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Two-fluid Dynamics in Clusters of Galaxies *

Yu-Qing Lou[1,2,3]

[1] National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012
[2] Department of Physics & Tsinghua Center for Astrophysics (THCA), Tsinghua University, Beijing 100084; *lou@oddjob.uchicago.edu* / *louyq@tsinghua.edu.cn*
[3] Department of Astronomy and Astrophysics, The University of Chicago, 5640 South Ellis Avenue, Chicago, IL 60637 USA

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Abstract We develop a theoretical formulation for the large-scale dynamics of galaxy clusters involving two spherical 'isothermal fluids' coupled by their mutual gravity and derive asymptotic similarity solutions analytically. One of the fluids roughly approximates the massive dark matter halo, while the other describes the hot gas, the relatively small mass contribution from the galaxies being subsumed in the gas. By properly choosing the self-similar variables, it is possible to consistently transform the set of time-dependent two-fluid equations of spherical symmetry with self-gravity into a set of coupled nonlinear ordinary differential equations (ODEs). We focus on the analytical analysis and discuss applications of the solutions to galaxy clusters.

Key words: dark matter – hydrodynamics – ISM: general – galaxies: clusters – outflows, winds – shocks

1 INTRODUCTION

A cluster of galaxies with a size of several Mpcs across may consist of a few tens to several hundreds or a thousand of galaxies bound together by gravity. Extensive X-ray observations reveal that a typical galaxy cluster is pervaded with a hot fully ionized gas medium of a temperature ranging from $\sim 10^7 - 10^8\,$K (e.g. Fabian 1988, 1994; Sarazin 1986, 1988; Fabian et al. 2003). The confinement of such extended hot gas medium in space, the high velocity dispersion of galaxies in clusters, and independent observations of gravitational lensing effects point to the presence of an unseen dark matter halo with a mass typically ten times larger than that of the gas medium in a galaxy cluster. In comparison, the mass of all cluster galaxies takes up only a few percent. In addition, numerous observations have indicated the presence of intracluster magnetic field of strengths $\gtrsim 1\,\mu$G (Fabian 1994; Clarke et al. 2001; Carilli &

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Taylor 2002; Hu & Lou 2004a). By the equipartition argument, magnetic field strengths have been estimated to be as high as $\sim 30-40\,\mu$G in the central core regions of some galaxy clusters.

It is of considerable interest to study and understand the large-scale hydrodynamics or magnetohydrodynamics (MHD) of the hot gas medium, the gravitational interplay between the gas medium and the dark matter halo, the heating of the gas and various roles of the intracluster magnetic field. In particular, diagnostics of intracluster gas and magnetic field can be utilized to probe the underlying large-scale dynamics of dark matter halo which is only known through its Newtonian gravitational effects. In this paper, we formulate a highly idealized model yet with plausible applications to galaxy clusters in mind. We hope that this much simplified approach can offer useful concepts and insights for understanding certain large-scale aspects of galaxy clusters. The hot intracluster gas medium is treated here as a fluid (e.g. Pringle 1989; Balbus & Soker 1990). While we shall mainly focus on large-scale hydrodynamics in this paper, we note in passing that when intracluster magnetic fields are involved, it should be a fairly good approximation to apply the MHD equations for a magnetized hot gas medium. In particular, the presence of magnetic field can make a collisionless gas medium to behave more like a fluid An immediate example in mind is the successful MHD description for a collisionless plasma of the magnetized solar wind in the interplanetary space. As for the dark matter halo, we also invoke the 'hydrodynamic approximation' in the spirit of Jeans equation (e.g. Binney & Tremaine 1987; Peebles & Vilenkin 1999; Peebles 2000; Subramanian 2000) as the first step. We further introduce additional assumptions to simplify the mathematical treatment while retaining the essential spirit of this fluid approach. For example, the random velocity dispersion of 'dark matter particles' is associated with an 'effective pressure'. Moreover, such a velocity dispersion is presumed to be constant, equivalent to an 'isothermal' approximation for the 'dark matter fluid'. Conceptually, all these assumptions can be modified as we know little about dark matter particles or fluids except for their ubiquitous manifestation of gravitational effects. We are using a simple formulation to study large-scale dynamical interactions between the gas and dark matter 'fluids' through self-gravity and mutual gravity. To make the problem manageable, we assume spherical symmetry but allow for a time dependence. Our main goal is to derive and construct possible similarity solutions for collapses, expansions, possible radial oscillations and shocks in a composite spherical system of two coupled isothermal fluids (i.e. the hot gas and dark matter halo). In addition to applications to galaxy clusters, this idealized model formulation with proper adaptations should be of interests to other astrophysical systems as well.

We do not yet know the exact physical nature of dark matters except for their Newtonian gravitational effects. Currently, the most common approach is to resort to the N-body numerical simulation by assigning each 'particle' a mass and by computing mutual gravitational interactions consistently for a large number (N) of 'particles' with prescribed initial and boundary conditions for the entire system. In essence, this problem is quite similar to the dynamical treatment of a collection of $\sim 10^{11}-10^{12}$ stars in a galaxy. These visible stars are collisionless and have mutual gravitational interactions. Depending on the level of physical information that one would like to extract, one may use a N-body numerical simulation, a distribution function approach, and a fluid approximation to model the collection of stars in a galactic system. The most prominent example in mind is the density wave theory in spiral galaxies that has been developed, analyzed and tested in N-body numerical simulations (e.g., Miller et al. 1970; Hohl 1971; Ostriker & Peebles 1973), in distribution function approach (e.g., Shu 1968; Binney & Tremaine 1987) and in fluid approximation (e.g., Lin & Shu 1964; Goldreich & Tremaine 1978; Lin 1987; Bertin & Lin 1996), respectively. For large-scale and relatively slow dynamics of a galaxy cluster without resonances and singularities, fluid approximation for gravitationally interacting particles is justifiable, much simpler and should be sufficient to provide basic results. As a result of gravitational stratification, the density of 'dark matter' in the central region should be higher than in the outer region; a fluid approximation is therefore expected to be better in the central region.

