

## The $Q$ Values of the Galilean Satellites and their Tidal Contributions to the Deceleration of Jupiter's Rotation \*

Hong Zhang and Cheng-Zhi Zhang

Department of Astronomy, Nanjing University, Nanjing 210093; zhangh@nju.edu.cn

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**Abstract** The relationship between the  $k_2/Q$  of the Galilean satellites and the  $k_{2J}/Q_J$  of Jupiter is derived from energy and momentum considerations. Calculations suggest that the Galilean satellites can be divided into two classes according to their  $Q$  values: Io and Ganymede have values between 10 and 50, while Europa and Callisto have values ranging from 200 to 700. The tidal contributions of the Galilean satellites to Jupiter's rotation are estimated. The main deceleration of Jupiter, which is about 99.04% of the total, comes from Io.

**Key words:** astrometry — celestial mechanics — planet and satellites: individual: (Io, Europa, Ganymede, Callisto)

### 1 INTRODUCTION

Departure of a tidally distorted body from perfect elasticity or fluidity can be neatly summarized in terms of the tidal dissipation function,  $Q^{-1}$ , defined by

$$\frac{1}{Q} = \frac{1}{2\pi E_0} \oint \left( -\frac{dE}{dt} \right). \quad (1)$$

Here  $E_0$  is the maximum energy stored in the total distortion, and the integral over  $-dE/dt$ , the rate of dissipation, represents the energy dissipated during one complete cycle. The dimensionless parameter  $Q$  is simply related to the phase lag  $\epsilon$ . The relation of  $Q$  to this phase lag  $\epsilon$  is (MacDonald 1964),

$$\frac{1}{Q} = \tan 2\epsilon, \quad (2)$$

or, since  $Q$  is generally large,  $Q^{-1} \approx 2\epsilon$ . The rates of change of the orbital elements are determined by the phase lag in the elastic component of the tidal bulge (MacDonald 1964).

Goldreich & Soter (1966) summarized the relevant information about  $Q$  in the solar system and suggested that the lower bound for Jupiter's  $Q_J$  is  $(1 \sim 2) \times 10^5$ . Yoder (1979) discussed in more detail how the orbital resonance locks among the three inner Galilean satellites are maintained and described the effects of the dissipative tides in both Jupiter and Io on their

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establishment and evolution. Arguments were given which limited  $Q_J$  to the range,  $2 \times 10^5 < Q_J \ll 2 \times 10^6$ . Based on calculations for a tidal origin of the Laplace resonance, the observations of the thermal output of Io (Maston et al. 1980; Sinton 1981) and the secular change in the mean motion of Io (Lieske 1987), a possible range of  $Q_J$  ( $4 \times 10^4 < Q_J < 5 \times 10^5$ ) was given (cf. Greenberg 1981, 1987; Malhotra 1991). Ioannou & Lindzen (1993) formulated the theory of excitation of tidal oscillations in a fluid planetary body and verified that  $Q_J$  is of the order of  $10^5$ . On the other hand, direct determinations of  $Q_J$  based on estimates of turbulent viscosity within Jupiter yielded much larger values of  $Q_J$ —the most extreme being  $Q_J \approx 10^{13}$  (Goldreich & Nicholson 1977). However, this figure implies that tidal interactions between Jupiter and its satellites have played a negligible role in the evolution of the satellites' orbits. Moreover, such a value of  $Q_J$  means there would be insufficient torque from Jupiter to assemble the resonances and to maintain the current hypothesized equilibrium (Peale 1999). Based on these considerations, the bound  $2 \times 10^5 < Q_J < 5 \times 10^5$  is preferred.

It is well known that the tidal energy dissipation rate is related to the tidal dissipation factor for the Galilean satellites (Peale & Cassen 1978). Now, experimental values of  $Q$  ranging between 40 and several thousand have been determined for various rocks and metals, but we do not know the outer composition and the state of other planets and satellites (Goldreich & Soter 1966).

