The Synchrotron Emission of Jets with Transverse Velocity Discrepancy *

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Abstract It has been commonly accepted that the bulk velocity of extragalactic jets varies in all directions. We examined the synchrotron radiation of a jet with velocity structure in the direction perpendicular to its axis and found that the spectral energy distribution (SED) is not strongly influenced by this circumstance, that there is only a small increase in the emission intensity and almost no shift in the peak frequency. For objects with smaller inclined angles θ_0 between the jet axis and the line of our sight, such as Blazars, the effect is more important. When θ_0 exceeds a critical value there is no longer any change in the SED. To compare the bulk speed with different velocity structure, an equivalent speed $\langle \beta \rangle$ is defined which would reproduce the same spectral profile. There possibly exists a stress $f_{\mu}(y)$ between layers of the outflow when the velocity is not the same in the jet.

Key words: galaxies: jets — radiation mechanisms: non-thermal — methods: numerical

1 INTRODUCTION

Many active galactic nuclei (AGN) such as radio galaxies and quasars display jet-like objects in their VLBI images. In these compact radio sources, energetic particles (electrons, positrons and/or protons), possibly driven by gas, radiation or magnetic pressure near the center, move at a relativistic velocity and along a highly collimated cone, forming a jet. As shown up in high-resolution images, these jets occur on pc scales as well as on kpc scales.

It is known that the jets radiate a broad spectrum with two pronounced continuum components which are interpreted by two basic radiation mechanisms. Nonthermal relativistic electrons with a power-law energy distribution in a tangled magnetic field produce the lower energy synchrotron emission from radio to X-ray. Ultra-relativistic electrons can also upper-scatter (inverse Compton scattering) soft photons from different sources to higher energy photons, especially to γ -ray photons. However, perfect matching of the observed multi-wave spectrum with

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these two mechanisms still remains to be achieved. In some earlier papers, a uniform blob or a sphere was adopted as the emission region, and some quite good results have been obtained (Gould 1979). Although this model is convenient in discussion, it may not be realistic enough. Marscher (1980) first used an accelerating jet model with nonlinear expansion, similar to the models of Blandford & Rees (1974) and Reynolds (1982), to calculate the synchrotron emission, dividing the jet into three segments. Of the three parts, Georganopoulos & Marscher (1998, GM98 hereafter) focused on the inner region beyond the 'nozzle' (a sonic point), and gave a unification scheme for BL Lac objects based on their simulated SEDs. For simplicity, a conical (linearly expanding) jet is also sometimes adopted as an approximation (Blandford & Königl 1979; Königl 1981; Pioner et al. 2003).

In all these inhomogeneous models, the velocity of the fluid is assumed to vary along the jet. In a more realistic jet, however, it is expected that the velocity should vary in all directions both parallel and perpendicular to the jet axis (Blandford & Levinson 1995). In order to explain the strange structure of the large-scale magnetic field and asymmetries in the jets of FR I radio galaxies, a two-speed jet model, in which a fast spine is surrounded by a slower layer, was introduced (Laing 1993, 1999; Ghisellini 2001). Chiaberge et al. (2000) also suggested this velocity structure in their reconciled BL Lac/FR I unification. As they showed, the faster component dominates the emission in BL Lacs while the slower component dominates the emission in misaligned sources.

In this paper, we consider the synchrotron emission of an inhomogeneous jet with a transverse velocity structure. We first introduce our stationary model in Sect. 2, based on an assumed relation between the speed and the radius, then present the luminosity of the synchrotron emission in the form of an integral involving the optical depth. Calculations and a discussion are given in Sect. 3, and a summary in Sect. 4.

2 A STATIONARY MODEL

As generally done in inhomogeneous jets, it is assumed that electrons with a power-law distribution are injected continuously at the base of the jet and, as they move downstrean, are cooled through synchrotron radiation, adiabatic expansion and inverse Compton scattering (to be considered when the energy density of soft photons becomes comparable to that of the magnetic field), with an ever decreasing Lorentz factor γ . As the energy falls, emission at lower frequencies becomes more important. At the same time, more of the radio emission is self-absorbed by the electrons. This process produces a steep profile of the temporal spectrum.

Our jet model has some significant differences from the previous ones. First, because the opening angle of the jet is very small (~ $1/\Gamma$) and the radius R increases slowly with longitudinal height z in the inner jet (Marscher 1980), taking a cylindrical jet (Ma et al. 2003) instead of a conical jet is a good approximation, which is also convenient for later discussion. So the energy loss due to adiabatic expansion which is important in the conical model is not included here. Secondly, the inverse Compton loss is certainly serious in the inner jet, but is also ignored in our consideration. Moreover, we will not consider any re-acceleration processes (e.g. the shock front and magnetic dissipation) throughout the jet. Lastly, differing from a simple two-speed model, the most important difference is that we assume the bulk velocity changes continuously with the transverse radius from the axis to the edge, while the bulk Lorentz factor is kept constant along the jet.