We venture into this more challenging theoretical problem of a composite system of two gravitating fluids in spherical geometry because we have gained considerable knowledge and experience regarding a closely related problem of self-similar collapses and outflows in a single isothermal fluid of spherical symmetry (Lou & Shen 2004; Shen & Lou 2004). In fact, this classical single fluid problem has been extensively examined in the past (Larson 1969; Penston 1969; Lazarus 1981; Shu 1977; Hunter 1977, 1986; Whitworth & Summers 1985; Ori & Piran 1988; Foster & Chevalier 1993; Tsai & Shu 1995; Shu et al. 2002) in various contexts of star or cloud formation (Hayashi 1966; Hunter 1967; Shu et al. 1987). Replacing the isothermal assumption by the polytropic approximation (Cheng 1978; Goldreich & Weber 1980; Yahil 1983; Bouquet et al. 1985; Suto & Silk 1988; Antonova & Kazhdan 2000), this single fluid problem has been investigated in various contexts of core collapses and supernova explosions (e.g., Lattimer & Prakash 2004). We have recently pursued this 'polytropic problem' (Lou & Gao in preparation) for constructing global similarity solutions of envelope expansion with core collapse (EECC) types (Lou & Shen 2004) and for inserting a similarity shock in an appropriate place (Landau & Lifshitz 1959; Tsai & Hsu 1995; Shu et al. 2002; Shen & Lou 2004). One important distinction between the 'isothermal problem' and the 'polytropic problem' is that the asymptotic flow at large x can be constant in the former and always approaches zero in the latter. Furthermore, it is possible to incorporate effects of radiative losses in the latter in a more realistic manner (e.g., Rybicki & Lightman 1979; Boily & Lynden-Bell 1995).

In the context of galaxy cluster formation and evolution in an expanding universe, similarity flow solutions have been sought for in the past (Gunn & Gott 1972; Fillmore & Goldreich 1984; Bertschinger 1985). With more detections of high-redshift ($z \gtrsim 6$) quasars suspected to be powered by supermassive black holes of mass $\sim 10^9 M_\odot$ (Fan et al. 2001, 2003; Willott et al. 2003; Vestergaard 2004), accretion shocks in an infalling gas have been modeled around a supermassive black hole of mass $\gtrsim 10^9 M_\odot$ to form the first generation of quasars (e.g., Wandel et al. 1984; Barkana & Loeb 2003) and produce characteristic radiative spectral features for resonant Lyman α absorption. It was suggested that this happens in response to the gravity pull of massive dark matter haloes ($\gtrsim 10^{12} M_\odot$).

Although in a distinctly different geometry, we have been investigating a class of related problems (with or without magnetic field) involving global perturbation structures in a composite system of two disks coupled by mutual gravity (e.g., Lou & Fan 1998; Lou & Shen 2003; Shen & Lou 2003, 2004a; Lou & Zou 2004; Lou & Wu 2005 submitted). While these composite disk problems have applications in other astrophysical contexts such as magnetized spiral galaxies, they also pave the way of thinking and attacking the current problem conceptually and technically. We now turn to Section 2 for the theoretical formulation and mathematical development of the model analysis of two-fluid dynamics with spherical symmetry. In Section 3, we derive various analytical similarity solutions and examine their properties. Applications of these solutions to galaxy clusters will be discussed in Section 4.

2 TWO-FLUID DYNAMICS OF SPHERICAL SYMMETRY

The dynamical formulation of spherical collapse and outflows for a composite system of two isothermal fluid spheres coupled by mutual gravity involves two mass conservation equations, two radial momentum equations; the two isothermality conditions replace the two energy equations; and the Poisson equation is automatically satisfied by introducing the enclosed mass $M^i(r,t)$ within r at time t. One major approximation here is the fluid approach for the collection of dark matter particles which might be collisionless. It is hoped that this simple approach could catch some essential features of the large-scale dynamics. We also tacitly assume that radiative losses are compensated by compressional heating of the gas medium. For a dynamical system of spherical symmetry, there is no gravitational wave losses from the dark matter fluid. In other words, it may be plausible to regard both the gas and dark matter fluids as approximately isothermal. It follows that both the gas pressure and the effective pressure of the dark matter fluid are proportional to their respective densities. By the symmetry of the problem, the relative roles of the two fluid spheres are identical − both involve spherical radial flows of isothermal and self-gravitating fluids. At this stage, we ignore other factors, such as magnetic field, radiation losses, rotation, turbulence and so forth for the sake of simplicity.