Now, many previous studies on the tidal evolution of the Galilean satellites depended on some conjectured values of the tidal lags or  $Q$  (e.g. Ross & Schubert 1987; Malhotra 1991; Showman & Malhotra 1997; Showman et al. 1997). As a result the time scales for these processes remained uncertain. In this paper, the relationship between  $k_2/Q$  of the Galilean satellites and  $k_{2J}/Q_J$  of Jupiter will be derived from energy and momentum considerations. Using the range of  $Q_J$  values of Jupiter and the possible ranges of  $Q$  values of the Galilean satellites, the tidal contributions by the Galilean satellites to the deceleration of Jupiter's rotation are estimated.

Some parameters of the Galilean satellites are listed in Table 1 (Yoder 1995; Seidelmann et al. 2002), (where  $M$  is the mass,  $R$  the mean radius,  $a$  the orbital semi-major axis,  $e$  the orbital eccentricity,  $\omega$  the angular velocity of rotation ( $\omega$  is equal to the mean orbital motion  $n$  because of the synchronous rotation)).

**Table 1** Parameters of the Galilean Satellites

Parameters	Io	Europa	Ganymede	Callisto
$M$ ( $10^{22}$ kg)	8.9319	4.7910	14.817	10.762
$R$ (km)	1821.3	1560.7	2634.1	2408.4
$a$ ( $10^5$ km)	4.216	6.709	10.70	18.83
$e$	0.0041	0.0101	0.0006	0.00736
$\omega$ ( $10^{-5}$ rad s $^{-1}$ )	4.1106	2.0479	1.0164	0.43575

Note: The mass of Jupiter is  $M_J = 1.89861 \times 10^{27}$  kg; the gravitational constant  $G$  is  $6.67259 \times 10^{-11}$  m $^3$  kg $^{-1}$  s $^{-2}$

## 2 $Q$ VALUES OF THE GALILEAN SATELLITES

Jupiter has many natural satellites, of which the four largest, Io, Europa, Ganymede and Callisto, were discovered by Galileo in January 1610 and named Galilean satellites. The orbits of the Galilean satellites are nearly in Jupiter's equatorial plane ( $i \approx 0$ ), are nearly circular ( $e \approx 0$ ) and are synchronous (orbital periods equal to the rotational periods).

Jupiter and its Galilean satellites form a complex system. The most striking characteristic of the Galilean satellites is the set of orbital resonances where the orbital mean motions satisfy the relations (cf. Peale 1999)

$$\begin{aligned} n_I - 3n_E + 2n_G &= 0, \\ n_I - 2n_E + \dot{\tilde{\omega}}_I &= 0, \\ n_I - 2n_E + \dot{\tilde{\omega}}_E &= 0, \\ n_E - 2n_G + \dot{\tilde{\omega}}_E &= 0, \end{aligned} \tag{3}$$

which lead to the following constraints on the longitudes:

$$\begin{aligned} \lambda_I - 3\lambda_E + 2\lambda_G &= 180^\circ, \\ \lambda_I - 2\lambda_E + \tilde{\omega}_I &= 0^\circ, \\ \lambda_I - 2\lambda_E + \tilde{\omega}_E &= 180^\circ, \\ \lambda_E - 2\lambda_G + \tilde{\omega}_E &= 0^\circ. \end{aligned} \tag{4}$$

The subscripts I, E, G refer to Io, Europa and Ganymede, respectively, and  $\tilde{\omega}_i$  are the longitudes of periaipse with the dot indicating the time differentiation. The Laplace relation refers to the first equations of Eqs. (3) and (4). The possible origins of the Laplace relation have been discussed by many authors (Yoder 1979; Peale et al. 1979; Yoder & Peale 1981; Malhotra 1991; Showman & Malhotra 1997; Showman et al. 1997; Peale & Lee 2002). In these work, the *Q* values of the Galilean satellites must be postulated. To qualitatively estimate the *Q* values, we concentrate on the momentum and energy of the system. In such a complex system, it is still not known how the decrease of the rotational angular momentum of Jupiter is allotted to the four moons. In addition, the tidal energy dissipation rate was derived just for a system consisting of two bodies (cf. Peale & Cassen 1978). In order to simplify the problem, an isolated system formed by Jupiter and one of the Galilean satellites will be considered (cf. Peale et al. 1979; Burša 1991), and the satellite is treated as orbiting in Jupiter's equatorial plane ( $i = 0$ ).

