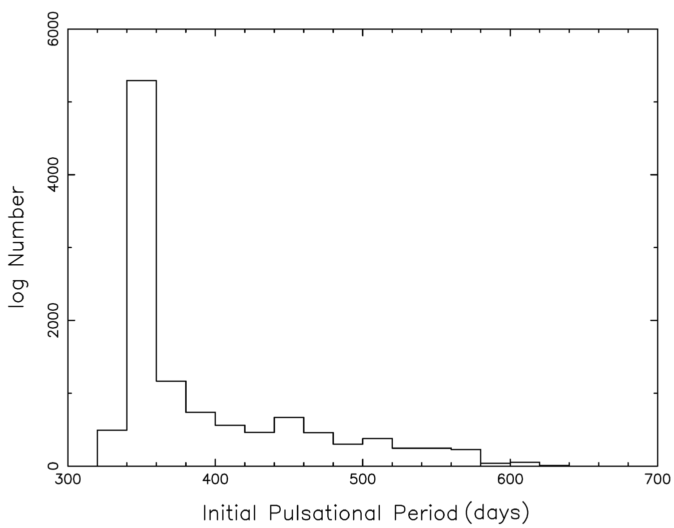

Fig. 3 Distribution of pulsating periods on stars evolving to the stage of Mira variables.

1000 days. Wood (1990) considered that Mira variables with very long periods ($P > 1000$ days) can only be produced by relatively massive AGB stars with initial masses $\geq 1.5\,M_\odot$. This is consistent with our results. From Figs. 3 and 4, our minimal pulsating period is larger than 320 days, which is not identical with the observations. In Wood & Cahn (1977) and Wyatt & Cahn (1983), there are some periods of Mira variables shorter than 300 days. The main reason is that our blue edge of the Mira instability strip is a crude one: in Fig. 5, we take $\log T_{\rm eff} = 3.50$ as the blue edge, but the minimal pulsating period is very sensitive to a change in the selected temperature.

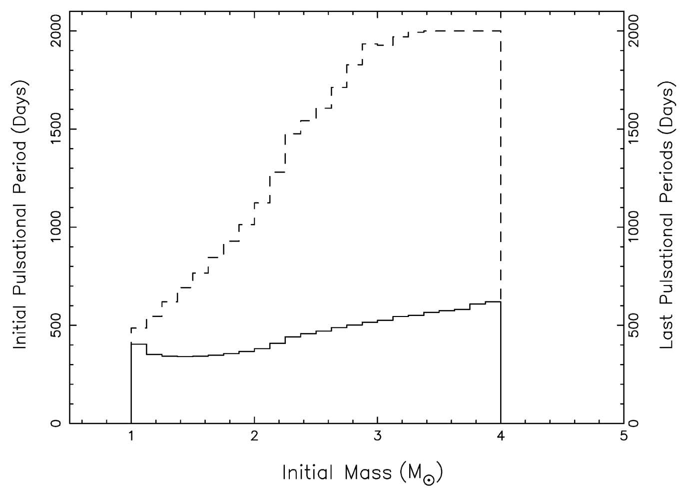

Fig. 4 Pulsating periods given by different initial masses. The solid line refers to the time of entry into the Mira instability strip; the dash line, the time of exit.

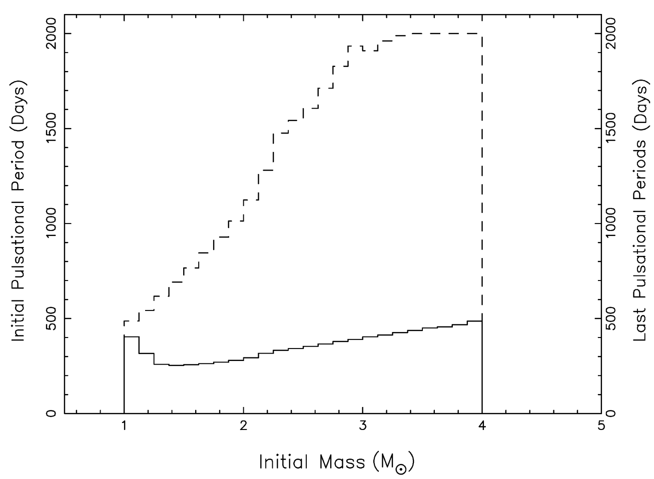

Fig. 5 Same as Fig. 4, but with the blue edge of the Mira instability strip ending at log $T_{\rm eff} = 3.50$.

From Fig. 6, the mass loss during the Mira stage is dominant over the whole life of the star although Mira instability strip only last $\sim 10^5$ years. The determination of the amount of material injected by stars into the interstellar medium is a pre-requisite for any attempt to model the evolution of the interstellar matter of the galaxy and Mira variables play a key role here.