We can readily write down the two-fluid equations (in the Eulerian form) of spherical symmetry in spherical polar coordinates (r, ϑ, φ). Mass conservation separately for the two fluids is given by

$$\frac{\partial\rho^i}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho^i u^i)=0\ , \tag{1}$$

where the superscript $i=g$ or d stands for gas ('g') and dark matter ('d') 'fluids', respectively; $\rho^i(r,t)$ is the mass density; $u^i(r,t)$ is the bulk radial flow speed. Equivalently, we can write

$$\frac{\partial M^i}{\partial t}+u^i\frac{\partial M^i}{\partial r}=0\ , \qquad\qquad \frac{\partial M^i}{\partial r}=4\pi r^2\rho^i\ , \tag{2}$$

where $M^i(r,t)$ is the total mass of the gas or dark matter, enclosed within r at time t . The radial momentum equation is,

$$\frac{\partial u^i}{\partial t}+u^i\frac{\partial u^i}{\partial r}=-\frac{(a^i)^2}{\rho^i}\frac{\partial\rho^i}{\partial r}-\frac{GM}{r^2}\ , \tag{3}$$

where $M=M^g+M^d$ and GM/r^2 gives rise to the gravitational coupling between the gas medium and the dark matter fluid with M denoting the sum of the enclosed gas and dark matter masses; the flows here are taken to be isothermal and the pressure term here has been replaced with $(a^i)^2\rho^i$ according to the isothermal approximation; a^i is the isothermal sound speed. [1] In Eq. (3), we naturally identify

$$-\frac{\partial\Phi}{\partial r}\equiv-\frac{GM(r,t)}{r^2}\ , \tag{4}$$

where Φ is the total gravitational potential of the composite system of two fluids. Here, the Poisson equation for the gravitational potential Φ is automatically satisfied. Meanwhile, the

[1] For the dark matter 'fluid', the velocity dispersion is mimicked by an effective 'sound speed'.

energy conservation equation can be readily derived by combining Eqs. (1) and (3), namely

$$\frac{\partial}{\partial t}\left\{\frac{\rho^g(u^g)^2}{2}+\frac{\rho^d(u^d)^2}{2}-\frac{1}{8\pi G}\left(\frac{\partial\phi}{\partial r}\right)^2+(a^g)^2\rho^g\left[\ln\left(\frac{\rho^g}{\rho^g_c}\right)-1\right]+(a^d)^2\rho^d\left[\ln\left(\frac{\rho^d}{\rho^d_c}\right)-1\right]\right\}$$
$$+\frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2\rho^g u^g\left[\frac{(u^g)^2}{2}+(a^g)^2\ln\left(\frac{\rho^g}{\rho^g_c}\right)\right]\right\}+\frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2\rho^d u^d\left[\frac{(u^d)^2}{2}+(a^d)^2\ln\left(\frac{\rho^d}{\rho^d_c}\right)\right]\right\}$$
$$+\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2(\rho^g+\rho^d)(u^g+u^d)\Phi+\frac{r^2\Phi}{4\pi G}\frac{\partial^2\Phi}{\partial r\partial t}\right]=0\ , \tag{5}$$

where ρ^g_c and ρ^d_c are two arbitrary constant density scales (Fan & Lou 1999; Lou & Shen 2004). For this specific problem, the last term on the left-hand side of Eq. (5) explicitly related to $\Phi(r,t)$ vanishes, as in the case of a single isothermal sphere (Lou & Shen 2004). In equation (5), we can further identify the expressions for the energy density and the radial energy flux density, respectively.

With the assumption that the central object has a negligible dimension in comparison with the region of large-scale radial flows, the dimensional quantities in the problem under consideration here are the gravitational constant G, the 'isothermal sound speeds' of gas and dark matter a^i, the radius r and the time t. We therefore define two independent similarity variables in the form of

$$x^g\equiv r/(a^g t)\qquad\text{and}\qquad x^d\equiv r/(a^d t)\ . \tag{6}$$

For constructing global self-similar solutions, we introduce the following similarity transformations for the dependent physical variables ρ^i, M^i and u^i with characteristic length scales $a^i t$, mass scales $(a^i)^3t/G$ and speed scales a^i:

$$\rho^i(r,t)\equiv\frac{\alpha^i(x^i)}{4\pi Gt^2}\ ,\qquad M^i(r,t)\equiv\frac{(a^i)^3t}{G}m^i(x^i)\ ,$$
$$u^i(r,t)\equiv a^i v^i(x^i)\ ,\qquad \Phi(r,t)\equiv(a^g)^2\phi(x^i)\ , \tag{7}$$

where the dimensionless dependent variables $\alpha^i(x^i)$, $m^i(x^i)$ and $v^i(x^i)$ are the reduced densities, enclosed masses and radial flow speeds of gas or dark matter fluid, respectively. Our choice of reduced gravitational potential variable is somewhat arbitrary here; it would be almost the same should we set $\Phi(r,t)=\phi(x^i)(a^d)^2$.

With the similarity transformations (7) in Eqs. (1) and (2), we have the following coupled nonlinear ordinary differential equations (ODEs),

$$m^i+(v^i-x^i)\frac{dm^i}{dx^i}=0\ ,\qquad \frac{dm^i}{dx^i}=(x^i)^2\alpha^i\ .$$

By eliminating the term dm^i/dx^i, the reduced mass m^i can be obtained in the form of

$$m^i=(x^i)^2\alpha^i(x^i-v^i)\ , \tag{8}$$

where x^i must be larger than v^i in order to guarantee a positive m^i for $t>0$.

A direct substitution of Eqs. (7) and (8) and some manipulations allow us to transform Eqs. (2) and (3) into the following coupled sets of ordinary differential equations,

$$[(x^g-v^g)^2-1]\frac{1}{\alpha^g}\frac{d\alpha^g}{dx^g}=\left[\alpha^g-\frac{2}{x^g}(x^g-v^g)\right](x^g-v^g)+\alpha^d(x^d-v^d)\frac{a^d}{a^g}\ , \tag{9}$$

$$[(x^g-v^g)^2-1]\frac{dv^g}{dx^g}=\left[\alpha^g(x^g-v^g)+\alpha^d(x^d-v^d)\frac{a^d}{a^g}-\frac{2}{x^g}\right](x^g-v^g)\ , \tag{10}$$

$$[(x^d-v^d)^2-1]\frac{1}{\alpha^d}\frac{d\alpha^d}{dx^d}=\left[\alpha^d-\frac{2}{x^d}(x^d-v^d)\right](x^d-v^d)+\alpha^g(x^g-v^g)\frac{a^g}{a^d}\ , \tag{11}$$

$$[(x^d-v^d)^2-1]\frac{dv^d}{dx^d}=\left[\alpha^d(x^d-v^d)+\alpha^g(x^g-v^g)\frac{a^g}{a^d}-\frac{2}{x^d}\right](x^d-v^d)\ . \tag{12}$$