The momentum of the isolated system is considered first. If the rotational angular momentum of the satellite is ignored, the total momentum *L* of the system would include the rotational angular momentum of Jupiter and the orbital momentum  $L_{JS}$  of the system in the barycentric coordinate system. The total momentum can be written as

$$L = L_{JS} + C_J\omega_J. \tag{5}$$

Here  $C_J$  is the largest principal moment of inertia of Jupiter,  $\omega_J$  the rotational angular velocity of Jupiter and  $L_{JS} = \frac{MM_J}{M+M_J}\sqrt{G(M+M_J)}a(1-e^2) \approx \frac{MM_J}{M+M_J}na^2$ . If all external influences on the planet-satellite system are ignored, then the total angular momentum remains a constant:

$$\frac{dL_{JS}}{dt} = -C_J \frac{d\omega_J}{dt}. \tag{6}$$

Using Kepler's third law, we have

$$\frac{dL_{JS}}{dt} = -\frac{1}{3} \frac{MM_J}{M+M_J} a^2 \frac{dn}{dt}. \tag{7}$$

Substituting Eq. (7) into Eq. (6), the variation in *n* can be derived:

$$\frac{dn}{dt} = \frac{3(M+M_J)}{MM_J} \frac{1}{a^2} C_J \frac{d\omega_J}{dt}. \tag{8}$$

The tidal torque raised on Jupiter by the satellite can be expressed by (Jeffreys 1962)

$$\Gamma = C_J \frac{d\omega_J}{dt} = -\frac{3}{2} \frac{GM^2}{a} \left(\frac{R_J}{a}\right)^5 \frac{k_{2J}}{Q_J}, \quad (9)$$

where  $R_J = 69911$  km (Seidelmann et al. 2002) is the mean radius of Jupiter and  $k_{2J} = 0.474$  the second Love number of Jupiter (Yoder 1995). So the following equation can be obtained:

$$\frac{dn}{dt} = -\frac{9}{2} \frac{M}{M_J} n^2 \left(\frac{R_J}{a}\right)^5 \frac{k_{2J}}{Q_J}. \quad (10)$$

On the other hand, the total mechanical energy of the system decreases through tidal dissipation. The mechanical energy of the system in the barycentric coordinate system is

$$E = -\frac{GMM_J}{2a}. \quad (11)$$

Since the rotational energy of a synchronously rotating satellite cannot be diminished, the energy dissipated by the nonzero eccentricity must come from the orbit, which would lead to an increase in the orbital motion (cf. Peale et al. 1979)

$$\frac{dn}{dt} = -\frac{3}{nMa^2} \frac{dE_{\text{tide}}}{dt}. \quad (12)$$

Moreover, the total tidal dissipation in a synchronously rotating homogenous satellite in an eccentric orbit is given by Peale & Cassen (1978)

$$\frac{dE_{\text{tide}}}{dt} = \frac{21}{2} \frac{k_2}{Q} \left(\frac{R}{a}\right)^5 \frac{GM_J^2}{a} ne^2. \quad (13)$$

We obtain

$$\frac{dn}{dt} = -\frac{63}{2} \frac{GM_J^2}{M} \frac{e^2}{a^3} \left(\frac{R}{a}\right)^5 \frac{k_2}{Q}. \quad (14)$$

If the rates given by Eqs. (10) and (14) exactly cancel, then the relationship between  $k_2/Q$  of the satellite and  $k_{2J}/Q_J$  of Jupiter can be deduced to be

$$\frac{k_2}{Q} = \frac{1}{7e^2} \left(\frac{M}{M_J}\right)^2 \left(\frac{R_J}{R}\right)^5 \frac{k_{2J}}{Q_J}. \quad (15)$$

The possible ranges of  $k_2/Q$  of the Galilean satellites can be estimated and are listed in Table 2. For comparison, some values of  $k_2/Q$  of the Galilean satellites adopted by other authors are also included in Table 2.

