Golden Ratio Sinusoidal Sequences and the Multimode Pulsation of the δ Sct Star V784 Cassiopeiae

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Abstract   Non periodic ordered sequences obtained by production rules in formal grammars are applied to the analysis of the multi-periodicity of the δ-Scuti type variable star V784 Cas. An artificial light curve for V784 Cas is generated by a non deterministic derivation in a context-sensitive grammar by concatenation of two sinusoidal fragments following certain word sequences. The two basic building blocks represent temporal segments in a golden ratio and the number of long and short segments in a word are also in a golden ratio.

Key words: stars: variables: δ Scuti — stars: individual: V784 Cas

1 INTRODUCTION

In recent works we have proposed applying methods that are usual in the study of quasi-crystal structures in condensed matter physics to the analysis of variable stars with multi-periodic pulsating behavior. Quasi-crystal structures are structures intermediate between the periodic structures of classical crystals and the random structures of amorphous materials (Senechal 1995). They are modeled mainly with the help of quasi-periodic tilings which can be described in terms of formal grammars (Escudero 1995). The Fourier analysis of 1D quasicrystal structures obtained by substitutional sequences can be done with the help of standard methods (Bombieri & Taylor 1986; Baake & Moody 1998). Techniques based on substitutional sequences generating 1D aperiodic ordered structures have been proposed recently (Escudero 2002, 2003) for the study of the multi-periodicity of semiregular variables (Kiss et al. 1999). The analysis has been applied to a light curve of the semiregular UW Herculis (HD 156163). Stochastic Fibonacci sequences were used for the construction of an artificial light curve by concatenation of simple sinusoids with two different lengths in a golden ratio. In this work we discuss the use of similar methods for the analysis of the light curves of δ Scuti stars which exhibit multimode pulsation. The analysis is applied to the δ Scuti star V784 Cassiopeiae (HD 13122). According to the Hipparcos astrometric data, the main period of V784 Cas is 0.1092 d. Kiss et al. (2002) have analysed the spectroscopic data obtained at the University of Toronto and photometric observations at the Sierra Nevada Observatory, IAA-CSIC. Their period analysis resulted in the detection of four frequencies ranging from $9.15\,\mathrm{d}^{-1}$ to 15.90 d^{-1} and there is a suggestion of more unresolved frequency components. In this paper the method for the derivation of the models is introduced (Sect. 2) by using formal grammars. In Sect. 3 the models are studied from the point of view of their period analysis in order to compare with the results obtained by Kiss et al.

2 THE ARTIFICIAL LIGHT CURVES

The geometric Fibonacci sequence is a very well known example of a 1D quasicrystal structure and can be generated by means of Lindenmayer systems. A D0L-system (Rozenberg & Salomaa 1980) is a triple $G = (\Sigma, \phi, \omega)$ where Σ is an alphabet, ϕ is an endomorphism defined on the set Σ^* of all the words of Σ, and ω, referred to as the axiom, is an element of Σ^*. For the Fibonacci sequence the alphabet is $\{L, S\}$ and $\phi : \{L \longmapsto \phi(L) = LS, S \longmapsto \phi(S) = L\}$. If we choose the axiom L, the sequence consists in the words $L, LS, LSL, LSLLS, \ldots$ A 1D self-similar non periodic geometric structure can be obtained if L and S represent two segments with a ratio equals the golden number $\tau = 2\cos(\pi/5)$. The Fibonacci numbers $F(n)$ can be defined with the help of the recurrence relation $F(n+2) = F(n+1) + F(n), F(0) = F(1) = 1$. By iterating this relation we obtain the sequence 1, 1, 2, 3, 5, 8, 13, 21... and the quotient of two successive Fibonacci numbers approaches the golden number when n increases. The length of the word $\phi^n(L)$ is $F(n)$ with $F(n-1)$ symbols L and $F(n-2)$ symbols S therefore the quotient (number of symbols L)/(number of symbols S) approaches also the golden ratio.