These coupled nonlinear ODEs bear a strong resemblance to the self-similar nonlinear ODEs for a single isothermal sphere (Larson 1969; Penston 1969; Shu 1977; Lou & Shen 2004; Shen & Lou 2004). Here, the mutual gravitational interaction couples the collapse and outflow behaviors of the two isothermal fluid spheres.

As the two independent similarity variables x^d and x^g are related by $x^d=x^g a^g/a^d\equiv x^g\beta$, where β is defined to be the ratio of a^g to a^d, we can rewrite Eqs. (9)$-$(12) above in terms of one single independent similarity variable x^g as

$$[(x^g-v^g)^2-1]\frac{1}{\alpha^g}\frac{d\alpha^g}{dx^g}=\left[\alpha^g-\frac{2}{x^g}(x^g-v^g)\right](x^g-v^g)+\alpha^d(x^g-v^D)\ , \tag{13}$$

$$[(x^g-v^g)^2-1]\frac{dv^g}{dx^g}=\left[\alpha^g(x^g-v^g)+\alpha^d(x^g-v^D)-\frac{2}{x^g}\right](x^g-v^g)\ , \tag{14}$$

$$\left[(x^g-v^D)^2-\frac{1}{\beta^2}\right]\frac{1}{\alpha^d}\frac{d\alpha^d}{dx^g}=\left[\alpha^d-\frac{2}{x^g}(x^g-v^D)\right](x^g-v^D)+\alpha^g(x^g-v^g)\ , \tag{15}$$

$$\left[(x^g-v^D)^2-\frac{1}{\beta^2}\right]\frac{dv^D}{dx^g}=\left[\alpha^d(x^g-v^D)+\alpha^g(x^g-v^g)-\frac{2}{x^g\beta^2}\right](x^g-v^D)\ , \tag{16}$$

where $v^D\equiv v^d/\beta$ is a rescaled value of the reduced radial flow speed of the dark matter. Here β^2 represents the ratio of the 'temperatures' of gas to dark matter fluid. In fact, physical variables of gas and dark matter are symmetric in Eqs. (9)$-$(12). We explore possible solutions for $\beta>1$ and then derive the corresponding solutions for $\beta<1$ by interchanging the roles of the two 'fluids'. From now on, we write $x\equiv x^g$ for convenience.

We can also derive two Bernoulli relations along streamlines from the basic two-fluid equations (1)$-$(3), or equivalently, from the self-similar nonlinear ODEs (13)$-$(16), namely

$$f^g-x\frac{df^g}{dx}+\frac{(v^g)^2}{2}+\ln\alpha^g+\phi=\text{const.}\ , \tag{17}$$

$$f^d-x\frac{df^d}{dx}+\frac{(v^d)^2}{2}+\ln\alpha^d+\beta^2\phi=\text{const.}\ , \tag{18}$$

where $f^i(x)$ and $\phi(x)$ are functions of x only and satisfy the following equations

$$\frac{df^i(x)}{dx}=v^i\ , \tag{19}$$

$$\frac{d\phi(x)}{dx}=\frac{m^g}{x^2}+\frac{m^d}{x^2\beta^3}=\alpha^g(x-v^g)+\alpha^d(x-v^D)\ . \tag{20}$$

3 SIMILARITY SOLUTIONS FOR COUPLED FLOWS

In Sect. 3 here, we derive various analytical similarity solutions and examine their properties. Applications of these solutions to galaxy clusters will be discussed in Sect. 4.

3.1 Global Analytical Dynamic Solutions

For spherical collapses and outflows in a single isothermal self-gravitating fluid, there exist several known exact global analytical solutions. The first one is the self-similar hydrostatic state (Ebert 1955; Bonner 1956; Chandrasekhar 1957; Shu 1977). While a hydrostatic solution in a composite system does exist under a special constraint between ρ^d and ρ^g, there is no self-similar counterpart of this first solution for a composite system of two fluids coupled by their mutual gravity (see discussions after solution (13) in Lou & Shen 2004). The second self-similar solution describes a nonrelativistic 'Hubble expansion' in the Einstein-de Sitter universe (Whitworth & Summers 1985; Shu et al. 2002; Lou & Shen 2004). The self-similar counterpart of this second solution in a composite system of two isothermal fluids is

$$\begin{aligned} &v^g=\frac{2}{3}x\ , && v^D=\frac{2}{3}x\ , && \alpha^g+\alpha^d=\frac{2}{3}\ ,\\ &m^g=\frac{\alpha^g x^3}{3}\ , && m^d=\frac{\beta^3\alpha^d x^3}{3}\ , && \frac{d\phi}{dx}=\frac{2x}{9}\ , \end{aligned} \tag{21}$$

where α^g and α^d are two positive constants constrained by the third condition in Eq. (21). Note that this solution represents global similarity outflows that may be relevant in various astrophysical systems (e.g. possible winds from clusters of galaxies).

We now proceed to demonstrate why the self-similar hydrostatic solution does not exist in a composite system of two coupled fluids below.

