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Wavelet Analysis of the Schwabe Cycle Properties in Solar Activity *

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Abstract Properties of the Schwabe cycles in solar activity are investigated by using wavelet transform. We study the main range of the Schwabe cycles of the solar activity recorded by relative sunspot numbers, and find that the main range of the Schwabe cycles is the periodic span from 8-year to 14-year. We make the comparison of 11-year's phase between relative sunspot numbers and sunspot group numbers. The results show that there is some difference between two phases for the interval from 1710 to 1810, while the two phases are almost the same for the interval from 1810 to 1990.

Key words: Sun: sunspot — Sun: activity — method: data analysis

1 INTRODUCTION

Many papers have been devoted to the periodical analysis of the solar activity. The data mostly used are the relative sunspot numbers and the sunspot group numbers. The Schwabe cycles of solar activity have been paid great attention. Friis-Christensen & Lassen (1991) studied the relationship between the solar activity and the climate on the Earth. Ochadilick et al. (1993) first used the wavelet transform to obtain the Schwabe cycles' ridge line of solar activity recorded by relative sunspot numbers. Frick et al. (1997) acquired the sunspot group number's ridge line by using the same method. Fliggle et al. (1999) showed ridge lines of several parameters describing the solar activity. Polygiannakis et al. (2003) also presented the ridge line of the Schwabe cycle of solar activity recorded by sunspot index by using the technique of instantly maximal wavelet skeleton spectrum. Feng et al. (1998) studied the length of a solar cycle by detecting the singular points in the successive solar cycles. From solar cycle 10 to solar cycle 21, the solar cycle length obtained by Mursula (1998) ranges from longer than 9-year to longer than 12-year but shorter than 13-year. Han (2002a) also made the wavelet transform of the monthly mean relative sunspot numbers. Le & Wang (2003) analyzed the amplitude of 11-year period for solar activity evolution with time and compared it with that of the 101-year. Every given period signal has its amplitude at different moment. The amplitude

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evolution with time for any given period can be obtained by using wavelet transform. The square of the amplitude of a signal with a given period determines the signal's power evolution with time. In this paper, regardless of the solar cycle length, we study the main range of the Schwabe cycles of the relative sunspot numbers by calculating the wavelet power over a periodic band. In addition, as we know, the Schwabe cycles for the relative sunspot numbers and the sunspot group numbers are the periods describing the solar activity. Is there any difference between them? We will compare the well known 11-year periodicity's phase of the relative sunspot numbers with that of the sunspot group numbers to check whether the two parameters are equally effective in describing the solar activity.

2 DATA ANALYSIS

The continuous wavelet transform of a discrete sequence x_n is defined as the convolution of the x_n with wavelet function, i.e.,

$$W_n = \sum_{n'=0}^{N-1} x_{n'} \psi^* \left[\frac{(n'-n)\delta t}{s} \right], \tag{1}$$

where s is the scale size, δt is the time interval, and the superscript $*$ indicates the complex conjugate. The wavelet selected in this paper is the Morlet wavelet defined as the production of complex exponential wave and Gaussian envelope, i.e.,

$$\psi_0(t) = \pi^{-1/4} e^{i\omega_0 t} e^{-t^2/2}, \tag{2}$$

where ω_0 is the wave number taken to be 6 (Torrence & Campo 1998).

Yearly mean relative sunspot numbers from 1700 to 2003 and yearly mean sunspot group numbers used in this paper are shown in Fig. 1a and Fig. 1b.