The Fibonacci sequence is deterministic in the sense that only one word is allowed with a given length. In Escudero (2003) stochastic L-systems were considered for the generation of the artificial light curves. More general languages must be used in order to include several types of variable stars in the study. In this work a different type of formal system is examined. A phrase structure grammar (Rozenberg & Salomaa 1980) is an ordered quadruple (Σ, P, s, Δ) where Σ and Δ are alphabets and $\Delta \subset \Sigma$ (Δ is called the alphabet of terminals and $\Sigma \setminus \Delta$ the alphabet of nonterminals), s is in $\Sigma \setminus \Delta$ (the initial letter) and P is a set of production rules. In this paper each production in P is of the form $w_1 l w_1 \longmapsto w_1 w w_1$ where w_1 and w_2 are arbitrary words, l is a nonterminal letter and w is a nonempty word. In the Chomsky hierarchy the generated language is called type 1 or context-sensitive. We analyze observations that spanned the period JD 2452151.5–2452162.7. The Stroemgren *ubvy* photometric observations were acquired on 12 nights in August and September 2001, using the 0.9m telescope of the Sierra Nevada observatory (Spain) (Kiss et al. 2002). The differential photometric data were obtained by using the comparison star HD 14172 and the check star HD 14173.

The artificial light curve has been obtained by matching with the observations. The curve is defined in the whole length of the dataset. Within the gaps in the empirical dataset we have assumed that the oscillations can be described also with the same two basic building blocks. Although the number of long and short segments is fixed their ordering inside a gap is arbitrary. As general criteria we have tried to mimic a property of the Fibonacci sequence, namely, neither three consecutive long segments nor two consecutive short segments appear in a deterministic sequence. Although for some intervals this was not possible, the number of long and short segments in the word w describing the main part of the sequence have been chosen to be in a golden ratio. Also we have tried to obtain a word that is generated by a formal grammar as simple as possible. A context sensitive grammar is the tool for the generation of the artificial light curve. In this case the alphabets are $\Sigma = \{a, b, L, S\}, \Delta = \{L, S\}$ and the set of productions P is

$$
\begin{aligned}
a &\longmapsto \phi_1[a] = ab \mid \phi_2[a] = ba \mid \Phi_1[a] = LS \mid \Phi_2[a] = SL\,, \\
b &\longmapsto \phi_1[b] = \phi_2[b] = a \mid \Phi_1[b] = L\,, \\
aba &\longmapsto \omega_1[aba] = b^2 a^3 \mid \Omega_1[aba] = L^3 S^2\,, \\
aba^2 b &\longmapsto \Omega_1[aba^2 b] = L^5 S^3 \mid \Omega_2[aba^2 b] = L^3 S L^2 S^2\,.
\end{aligned}
\tag{1}
$$

The vertical bars separate alternative possible images under the map given by the arrows. If we choose $s = a$ the main part of the artificial light curve is obtained with the following derivation:

$$a \longmapsto ab \longmapsto aba = \phi_1^2[a] \longmapsto ababa = \phi_1^2[a]\phi_2[a] \longmapsto abaabaab = \phi_1^2[ab]\phi_1\phi_2[a]$$

$$\longmapsto \phi_1^4[a]\phi_1[a]\phi_2[a]\phi_1[b] \longmapsto \phi_1[aba^2b]\phi_2[aba]\phi_1[ab^2a^2]$$

$$\longmapsto \phi_1[\phi_1[aba^2b]ba^2b]\phi_2[a]\omega_1[aba]\phi_1[a(ab)^2]$$

$$\longmapsto \phi_1[aba(ab)^2aab]\phi_2[ab]\phi_2[a(ab)^2]\phi_1[aba]\phi_1[b^2a^3(ab)^2]\phi_2[a]\phi_1[aba] \qquad (2)$$

$$\longmapsto \Phi_1[\phi_1^4[a]]\Omega_1[\phi_1^3[a]]\Omega_2[\phi_1^3[a]]\Phi_1[\phi_2\phi_1[a]]\Phi_1[(ba)^2ab]\Phi_2[a^2]\Phi_1[aba]\Phi_2[ab]\Phi_2[a^3bab^2]$$

$$\Phi_1[a]\Omega_1[aba]\Phi_1[(ab)^2(a^2b)^2] = \mathbf{L}SLLSLSL\mathbf{LS}LLSLLL\mathbf{LL}SSSLLLS\mathbf{LL}SSLLSLSLL$$

$$SLLSLSL\mathbf{SLS}LLSLLS\mathbf{SL}LSLSLSLL\mathbf{SL}LLLSLLL\mathbf{LS}SLSLLSLL\mathbf{SL}SLLSLS\mathbf{L} = w$$