3.2 Hydrostatic Equilibria

Static solutions represent a very special subclass. In contrast to a single isothermal self-gravitating fluid, a 'hydrostatic self-similar solution' does not exist, because when $v^g=v^D=0$, Eqs. (14) and (16) cannot be satisfied except for the special $\beta=1$ case for a single isothermal fluid. However, based on the original radial momentum equations, hydrostatic equilibrium for a composite system of two isothermal self-gravitating fluids can be constructed by solving the following ODEs [see Eqs. (1)−(3)],

$$\frac{\partial\rho^g}{\partial t}=\frac{\partial\rho^d}{\partial t}=0\ , \tag{22}$$

$$-\frac{(a^g)^2}{\rho^g}\frac{\partial\rho^g}{\partial r}-\frac{GM}{r^2}=-\frac{(a^d)^2}{\rho^d}\frac{\partial\rho^d}{\partial r}-\frac{GM}{r^2}=0\ . \tag{23}$$

By the similarity transformations (7) of ρ^i, the above condition (23) indicates two apparently inconsistent relations

$$\frac{d\ln\alpha^g}{dx^g}=\frac{d\ln\alpha^d}{dx^d}\beta$$

and

$$\frac{d\ln\alpha^g}{dx^g}\beta=\frac{d\ln\alpha^d}{dx^d}\ ,$$

separately, unless $\beta=1$ for a single isothermal fluid.

On the other hand, by making the substitution of a specific density relation $\rho^d=K(\rho^g)^{\beta^2}$ derived directly from the static equilibrium condition (23), where K is a constant parameter, into Eq. (23), the radial force balance can be readily cast in the form

$$\frac{(a^g)^2}{r^2}\frac{d}{dr}\left(r^2\frac{d\ln\rho^g}{dr}\right)=-4\pi G[\rho^g+K(\rho^g)^{\beta^2}]\ , \tag{24}$$

in terms of the gas density $\rho^g(r)$ and radius r. By a proper variable transformation, we introduce a new pair of dependent and independent variables $\varphi(\zeta)$ and ζ such that

$$\rho^g = \lambda e^{-\varphi} \ , \qquad\qquad r = \left[\frac{(a^g)^2}{4\pi G\lambda}\right]^{1/2}\zeta \equiv \sigma\zeta \ , \tag{25}$$

where λ is, for the present, an arbitrary constant and $\sigma \equiv [(a^g)^2/(4\pi G\lambda)]^{1/2}$ represents a length scale. It follows immediately that Eq. (24) with $\delta = 0$ reduces to

$$\frac{1}{\zeta^2}\frac{d}{d\zeta}\left(\zeta^2\frac{d\varphi}{d\zeta}\right) = e^{-\varphi} + K\lambda^{\beta^2-1}e^{-\beta^2\varphi} \ . \tag{26}$$

If the central densities remain finite as $r \to 0$, we may choose λ and $K\lambda^{\beta^2}$ as the central densities of the gas and the dark matter 'fluids', respectively. In terms of an asymptotic series expansion near $\zeta = 0$, we show a few leading terms sufficiently accurate to compute $\varphi(\zeta)$, namely

$$\varphi = \frac{(1+Q)}{6}\zeta^2 - \frac{(1+Q)(1+Q\beta^2)}{120}\zeta^4 + \cdots \tag{27}$$

where $Q \equiv K\lambda^{\beta^2-1}$ is the ratio of the central density of dark matter fluid to the central density of gas. We emphasize that without the special density relation $\rho^d = K(\rho^g)^{\beta^2}$, a static equilibrium of two isothermal fluids under self-gravity and with spherical symmetry is impossible. In modeling galaxy clusters both theoretically and observationally, it is most common to invoke a static equilibrium to begin with. As ρ^g and ρ^d can be inferred observationally and by the law of gravity, it is then possible to test the applicability of various assumptions involved in the formulation. The main point here is not necessarily the reality of the isothermal assumption for the two 'fluids' as this is taken for simplicity and illustration; if two different temperature profiles are prescribed for the two 'fluids', another ρ^d and ρ^g relation of the sort would be implied and can be tested by observational and theoretical inferences. This appears to be a practical way of learning more physical properties of dark matters in galaxy clusters.

3.3 Asymptotic Behaviors of Dynamic Solutions

It is plausible to presume that the reduced radial speeds $v^i(x)$ approach finite values at the 'initial instant' or $x \to +\infty$. Such solutions can describe either outflows when $v^i(x) > 0$ or inflows when $v^i(x) < 0$ as $x \to +\infty$. Based on our knowledge and experience with similarity flow solutions in a single isothermal fluid (Lou & Shen 2004), we further anticipate global similarity solutions of envelope expansion with core collapse (EECC) for a composite system of two coupled isothermal flows in appropriate parameter regimes. With these possibilities in mind, we derive the following asymptotic behaviors at large x of the similarity solutions:

$$\begin{aligned}
\alpha^g &\to \frac{A}{x^2} - \frac{(A+B-2)A}{2x^4} + \cdots \ , \\
v^g &\to C - \frac{A+B-2}{x} - \frac{(A+B-2)C-(AC+BD)}{2x^2} + \cdots \ , \\
\alpha^d &\to \frac{B}{x^2} - \frac{(A+B-2/\beta^2)B}{2x^4} + \cdots \ , \\
v^D &\to D - \frac{A+B-2/\beta^2}{x} + \frac{(AC+BD)-(A+B-2/\beta^2)D}{2x^2} + \cdots \ ,
\end{aligned} \tag{28}$$

where A and B are two constants representing the leading terms of the asymptotic densities of the gas and dark matter fluids at large x, and C and D are two other constants for the leading terms of the asymptotic radial flow speeds of the gas and dark matter fluids at large x. To the leading order, we have in the limit of $t \to 0^+$

$$\rho^i(r,0) = \frac{(a^i)^2\Lambda^i}{4\pi G} r^{-2} \ , \tag{29}$$

where we write $\Lambda^g = A$ and $\Lambda^d = B$. Apparently, such r^{-2} scalings of mass densities remain valid for a composite system of two mutually gravitating fluids at large x.