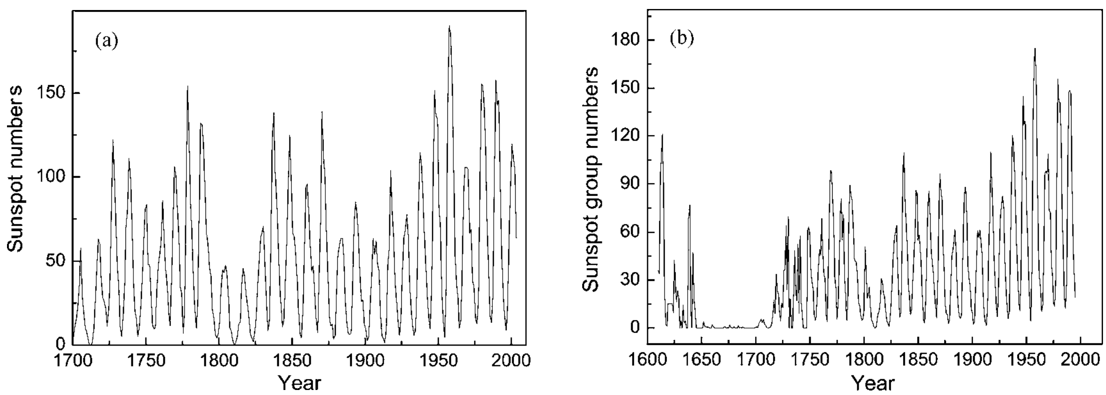

Fig. 1 (a) Relative sunspot numbers from 1700 to 2003 and (b) Sunspot group numbers from 1700 to 1994.

Because a vertical line through a wavelet figure like the fig. 2 in Le & Wang (2003) gives the local spectrum, so the wavelet spectrum at some moment over a certain interval (here we call it scaled wavelet power: SWP) is

$$\mathrm{SWP} = \sum_{n=n_1}^{n=n_2} |W_n(s)|^2, \tag{3}$$

where the index n is arbitrarily assigned from n_1 to n_2. By repeating Eq. (3) at each time step, we can create a scaled wavelet power from n_1 to n_2.

The conventional solar cycle length is defined as the time span between two successive sunspot minima. Mursula (1998) presented more precise solar cycle length by calculating the difference between the median activity times of two successive sunspot cycles. We know that the relative sunspot numbers for one by one solar cycle constitute the relative sunspot number series. Here, we concentrate on the main range of the Schwabe cycles of the solar activity (SCSA) of the relative sunspot numbers by calculating the scaled wavelet power over a periodic band. The main range of SCSA is determined by the difference between the lower value and higher value of the main range. First, we determine the lower value of the main range. We assume some lower values for the main range, while the higher value is set to be 12 (can also be 13 or other higher value). By comparing the values of the SWP for different periodic interval, we can determine the lower value of the main range. Several SWP values for different periodic interval are shown in Fig. 2, from which we can see that the difference between SWP for the range from 8 to 12 and SWP for the range from 9 to 12 is obvious, while the difference between SWP for the range from 8 to 12 and SWP for the range from 7 to 12 is very small. So we can define that the lower value for the main range of the Schwabe cycle of Solar activity is 8. We can also define the higher value of the SCSA by using the same method. Several SWP values for different periodic interval to determine the high value of the main range of the SCSA are shown in Fig. 3, from which one can find that the higher value of the main range of the SCSA is 14 because the difference between the SWP for the range from 8-years to 14-years and the SWP for the range from 8-years to 15-years is very small. Thus we can conclude that the main range of SCSA is the periodic span from 8-years to 14-years.

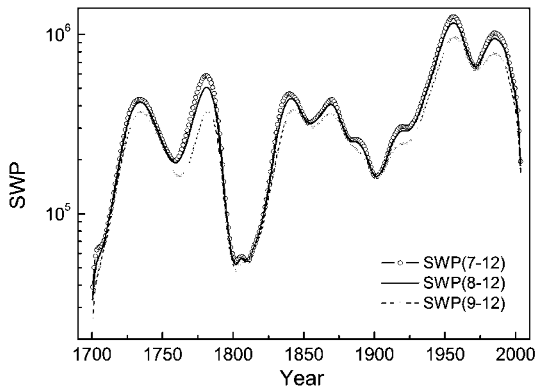

Fig. 2 SWP for different periodic interval to determine the low value of the main range of the SCSA.

Now we check the phase of 11-years of the relative sunspot numbers and sunspot group numbers and make a comparison between them. We have known a signal with a definite frequency will have the form,

$$f_\omega(t) = A(t)e^{i(\omega t+\varphi(t))}, \tag{4}$$

where $A(t)$ is the amplitude of the signal, $\omega = 2\pi f$, f is the frequency of the signal, $\omega t + \varphi(t)$ is the phase of the signal.

If

$$\omega t_1 + \varphi(t_1) = \omega t_2 + \varphi(t_2) + 2k\pi, k \in \text{int} \tag{5}$$

then we have

$$e^{i(\omega t_1 + \varphi(t_1))} = e^{i(\omega t_2 + \varphi(t_2))}. \tag{6}$$

So we only need to consider the part lower than 2π in $\omega t + \varphi(t)$. Figure 4 presents the phase evolution with time for the signal with period 11-years of the relative sunspot number and sunspot group number.