3.3 Mira Variables in the Vicinity of the Sun

Many observed Mira variables are located near the Sun. In Fig. 7 the observed periods are plotted against the calculated initial masses from kinematic properties, using the data in Wyatt

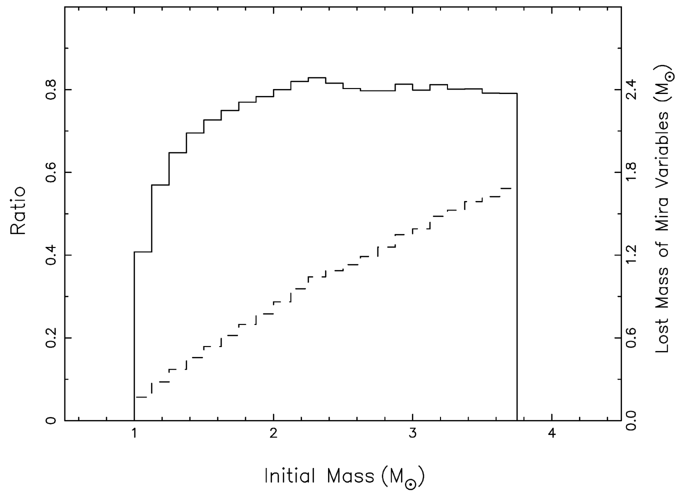

Fig. 6 Mass loss for different initial masses. The dash line shows the mass loss on the Mira instability strip, the solid line, the same relative to the total mass loss.

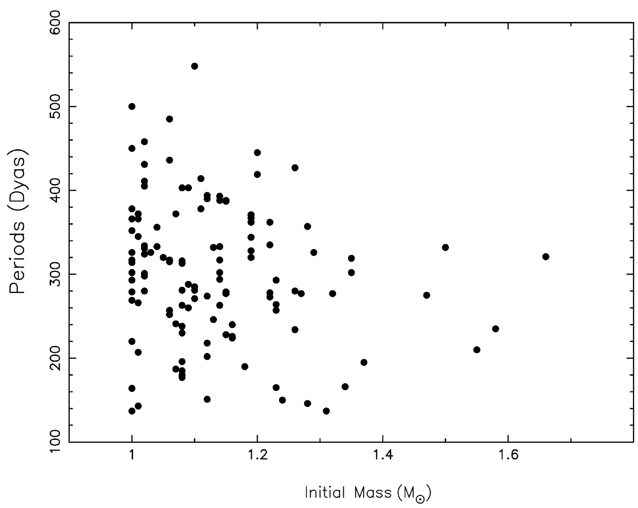

Fig. 7 Periods and the initial masses of 124 Mira variables taken from Wyatt & Cahn (1983). Details can be seen in the text.

& Cahn (1983). We assume that all stars near the Sun have about the same metallicity ($Z \sim 0.02$). The Mira variables in the solar neighborhood probably have the following properties:

(1) According to Figs. 1 and 2, most of initial masses are between $\sim 1.0\,M_\odot$ and $2.0\,M_\odot$. Wyatt & Cahn (1983) derived a mass range between $1.00\,M_\odot$ and $1.66\,M_\odot$ for the progenitor main sequence stars of 124 Miras in the greater solar neighborhood from an analysis of their kinematic properties (only 10 percent of these are larger than $1.4\,M\odot$, see also Fig. 7).

(2) According to Figs. 3, 4 and 5, the pulsating periods should be between ~ 200 days and ~ 600 days. Wood & Cahn (1977) found that most Mira variables have periods between 320 days and 420 days in the solar neighborhood.

(3) Based on the distribution of Mira variables observed for a region near the galactic plane, Wood & Cahn (1977) estimated that the space density of Mira variables is 245 Miras kpc^{-3}, the mean lifetime is 7×10^5 yr, and the birth rate is 3.0×10^{-4} Miras yr^{-1} kpc^{-1}. If we crudely assume that the stars are distributed in a galactic plane volume of $\sim 10^3$ kpc^3, then the birth rate of stars per kpc^3 is $\sim 10^{-3}$ and the birth rate of Mira variables is $\sim 6 \times 10^{-4}$ kpc^{-3}. In the present work, the mean lifetime of Mira variables is $\sim 2 \times 10^5$ yr, then the space density is $\sim 10^2$ Miras kpc^{-3}. Wood & Cahn (1977) may have overestimated the mean lifetime. Oort & van Tulder (1942) derived a space density of ~ 100 Miras kpc^{-3}. If we take 4000 kpc^3 as the volume of the galactic plane, then near the Sun we have 2.5 times the birth rate of Mira variables. From the above comparison, it can be verified that our model is valid in the solar neighborhood.

4 CONCLUSIONS

Basing on the stellar evolution given by the evolutionary population synthesis of a single stellar population, we derive some properties of Mira variables. It is crucial for our model to precisely pin down the Mira instability strip. Because of limited data, the blue edge, lower border and the red edge of the Mira instability strip in Gautschy (1999) need to be revised. With more observations and theoretical study, our model would become more realistic.

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