2.1 The Velocity Structure

It is unimaginable that a jet, being a plasma fluid located in the complex galactic environments, to have a constant velocity over its whole extended length, but the variation in the longitudinal velocity has been discussed adequately in the literature. However, a more likely scenario would be that there is also a velocity structure in the transverse direction. This is also suggested by numerical simulations of relativistic jets (Aloy 2000). Such a velocity structure can be produced when the jet is formed in the central region of the galaxy, or when there is interaction between the intergalactic material or photons from external sources and the outer surface of the jet, and the effects are transmitted into the central region layer by layer. If such continual effects exist and are dominant along the jet, the speed difference will become large enough at some time to form intensive turbulence accounting for knotty structures of the jet other than caused by shock fronts.

The transverse distribution of the bulk velocity should be axis-symmetric on large scales. We introduce a simple relation between the speed β in units of c (velocity of light) and radius R in the stationary frame:

$$\frac{\beta_c - \beta(R)}{\beta_c - \beta_m} = \left(\frac{R}{R_0}\right)^{\varepsilon} = y^{\varepsilon},\tag{1}$$

the bulk velocity and the bulk Lorentz factor at radius y are then:

$$\beta(y) = \beta_c - y^{\varepsilon} (\beta_c - \beta_m), \qquad (2)$$

and

$$\Gamma(y) = \frac{1}{\sqrt{1 - \beta(y)^2}}.$$
(3)

Here β_c , β_m are speeds on the central axis and at the margin (edge), ϵ is the index of the velocity structure. When $\varepsilon = 1$, the flow is the Newtonian fluid; and when $\varepsilon = 2$, its velocity distribution is a form of paraboloid. This correlation can be obtained from a more general form: $\beta(y) = \sum_{n=0}^{\infty} a_n y^n$ (when n = 0, 1; *n* is zero or an integer), like the form presented in Cantó, Raga & Riera (2003).

The angle between the line of sight and the jet axis is θ_0 in the stationary frame and θ_0^* in the commoving frame at radius y, and they satisfy the relation $\cos \theta_0 = \frac{\cos \theta_0^* + \beta(y)}{1 + \beta(y) \cos \theta_0^*}$, which can be simply taken as a form $\cos \theta_0 = \frac{\cos \theta_0^* + \beta_c}{1 + \beta_c \cos \theta_0^*}$ for the negligible difference between β_c and β_m . So the Doppler factor at radius y is:

$$D(y) = \frac{1}{\Gamma(y)(1 - \beta(y)\cos\theta_0)}.$$
(4)

Thus, considering the difference of Doppler factors or beaming effects at different radii, we find that the observed radiation frequency ν and luminosity $L(\nu)$ radiated by electrons in different layers are also different:

$$\nu = D(y)\nu^*, \ L(\nu) = D(y)^3 L^*(D(y)\nu^*), \tag{5}$$

where ν^* and $L^*(\nu^*)$ are the values at radius y in the commoving frame. In fact, as will be seen in the next section, the energy loss rate also varies with the distance y because of the different local bulk speeds.

2.2 Evolution of Electron Energy Spectrum

We will adopt an approach like GM98 to obtain the evolutionary spectrum of electrons. The equation for electron energy-loss (measured in the stationary frame) at a distance z from the galactic center is given in the fluid frame:

$$\frac{d\gamma}{dz} = -C(y) \cdot \gamma^2,\tag{6}$$

where $C(y) = \frac{4}{3} \frac{\sigma_T(u_B+u_s)}{m_e c^2} \frac{1}{\Gamma(y)\beta(y)}$, u_B and u_s are the intrinsic energy densities of the turbulent magnetic field and the soft photons, respectively. The ratio between the two is approximately represented by the flux ratio of the two spectral components in pure synchrotron self-Compton (SSC) models (see Sambruna et al. 1997). The spectral analyses of some sources indicate u_B $\sim u_s$ (for example u_B/u_s is close to 1 for Mrk 421). The second component of the spectrum, including the energy duration of hard X-ray and γ -ray, presents short variability timescales and is estimated to be produced at a distance $z \sim 0.01 - 0.1$ pc. At this location, the soft photons are copious. However, this region is still very small compared with the characteristic length $\sim 10^{18}$ cm (see the discussion below). Here, we only consider the energy loss through synchrotron emission: $C(y) = \frac{4}{3} \frac{\sigma_T u_B}{m_e c^2} \frac{1}{\Gamma(y)\beta(y)}$. Solving the energy-loss equation, we obtain

$$\gamma(z, y) = \frac{\gamma(z_0)}{C(y)(z - z_0)\gamma(z_0) + 1}.$$
(7)