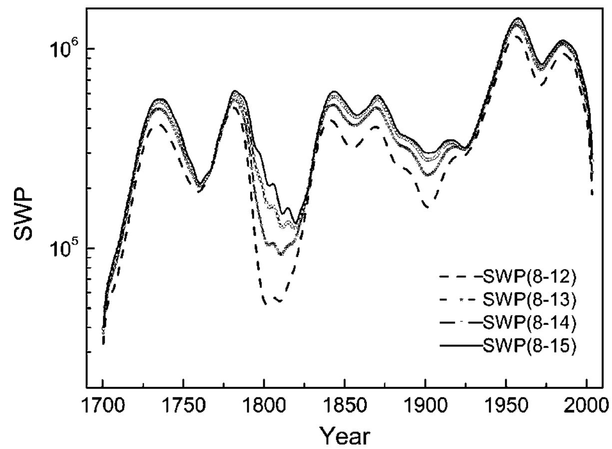

Fig. 3 SWP for different periodic interval to determine the higher value of the main range of the SCSA.

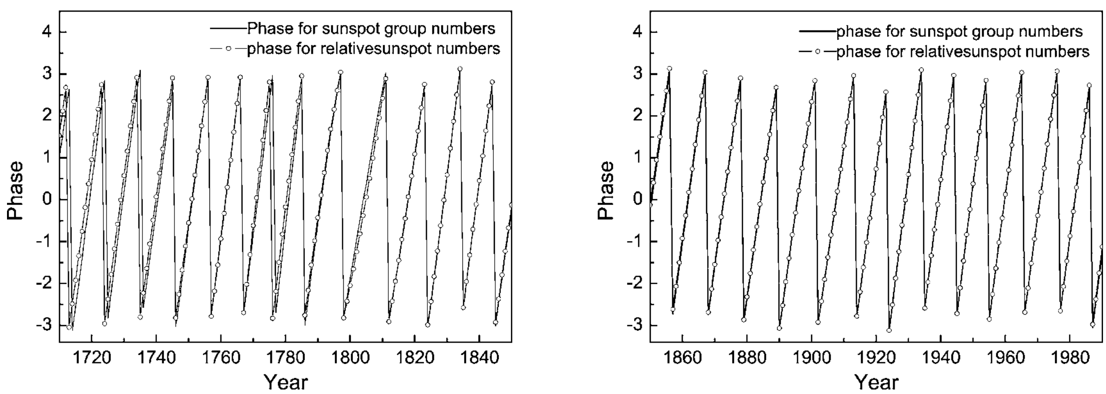

Fig. 4 Time evolution of the phase with the period of 11-years for the relative sunspot numbers and for the sunspot group numbers.

From Fig. 4 we can see that a difference does exist between the 11-year's phases for the relative sunspot numbers and the sunspot group numbers during the interval from 1710 to 1810, while the two phases are almost the same during the interval from 1810 to 1990. Also we can see that the phase evolution with time is not linear. Because ω can be a definite value for any given period, if the primary phase $\varphi(t)$ is a constant, then $\omega t + \varphi(t)$ must be a linear expression and the curve depicted by $\omega t + \varphi(t)$ must be constituted by some repetitive straight lines. From Fig. 4 we can deduce that the primary phase $\varphi(t)$ is not a constant but it varies with time.

3 CONCLUSIONS AND DISCUSSION

Properties of the Schwabe cycles of the solar activity are investigated by using wavelet transform. We find that the main range of the Schwabe cycles of solar activity recorded by relative sunspot numbers is the periodic span from 8-years to 14-years, and that there is some difference between the 11-year's phases for the relative sunspot numbers and the sunspot group numbers during the interval from 1710 to 1810. However, the 11-year's phases for the two sequences are almost the same during the interval from 1810 to 1990.

The fact that the difference of the 11-year phases of the two sequences occurs in the time earlier than 1810 and after then the two phases are almost the same may reveal two things: one is that the difference during the time earlier than 1810 may be due to the poor precision of the observational instrument; the other is that the paces of the 11-year phase for the two sequences are almost the same after 1810, which means that the two parameters are equally effective in describing the solar activity. The amplitude evolution with time given by Le & Wang (2003) for 11-year period shows that the 11-year amplitude varies with time. In this paper, we have clarified that the primary phase, $\varphi(t)$, also varies with time. These two phenomena indicate that the solar activity is very complicated.

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