Near the 'origin' ($x \to 0^+$), the asymptotic behaviors of the similarity solutions can be readily obtained from the same basic nonlinear ODEs (13)$-$(16). One possible asymptotic solution is that the core mass accretion rates of gas and dark matter fluids remain constant, while the density profiles and radial infall speeds diverge, that is,

$$\begin{aligned}
\alpha^g &\to \left[\frac{(m_0^g)^2}{2(m_0^g+m_0^d)x^3}\right]^{1/2} , & v^g &\to -\left[\frac{2(m_0^g+m_0^d)}{x}\right]^{1/2} , \\
\alpha^d &\to \left[\frac{(m_0^d)^2}{2(m_0^g+m_0^d)x^3}\right]^{1/2} , & v^D &\to -\left[\frac{2(m_0^g+m_0^d)}{x}\right]^{1/2} , \\
m^g &\to m_0^g \ , & m^d &\to m_0^d \ ,
\end{aligned} \tag{30}$$

where m_0^g and m_0^d determine the central mass accretion rates for the core collapse [see transformations (7)].

By Eq. (20), this solution further implies a diverging total gravitational potential $\phi(x) \propto x^{-1}$ as $x \to 0^+$ (see Lou & Shen 2004).

Another asymptotic solution of interest as $x \to 0$ is that the core mass accretion rates of both gas and dark matter fluids approach very small values as the reduced radial speeds vanish. Both reduced mass densities then approach certain constant values, namely

$$\begin{aligned}
\alpha^g &\to g_0 + [g_0/9 - g_0(g_0+d_0)/6]x^2 \ , & v^g &\to 2x/3 \ , \\
\alpha^d &\to d_0 + [d_0/9 - d_0(g_0+d_0)/6]x^2 \ , & v^D &\to 2x/3 \ , \\
m^g &\to \frac{g_0 x^3}{3} \ , & m^d &\to \frac{\beta^3 d_0 x^3}{3} \ ,
\end{aligned} \tag{31}$$

where g_0 and d_0 are two positive constant parameters (Lou & Shen 2004). The above asymptotic solutions are necessary for constructing global similarity solutions through numerical integrations from either $x \to +\infty$ radially inward or from $x \to 0^+$ radially outward.

By examining the coefficients of the derivatives in the nonlinear ODEs (13)$-$(16), it is clear that $x = 1 + v^g$ is a singular point of the ODEs (13) and (14), while $x = 1/\beta + v^D$ is yet another singular point of the ODEs (15) and (16). It should be noted that the conjugate pairs $x = -1 + v^g$ and $x = -1/\beta + v^D$ are also singular points of the nonlinear ODEs (13)$-$(16) at negative x.

For the similarity flows to pass smoothly (or analytically in mathematical terms) through such singular points, we must impose the conditions that $\alpha^g(x - v^g) + \alpha^d(x - v^D)$ be equal to $2/x$ at $x = 1 + v^g$ and to $2/(x\beta^2)$ at $x = 1/\beta + v^D$. More specifically, we shall refer to these points as 'critical points', namely, $\mathcal{G}$ critical point for the gas fluid and $\mathcal{D}$ critical point for the dark matter 'fluid'. Physically, these singularities are transonic points that separate regions of subsonic and supersonic flows in the similarity expansion profile. In order to construct global similarity solutions that are smooth everywhere, analytical solution behaviors in the vicinity of these transonic points are necessary. If weak discontinuities (Lazarus 1981; Whitworth & Summers 1985) or strong shocks (Courant & Friedrichs 1976; Spitzer 1978; Tsai & Hsu 1995; Shu et al. 2002; Shen & Lou 2004) are allowed across these sonic critical lines, then it would be technically much easier to construct the global similarity solutions. [2] More explicitly, for smooth global similarity solutions across the sonic critical lines, we must impose the following conditions,

$$\alpha^g(x-v^g)+\alpha^d(x-v^D)=2/x \qquad \text{for} \qquad x-v^g=1\ ; \tag{32}$$

$$\alpha^g(x-v^g)+\alpha^d(x-v^D)=2/(x\beta^2) \qquad \text{for} \qquad x-v^D=1/\beta\ . \tag{33}$$

For local solutions satisfying the critical conditions (32) or (33), we obtain the Taylor series expansions by applying the L'Hôpital rule in the neighborhood of $x=x_*$, where $x_*>0$ is the value of x at the critical point (Jordan & Smith 1977; Bender & Orszag 1978; Shu 1977; Whitworth & Summers 1985; Lou & Shen 2004) and analyze their properties in the vicinity of x_*.

4 APPLICATIONS TO CLUSTERS OF GALAXIES

Regarding the applicability of the two-fluid formalism to clusters of galaxies, we shall first use the example of large-scale galactic dynamics as a closely parallel analogy. As a first approximation, one may treat the interstellar gas medium as a fluid and the collection of $\sim 10^{11}-10^{12}$ stars as another 'fluid' on large scales (e.g. Bertin & Lin 1996). While mathematically complex and involved, a much better treatment for the latter would be a distribution function approach (e.g. Lin & Shu 1966; Lin 1987; Binney & Tremaine 1987). Usually for large-scale dynamics and away from singularities, a fluid formalism for the stellar disk would suffice (e.g. Goldreich & Tremaine 1981). The gas and stellar 'fluid' disks are coupled through mutual gravity (e.g. Lou & Shen 2003). In a galactic system, a third important component is the dark matter 'halo' that exerts a gravitational effect on both the gas and star disks. Except for the gravitational effect, the study of the dark matter halo has to rely on assumptions and approximations together with a clever use of observational diagnostics. Extensive numerical simulations have adopted the so-called N-body formulation for the dark matter halo by assigning a mass to each particle, the total particle number N being limited by the computer capability and the state-of-art algorithm. Only in the sense of very large N, might a fluid description for a dark matter halo be justifiable.