So, the electron energy distribution (EED) is the same at any y and satisfies a power-law form with energy cutoffs

$$n(\gamma(z_0)) = n_0 \gamma(z_0)^{-s}, \ \gamma_{\min}(z_0) \le \gamma(z_0) \le \gamma_{\max}(z_0).$$
 (8)

Then we can obtain the energy spectrum at an arbitrary height z by the number conservation of electrons

$$n(\gamma(z,y)) = n_0 \frac{\gamma(z,y)^{-s}}{\left[1 - c(y)(z - z_0)\gamma(z,y)\right]^{2-s}},$$
(9)

and energy cut-offs

$$\gamma(z,y)_{\min,\max} = \frac{\gamma(z_0)_{\min,\max}}{C(y)(z-z_0)\gamma(z_0)_{\min,\max}+1}.$$
(10)

2.3 Synchrotron Radiation and Self-absorption

To calculate the synchrotron emission of different parts of a jet with tangled magnetic fields, we adopt the pitch-angle-averaged synchrotron emissivity $j(\nu, \gamma)$ and self-absorbed cross-section $\sigma_s(\nu, \gamma)$ for a single electron (Crusius & Schlickeiser 1986; Ghisellini, Guilbert & Svensson 1988; Ghisellini & Svensson 1991):

$$j(\nu,\gamma) = \frac{3\sqrt{3}}{\pi} \frac{\sigma_T c u_B}{\nu_L} \left(\frac{x}{2}\right)^2 \left\{ K_{1/3} \left(\frac{x}{2}\right) K_{4/3} \left(\frac{x}{2}\right) - \frac{3}{5} \left(\frac{x}{2}\right) \left[K_{4/3}^2 \left(\frac{x}{2}\right) - K_{1/3}^2 \left(\frac{x}{2}\right) \right] \right\}, \quad (11)$$

and

$$\sigma_s(\nu,\gamma) = \frac{\sqrt{3\pi}}{10} \frac{\sigma_T}{\alpha_f} \frac{B_{cr}}{B} \frac{x}{\gamma^5} \left[K_{4/3}^2 \left(\frac{x}{2}\right) - K_{1/3}^2 \left(\frac{x}{2}\right) \right],\tag{12}$$

where $x = \nu/\nu_c = \nu/(\frac{3}{2}\gamma^2\nu_L)$, $\nu_L = \frac{eB}{2\pi mc}$, $B_{cr} = 4.4 \times 10^{13} \,\text{G}$ and $\alpha_f = e^2/\hbar c = 7.3 \times 10^{-3}$.

We first calculate the optical depth from every cell or 'point' of the jet of cylindrical coordinate (z, y, ϕ) ($\phi = 0$ in the direction of the line of sight). The total path length (in units of R_0) that radiated photons of frequency $\nu^* (= \nu/D(y))$ propagate through the jet from an arbitrary point (z, y, ϕ) is $l_{\max}(y, \phi) = \frac{\sqrt{1-(y\sin\phi)^2-y\cos\phi}}{\sin\theta_0}$. On the line (from 0 to $l_{\max}(y, \phi)$) the y and z values of an arbitrary point at a distance $l (= l(y, \phi)$; in units of R_0) from the original location (z, y, ϕ) are

$$z_1 - z = \Delta z = l \cdot R_0 \cdot \cos \theta_0, \tag{13}$$

and

$$y_1 = \sqrt{y^2 + (l\sin\theta_0)^2 + 2y(l\sin\theta_0)\cos\phi}.$$
 (14)

So we can obtain the optical depth of every point in the jet

$$\tau_{\nu} = \int_{z}^{z_{1}\max} \int_{\gamma_{\min}(z_{1})}^{\gamma_{\max}(z_{1})} D(y_{1})^{-1} n(\gamma(z_{1}, y_{1})) \sigma_{s} \Big(\frac{\nu}{D(y)}, \gamma(z_{1}, y_{1})\Big) d\gamma(z_{1}, y_{1}) \frac{dz_{1}}{\cos\theta_{0}},$$
(15)

where $z_{1 \max} = z + l_{\max}(y, \phi) R_0 \cos \theta_0$. In terms of the optical depth, we can determine whether the point is optically thin or thick in the direction of sight. Here we expect to find the minimum frequency above which the radiation is optically thin everywhere in the jet.