For a homogeneous and incompressible satellite, the second Love number can be estimated from the Kelvin equation (Munk & MacDonald 1960):

$$k_2 = \frac{3}{2} \cdot \frac{1}{1 + \frac{19\bar{\mu}}{2\bar{\rho}gR}}, \quad (16)$$

where  $\bar{\mu}$  is the mean coefficient of rigidity of the satellite,  $g$  the surface gravity and  $\bar{\rho}$  the mean density of the material in the tidal bulge (cf. Showman et al. 1997). Based on the internal structure models, the bulk moduli  $K(s)$  of the Galilean satellites, as functions of  $s$  ( $s = r/R$ ),

can be calculated (cf. Zhang 2003). Then the distributions of the coefficients of rigidity,  $\mu(s)$ , can be assessed by the following equation (Bullen 1975):

$$\mu(s) = \frac{3}{2}K(s) \left( \frac{1 - 2\sigma}{1 + \sigma} \right). \tag{17}$$

Here  $\sigma$  is the Poisson's coefficient ( $\sigma = 0.44$  for the core and  $\sigma = 0.40$  for the mantle). So the mean coefficients of rigidity of the Galilean satellites can be derived from the following equation:

$$\bar{\mu} = 3 \int_0^1 \mu(s)s^2 ds. \tag{18}$$

Hence the possible ranges of  $Q$  values of the Galilean satellites can be estimated. These are displayed in Table 3, which also includes the  $\bar{\mu}$  and  $k_2$  values.

**Table 2** The Range of  $k_2/Q$  of the Galilean Satellites

Satellite	Estimated value	Reference 1	Reference 2
Io	$(1.57 \sim 3.92) \times 10^{-3}$	$10^{-4}$	$4 \times 10^{-4} \sim 1.27 \times 10^{-3}$
Europa	$(1.59 \sim 3.97) \times 10^{-4}$	$10^{-3} \sim 10^{-2}$	$4.1 \times 10^{-3}$
Ganymede	$(0.51 \times 1.28) \times 10^{-2}$	$10^{-3} \sim 10^{-1}$	0.127
Callisto	$(1.74 \sim 4.35) \times 10^{-4}$		

Note: The value under Reference 1 is that adopted by Malhotra (1991); the value under Reference 2, that adopted by Showman & Malhotra (1997).

**Table 3**  $Q$  Values of the Galilean Satellites

	Io	Europa	Ganymede	Callisto
$\bar{\mu}$ (kbar)	300	205	110	210
$k_2$	0.067	0.084	0.147	0.119
$Q$	17 ~ 43	212 ~ 528	11 ~ 29	274 ~ 684

Goldreich & Soter (1966) reviewed the secular tidal changes in the solar system and found  $Q$  values in the range from 10 to 500 for the satellites of major planets. Our calculations suggest that the  $Q$  values of the Galilean satellites can be divided into two classes. The first class, containing values from 10 up to 50, includes Io and Ganymede; the second class, containing values larger than 200, consists of Europa and Callisto.

Our results depend on the  $Q_J$  value of Jupiter and the second Love numbers  $k_2$  of the Galilean satellites. However, the elastic characters of the Galilean satellites are still uncertain. Here, the chosen values of the second Love numbers, which suggest that the Galilean satellites behave mainly as rigid bodies, are derived from the internal structure models of the Galilean satellites. Calculating  $k_2$  from either the  $J_2$  or  $C_{22}$  observations, Anderson et al. (1996, 1998, 2001a, 2001b) proposed the larger values of the second Love numbers ( $k_2 > 1$ ) for the Galilean satellites, which indicate that the Galilean satellites mainly behave as elastic bodies. What really reflects the elastic character of the Galilean satellites awaits future research.