Modelling the dynamics of a cluster of galaxies can be based on a similar rationale. For a typical galaxy cluster, the mass in all the stars is much smaller than the mass in the gas which in turn is much smaller than the mass of the dark matter halo. As a first approximation, there is no problem of adopting an isothermal fluid for the hot gas component which is empirically justifiable by the extensive X-ray observations (e.g. Sarazin 1988; Fabian 1994). The use of a fluid approach to the dark matter halo is an approximation (e.g. Peebles 2000; Subramanian 2000) and the 'isothermal' assumption for the 'dark matter fluid' must be tested against observations for the properties of the hot gas component. One major point of this paper is to propose a way of probing the equation of state for the dark matter halo.

With the limitations and qualifications of our model analysis, we attempt to describe several applications pertinent to galaxy clusters and address relevant issues.

[2] Weak discontinuities are really weak shocks (Landau & Lifshitz 1959; Boily & Lynden-Bell 1995).

4.1 Hydrostatic Model for a Cluster of Galaxies

As a first approximation, one often invokes the so-called isothermal $\beta-$model to describe a static cluster of galaxies (e.g., Sarazin 1986, 1988; Fabian 1994). Given our formalism for two 'isothermal fluids' with spherical symmetry in hydrostatic equilibrium, we should require, for consistency,

$$\frac{(a^g)^2}{r^2}\frac{d}{dr}\left(r^2\frac{d\ln\rho^g}{dr}\right)=-4\pi G(\rho^g+\rho^d) \tag{34}$$

and

$$\frac{(a^d)^2}{r^2}\frac{d}{dr}\left(r^2\frac{d\ln\rho^d}{dr}\right)=-4\pi G(\rho^g+\rho^d)\ . \tag{35}$$

With a few simplifying assumptions, usual approach is to estimate the radial distribution of gas density $\rho^g(r)$ and $(a^g)^2$ from diagnostics of X-ray observations of a galaxy cluster. One can then use the static condition (34) to infer the radial distribution of the dark matter fluid mass density $\rho^d(r)$. Given our 'isothermal' assumption for the dark matter 'fluid', we can now use the condition (35) to readily estimate the value of $(a^d)^2$ and then infer the 'velocity dispersion' (assumed to be independent of r) of the dark matter fluid. Or the other way around, we can check whether the empirically inferred $\rho^d(r)$ can be fitted with the special density relation $\rho^d=K(\rho^g)^{\beta^2}$ as implied by the hydrostatic condition under spherical symmetry. From the perspective of utilizing effects of gravitational lensing, it might be eventually possible to infer $M(r)$ and then $\rho^d(r)+\rho^g(r)$ within a cluster of galaxies. This is just an illustrative example for probing the equation of state of dark matter halo on the scale of a galaxy cluster. On the basic assumption that dark matter 'fluids' on the galactic scale, the galaxy cluster scale and the cosmological scale are of the same physical nature, we can develop a systematic procedure to study the properties of dark matter 'fluids'. For example, in the context of galaxy clusters, we may relax the assumption of isothermality and introduce the polytropic or the barotropic approximation instead for the two gravitationally coupled 'fluids'. It is then possible to infer the properties of the dark matter 'fluid' in galaxy clusters or to test various hypotheses on them.

We note in passing that such a hydrostatic equilibrium when perturbed can, in general, support trapped acoustic waves ($p-$modes) and trapped internal gravity waves ($g-$modes) (Pringle 1989; Balbus & Soker 1990). In contrast to solar and stellar oscillations, a hot intracluster gas is optically thin and it is possible to reveal patterns of $g-$modes trapped around the core region. For a composite system of two gravitationally coupled fluid spheres, one would expect two classes of $p-$modes and two classes of $g-$modes (e.g. Lou & Fan 1998; Lou & Shen 2003). In the first class of modes, density perturbations in the gas and dark matter are in phase, while in the second class, they are out of phase (e.g., Lou & Shen 2003; Lou & Zou 2004). For the large-scale X-ray patterns revealed by *CHANDRA* observations in the central Perseus galaxy cluster (Fabian 2004), an acoustic waves interpretation has been proposed. We emphasize here that in addition to possible acoustic waves, internal gravity waves caused by nonspherical perturbations are equally possible. It appears that such large-scale wavy patterns are fairly generic in central regions of galaxy clusters (Fabian 2004, private communications). Another example is the X-ray triple rings in the central regions of the Virgo cluster seemingly associated with the M87 jet (Feng, Zhang, Lou & Li 2004; Forman et al. 2004); these may be caused by nonlinear acoustic waves, internal gravity waves and/or shocks. In 'cooling flow' clusters with unusual X-ray bright cores (e.g., Bertschinger 1989 and references therein), galaxy cluster $p-$mode and $g-$mode oscillations may be induced and sustained by an infalling gas of $\sim 10-15$ percent of mass fraction which is gravitationally coupled with the dark matter fluid such that the entire system oscillates globally. If the dark matter particles are collisionless, then either acoustic waves or internal gravity waves would be damped by the Laudau damping mechanism. With this process ongoing, the gravitational potential energy can then be utilized to generate waves and then compensate radiative losses.