For photons of low frequencies, the electron self-absorption cross-section increases with decreasing Lorentz factor. We select $z_{1 \min}$ as the maximum length z_{\max} of the jet used in integration of the total luminosity, where $\gamma(y_1, z_1)$ gets the minimum value and the cross-section of cooled electrons becomes the largest there at any radius. For $\phi = \pi$ and y = 1, $l_{\max}(y, \phi)$ gets the largest value $l_{\max}(1, \pi) = 2/\sin \theta_0$. Moreover, we get the maximum optical depth (a function of ν^*) (see Fig. 1).



Fig. 1 Logarithmic optical depth versus logarithmic frequency for jet with a velocity structure index $\varepsilon = 2$ and power-law spectrum of electrons $n(\gamma) = n_0 \gamma^{-s}$ (s = 1.8 and $n_0 = 1.0 \times 10^5$ cm⁻³).

From Fig. 1, we can estimate the maximum frequency ν_{max} to be below 10^{11} Hz. Photons with frequencies above 10^{11} Hz can not be heavily absorbed by the electrons when passing through the jet. Therefore, the total luminosity of the jet can be obtained by integrating the synchrotron emissivity over the whole emission region

$$L_{\nu} = 8\pi^2 R_0^2 \int D(y)^2 j\left(\frac{\nu}{D(y)}, \gamma(z, y)\right) n(\gamma(z, y)) y d\gamma(z, y) dy dz \,. \tag{16}$$

In this formula, we have taken into account the transformation of emissivity by multiplying $D(y)^2$ from the fluid frame to the observer frame in a volume element $dV = 2\pi R_0^2 y dy dz$.

3 CALCULATION AND DISCUSSION

Some of the physical parameters in our simulation are taken to be the same as in GM98. We take the energy cutoffs $\gamma_{\min} = 100$ and $\gamma_{\max} = 2.0 \times 10^5$, a stationary magnetic field with a constant $B \sim 0.1 \text{ G}$, and β_c and β_m corresponding to $\Gamma_c = 10$ and $\Gamma_m = 5$ which give negligible speed difference at different radii. The normalization factor N_0 in the power-law energy spectra of electrons $N(\gamma) = N_0 \gamma^{-s}$ can be determined as in the form of formula (11) of GM98: $N_0 = \frac{3(2-s)\Lambda_{\text{kin}}}{4\Gamma^2 \overline{\beta} c(\gamma_{\max}^{2-s} - \gamma_{\min}^{2-s})}$ (erg cm⁻¹), where Λ_{kin} is the electron kinetic luminosity, $\overline{\beta} = (\beta_c + \beta_c)/2$ and $\overline{\Gamma} = 1/\sqrt{1-\overline{\beta}^2}$. Here the normalization factor in the form of electron

 $\overline{\beta} = (\beta_c + \beta_m)/2$ and $\overline{\Gamma} = 1/\sqrt{1-\overline{\beta}^2}$. Here the normalization factor in the form of electron density is $n_0 = N_0/(\pi R_0^2 m_e c^2)$ (cm⁻³), and we set $n_0 \sim 10^5$ cm⁻³ for a jet with radius $R_0 = 10^{16}$ cm.

We will not take the entire length of the jet as the emission region for the time-consuming integration. For the innermost jet, there may be an increase of bulk velocity caused by internal energy converted into bulk kinetic energy (Marscher 1980). So, by considering the variability timescales, we set a not too small minimum height at $z_{\rm min} = 10^{16}$ cm. The maximum height $z_{\rm max}$ is given by the lengthscale in which electrons with the lowest energy lose half of their energy (for higher-energy electrons, the lost energy exceeds $\gamma/2$). It is taken to be about 5×10^{18} cm in our discussion.

Presented in Fig. 2 and Fig. 3 are the numerical results. It is shown that the total luminosity (assuming isotropy) L_{ν} is approximately the same when taking $\epsilon = 0, 1$ and 2, setting β_c and β_m to be constant. This can be described by an averaged bulk speed $\langle \beta \rangle$ over all y defined below, which produces a more Doppler boosted spectrum when ε increases. The Doppler factor D increases with θ_0 . When the inclination is smaller, the observed luminosity will be steeper and be more affected by changes in ε . In fact, the L_{ν} of different ε overlaps each other when θ_0 is large enough, see Fig. 2 for $\theta_0 = 15^{\circ}$. In a recent unification scheme, Blazars regarded as objects with smaller inclined angles. So the SEDs of Blazars are affected more strongly by the velocity structure. In Fig. 3, the SEDs for $\theta_0 = 10^{\circ}$ and $\varepsilon = 2$ are plotted with different energy spectral index s. The jet with smaller s, (containing more higher energy electrons), is much more luminous. The result is the same as when there is no velocity structure present. Another common characteristic of the SED is that the peak frequency remains almost the same, near 10^{16} Hz, although the synchrotron luminosity varies.