In many cases (e.g. Ross & Schubert 1987; Malhotra 1991; Showman & Malhotra 1997; Showman et al. 1997), different  $Q$  values of the Galilean satellites were used. Ross & Schubert (1987) made use of  $Q$  value less than 100 to explain the existence of the subsurface ocean of

Europa. It is obvious that larger  $Q$  values of Europa will lead to less tidal dissipation and will make it difficult to explain the subsurface ocean. Some authors (Malhotra 1991; Showman & Malhotra 1997; Showman et al. 1997) adopted the selected  $Q$  values to numerically simulate the orbital evolution of the Galilean satellites (see Table 2). However, the  $Q$  values provided in this paper are chosen, then different orbital evolution of the Galilean satellites may result. These points should be considered further in the future.

### 3 TIDAL CONTRIBUTION OF THE GALILEAN SATELLITES TO JUPITER'S ROTATION

Because of tidal friction, Jupiter's rotation must decelerate. The Galilean satellites, orbiting synchronously in the equatorial plane of Jupiter, are mainly responsible for this phenomenon, if exists in practice.

#### 3.1 The Secular Tidal Variations in $a$ and $n$ of the Galilean Satellites

Tidal variations in the orbital elements can be derived directly from the Lagrangian planetary equations, to which tidal force function should be inserted as the perturbing function (Kaula 1964). In particular, the variations of  $a$  and  $n$  are as follows:

$$\frac{da}{dt} = 3na \frac{k_{2J}}{Q_J} \frac{M}{M_J} \left( \frac{R_J}{a} \right)^5, \quad (19)$$

$$\frac{dn}{dt} = -\frac{9}{2} n^2 \frac{k_{2J}}{Q_J} \frac{M}{M_J} \left( \frac{R_J}{a} \right)^5. \quad (20)$$

The current values of the variations in the semi-major axes and mean motions due to tidal friction can be estimated; they are listed in Table 4, which includes also the corresponding values for the Moon (Yoder 1995).

**Table 4** Secular Tidal Variations of  $a$  and  $n$

Satellite	$da/dt$ (cm cy <sup>-1</sup> )	$dn/dt$ (arcsec cy <sup>-2</sup> )
Io	96 ~ 242	-92 ~ -230
Europa	4 ~ 10	-1.2 ~ -3.0
Ganymede	0.96 ~ 2.4	-0.089 ~ -0.22
Callisto	0.031 ~ 0.078	-0.0007 ~ -0.0018
Moon	384	-26.0

Our results are one order of magnitude larger than those provided by Burša (1991) because the  $Q_J$  value adopted by Burša (1991) is  $1.9 \times 10^6$ . Our calculations also suggest that the variation of  $n$  of the Moon is smaller than that of Io and much larger than those of the other three Galilean satellites.

#### 3.2 Tidal Deceleration in Jupiter's Rotation

For a rough estimate, we keep only the dominating terms and calculate the contributions by the individual satellites to the tidal variations in Jupiter's angular velocity of rotation according to the following equation derived from Kepler's equation and Eq. (8):

$$\frac{d\omega_J}{dt} = -\frac{1}{2C_J} \frac{MM_J}{M + M_J} \sqrt{\frac{G(M + M_J)}{a}} \frac{da}{dt}. \quad (21)$$

Here, the principal moment of inertia of Jupiter is  $C_J = 2.5 \times 10^{42} \text{kg m}^2$ .