4.2 Magnetohydrostatic Model for a Cluster of Galaxies

It is now known that the hot gas trapped in a galaxy cluster is permeated with magnetic field of strengths greater than $\sim 1\,\mu$G (Clarke et al. 2001). In the central regions of some galaxy clusters, the magnetic field has been estimated to be as high as $30 \sim 40\,\mu$G (Fabian 1994; Carilli & Taylor 2002; Hu & Lou 2004a). Here we qualitatively describe a magnetohydrostatic model for a cluster of galaxies to illustrate the essential points. Let us presume that a 'ball' of random magnetic fields is trapped in a galaxy cluster and provides a magnetic pressure. This random magnetic field directly interacts with the ionized hot gas but not the dark matter fluid. For magnetostatic equilibrium in a composite system, the condition (35) remains valid, while the condition (34) should be modified to

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho^g}\frac{dP_M}{dr}\right)+\frac{(a^g)^2}{r^2}\frac{d}{dr}\left(r^2\frac{d\ln\rho^g}{dr}\right)=-4\pi G(\rho^g+\rho^d)\ , \tag{36}$$

where $P_M(r)\equiv\langle B^2\rangle/(8\pi)$ stands for an isotropic magnetic pressure associated with the 'ball' of random magnetic fields. We now examine the equations (35) and (36) together. With $\rho^g(r)$ inferred empirically from X-ray observations of a galaxy cluster, we may solve the equation (35) for $\rho^d(r)$ with appropriate boundary conditions, treating $(a^d)^2$ as a parameter. With the derived solution of $\rho^d(r)$ and observationally estimated $(a^g)^2$ and $\rho^g(r)$ in Eq. (36), we can then infer information on the magnetic pressure $P_M(r)$. In this procedure, the unknown parameter $(a^d)^2$ may be estimated by assuming that the dark matter fluid has been virialized completely in the total gravitational potential well.

As global oscillations of a galaxy cluster, acoustic waves should now be replaced by magnetosonic waves while internal gravity waves, by internal magneto-gravity waves (Lou 1996). Moreover, the presence of cluster magnetic field should lead to anisotropic electron velocity distributions parallel and perpendicular to the local magnetic field. By assuming a bi-Maxwellian relativistic distribution for hot electrons, we were able to compute the magnetic Sunyaev-Zel'dovich effect in the galaxy cluster Abell 2163 (Hu & Lou 2004a) and inferred a core magnetic field strength of $\sim 36\,\mu$G. We have also applied the same analysis to the Coma galaxy cluster and estimated a core magnetic field strength of $\lesssim 10\,\mu$G, consistent with other magnetic field observations (Hu & Lou 2004b).

4.3 Galaxy Cluster Winds or Outflows with or without Shocks

During a certain evolutionary phase of a galaxy cluster, the system may approach a global self-similar behavior. Based on the similarity solution (28) for large x, it is possible for a composite fluid system to support outflows or 'galaxy cluster winds' with both density profiles $\propto r^{-2}$. This could happen in several ways. First, both the gas and dark matter may flow outward without crossing the sonic critical points. Secondly, the dark matter flows inward while the gas flows outward. Thirdly, asymptotic outflows with shocks across the sonic critical lines (Shen & Lou 2004). Observationally, such a spherical shock in the hot gas would appear as an X-ray bright ring or shell. According to the property of our self-similar solution, such a ring should travel outward with a constant speed. It should be noted that such outflows and central infalls or collapses are related to each other.

4.4 Classification of Galaxy Clusters

In terms of X-ray observations, there are broadly two classes of galaxy clusters, in one, the central or core X-ray brightness is relatively smooth and normal, in the other, it is unusually luminous or spiky (frequently referred to as 'cooling flow' clusters). The key physical question is to find a dynamical basis for such a bifurcation of qualitatively different galaxy clusters. For galaxy clusters (e.g., Sarazin 1988; Fabian 1994), if their evolutionary histories involve a self-similar phase of central collapse (e.g. Gunn & Gott 1972; Fillmore & Goldreich 1984; Bertschinger 1985; Navarro et al. 1997), then isothermal similarity solutions (30) and (31) suggest two possible classes of galaxy clusters that emit X-rays through hot gases virialized in gravitational potential wells, namely, those with steep gravitational potential wells and hence extremely high X-ray core luminosities, and those with relatively smooth and shallow gravitational potential wells and hence normal X-ray core luminosities. While we do not yet know from the theoretical point of view what initial or boundary conditions that would lead to the appearance of two different similarity evolutionary phases (30) and (31) for small x, it is indeed very tempting to propose, based on the properties of these two possible similarity solutions (30) and (31) for $x \to 0^+$, that they correspond to the two broad classes of galaxy clusters in X-ray brightness observations. In particular, as solutions (30) are characterized by diverging gravitational potentials, they describe galaxy clusters with unusually luminous central or core X-ray brightness. The main reason for this identification is that by virialization, a deeper gravitational potential well would give rise to a higher gas temperature and density around the central singularity and hence a stronger core X-ray luminosity. It should be noted that according to the similarity solutions (30), the central infall speed scales as $r^{-1/2}$ and the core density scales as $r^{-3/2}$; this core density scaling is steeper than the r^{-1} density scaling found in numerical simulations (e.g., Navaro, Frenk & White 1995, 1996, 1997). In comparison, the solutions (31) are characterized by finite and smooth gravitational potentials and they describe galaxy clusters with normal central or core X-ray brightness. The solutions in (31) can be matched with self-similar oscillations, shocks across sonic critical lines, and asymptotic inflows or outflows (Lou & Shen 2004; Shen & Lou 2004).

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