The total luminosity is composed of partial intensities $L_{\nu}(y)$ at radius y, each integrated over height z, i.e. $L_{\nu}(y) = 4\pi \int D(y)^2 j(\frac{\nu}{D(y)}, \gamma(z, y)) n(\gamma(z, y)) d\gamma(y, z) dz$. In Fig. 4, we present the SEDs at radius y = 0, 0.5, 0.75 and 1. They show little difference in shape, only a parallel increase in the luminosity and a small increase in the peak frequency as the radius decreases. This can be used to interpret the weak effect, noted above, of the transverse velocity structure on the luminosity. For stronger beaming effects in the inner region, the jet observed in lower frequency is more extended than that in higher frequency, but the difference can be neglected as in Fig. 4.



Fig. 2 Effect on the SED of velocity structure with different inclined angles. Left: $\theta_0 = 5^\circ$; Right: $\theta_0 = 15^\circ$.



Fig. 3 SEDs of different spectral index s with fixed $\theta_0 = 10^\circ$ and $\varepsilon = 2$.



Fig. 4 SED at different radius y for jet with $\varepsilon = 2, \ \theta_0 = 5^{\circ}$ and s = 1.8.

To compare the apparent bulk velocities of jets with velocity structure, we define an equivalent speed, averaged over \boldsymbol{y}

$$\langle \beta \rangle = \int_0^1 \beta(y) dy \Big/ \int_0^1 dy = \frac{\varepsilon \beta_c + \beta_m}{\varepsilon + 1},\tag{17}$$

and the equivalent Lorentz factor, $\langle \Gamma \rangle = 1/\sqrt{1-\langle \beta \rangle^2}$. For the general form of the velocity structure above, the averaged speed is $\langle \beta \rangle = \sum_{n=0}^{\infty} \frac{a_n}{n+1}$. The synchrotron radiation from jets with equal $\langle \beta \rangle$ is about the same in the observer frame (see Fig. 5).

Speed difference between different layers of the outflow will bring about an additional force of friction, which tends to eliminate the speed difference. If the velocity structure comes from the formation of the jet, the bulk velocity at different radius will be the same at some height. On the other hand, if it is caused by the ambient medium, then there will be friction and the equivalent speed will decrease along the jet. The internal friction stress is connected with the differentiation of the speed

$$f_{\mu}(y) \propto -\frac{d\beta(y)}{dy} = \varepsilon y^{\varepsilon - 1} (\beta_c - \beta_m).$$
 (18)

When $\varepsilon = 0$, i.e., the speed is the same at any radius, there is no internal friction. When $\varepsilon > 0$, there are three cases: (i) $\varepsilon < 1$, the force of internal friction is very large in the inner region and is infinite at y = 0, which is impossible in practice; (ii) $\varepsilon = 1$, the internal stress is the same everywhere; (iii) $\varepsilon > 1$, it increases with the radius y.



Fig. 5 SEDs with the same equivalent bulk Lorentz factor $\langle \Gamma \rangle = 6.5$, with $\theta_0 = 5^{\circ}$ and different values of ε .

4 SUMMARY

Simulated spectra of synchrotron emission are given for our jet model with transverse velocity structure. We find that there is only little difference for jets with the same bulk speed. The luminosity has an approximately parallel increase but the peak frequency almost keeps the same as ε increases where β_c and β_m are constant. It is expected that there is more obvious discrepancy in the SEDs if one takes a larger difference between β_c and β_m , as was done in Thompson (1997). If we decrease the angel θ_0 between the jet axis and the line of sight, then the effects of the velocity structure become more obvious on the spectrum. We decomposed the total luminosity into partial emissions at varying radii, but still found the difference to be

small. There should be a stress of friction since there is a velocity structure. The case $0 < \varepsilon < 1$ is impossible because of the singularity at radius y = 0. When $\varepsilon > 1$, the stress of internal friction becomes larger with increasing radius.

In summary, we find that the image taken in the higher frequency is more extended than that in the lower one due to the smaller Doppler factor at outer radius in a stationary jet. This will present little difference as discussed in the part intensity $L_{\nu}(y)$, however, which decreases by less than 1 order of magnitude in the surface than at the center.

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