To estimate the total deceleration in Jupiter's rotation, we should know the time interval  $\Delta T$  during which the tidal friction mechanism can be supposed to operate; here we put  $\Delta T = 4.5 \times 10^9 \text{ yr}$ . After integrating Eq. (19), the total change of the semi-major axis  $\Delta a = a(t = 0) - a$  over  $\Delta T$  can be figured out, where  $a$  is the current orbital semi-major axis. The orbital semi-major axis  $4.5 \times 10^9$  years ago is

$$a(t = 0) = \left\{ a^{\frac{13}{2}} - \frac{39}{2} \sqrt{G(M + M_J)} \frac{M}{M_J} R_J^5 \frac{k_{2J}}{Q_J} \Delta T \right\}^{\frac{2}{13}}. \quad (22)$$

Then the deceleration in Jupiter's rotation due to the individual satellite can be derived from Eq. (21):

$$\delta\omega_J = -\frac{1}{C_J} \frac{MM_J}{M + M_J} \sqrt{G(M + M_J)} (\sqrt{a} - \sqrt{a(t = 0)}). \quad (23)$$

Here  $\delta\omega_J = \omega_J - \omega_J(t = 0)$ ,  $\omega_J = 1.758533 \times 10^{-4} \text{ rad s}^{-1}$  ( $= 9^{\text{h}}55^{\text{m}}29.7^{\text{s}}$ ) is the angular velocity of rotation of Jupiter at present and  $\omega_J(t = 0)$  is its angular velocity of rotation  $4.5 \times 10^9$  years ago. For  $Q_J = 3.5 \times 10^5$ , the estimated numerical values are listed in Table 5.

**Table 5** Decelerations in Jupiter's Rotation due to the Individual Galilean Satellites

Satellite	$a(t = 0)$ ( $10^6 \text{m}$ )	$\Delta a$ ( $10^4 \text{m}$ )	$\delta\omega_J$ ( $\text{rad s}^{-1}$ )
Io	257.51	-16409.3	$-5.7 \times 10^{-8}$
Europa	668.28	-262.1	$-3.5 \times 10^{-10}$
Ganymede	1069.38	-61.6	$-2.0 \times 10^{-10}$
Callisto	1882.98	-2.0	$-3.5 \times 10^{-12}$

The total deceleration in Jupiter's rotation is

$$\Delta\omega_J = \sum \delta\omega_J = -5.75535 \times 10^{-8} \text{ rad s}^{-1}, \quad (24)$$

its absolute value is larger than that estimated by Burša (1991). It means that  $4.5 \times 10^9$  years ago the angular velocity of rotation of Jupiter is  $\omega_J(t = 0) = 1.759109 \times 10^{-4} \text{ rad s}^{-1}$ . The main contribution to the deceleration comes from Io, which accounts for about 99.04% of the total deceleration.

#### 4 CONCLUDING REMARKS

- 1) The  $Q$  values of the Galilean satellites can be divided into two classes. One, containing values from 10 to 50, includes Io and Ganymede. The other, containing values larger than 200, consists of Europa and Callisto.
- 2) The second Love numbers of the Galilean satellites, which are derived from the internal structure models, are 0.067 for Io, 0.084 for Europa, 0.147 for Ganymede and 0.119 for Callisto. These values suggest that the Galilean satellites behave mainly as rigid bodies.
- 3) The tidal variation of  $n$  of Io is at least about two orders of magnitude higher than those of the other three Galilean satellites. Incidentally, the variation of  $n$  of the Moon is less than that of Io but much larger than those of Europa, Ganymede and Callisto.

- 4) If  $Q_J = 3.5 \times 10^5$ , the total deceleration in Jupiter's rotation is  $-5.75535 \times 10^{-8} \text{ rad s}^{-1}$ . This means that  $4.5 \times 10^9$  years ago the period of rotation of Jupiter is about  $9^{\text{h}}55^{\text{m}}18.0^{\text{s}}$ .
- 5) The main contribution to the deceleration of Jupiter's rotation comes from Io, which is about 99.04% of the total deceleration.

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