where the letters in boldface type correspond to an approximate matching with the available data. The word w has length $F(10) = 89$ with $F(9)$ L segments and $F(8)$ S segments. The analyzed period of time is JD 2452151.5–2452162.7 which corresponds to the word $SwLSL^6SL$. In this case the light curve contains also two sinusoidal fragments in a golden ratio with durations $L=$ MJD 131 and $S=$ MJD 81 with MJD$=$(JD–2452150)$\times 10^3$. In Fig. 1 are seen the observed data and the curve corresponding to the fragment $\mathbf{SLS}LLSLLS\mathbf{SL}$.
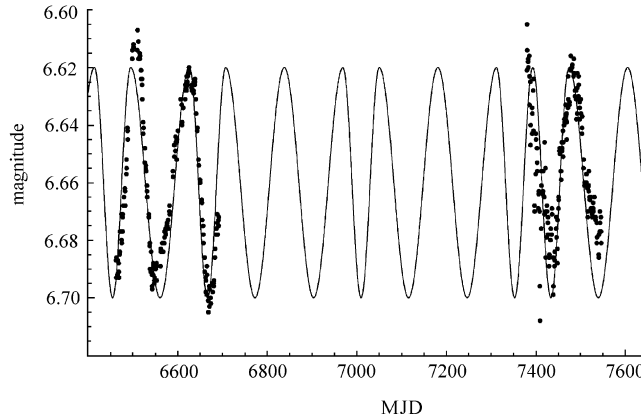

Fig. 1 Part of the empirical data set and a fragment of the artificial light curve. The ordinate is the magnitude. The abscise is the time (JD–2452150)$\times 10^3$.

3 PERIOD ANALYSIS

The Fourier spectrum of an artificial light curve with two sinusoidal fragments with lengths L and S in a golden ratio and following a Fibonacci sequence can be seen in Escudero (2003). The analysis shows that the main periods appear with a factor of $3 - \tau$ in relation with the durations of the sinusoidal fragments forming the light curve. The frequencies f of the Fourier spectrum of Fibonacci sequences belong to a set which consists of linear combinations with integer coefficients of two fundamental frequencies $f = m_1 f_1 + m_2 f_2$ where f_1 and f_2 are in a golden ratio and m_1, m_2 are integers (Bombieri & Taylor 1986). Due to the fact that τ is an

irrational number the real numbers f are dense in the real line. Although this spectrum is a dense set, above a certain amplitude threshold the number of frequencies in the spectrum below a fixed frequency is finite (Senechal 1995). The analysis of the merged data for the interval JD 2451433.4–245162.7 given in Kiss et al. (2002) yields the detection of the frequencies 9.15, 9.46, 15.4, 15.9 cpd with normalized amplitudes $1, 0.34, 0.22, 0.14$. In Fig. 2 it can be seen the amplitude spectrum of the model generated with $SwLSL^6SL$. The frequencies 9.15, 9.78, 14.2,
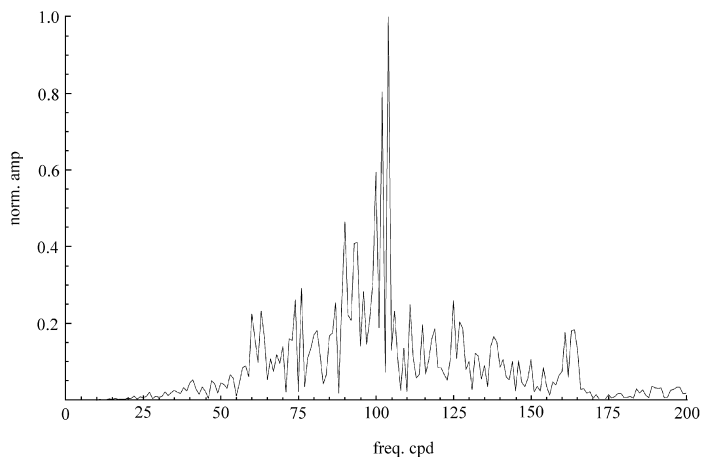

Fig. 2   Fourier analysis of the model for V784 Cas. The amplitude is normalized and the frequency is given in units of 1000/11250 cpd.
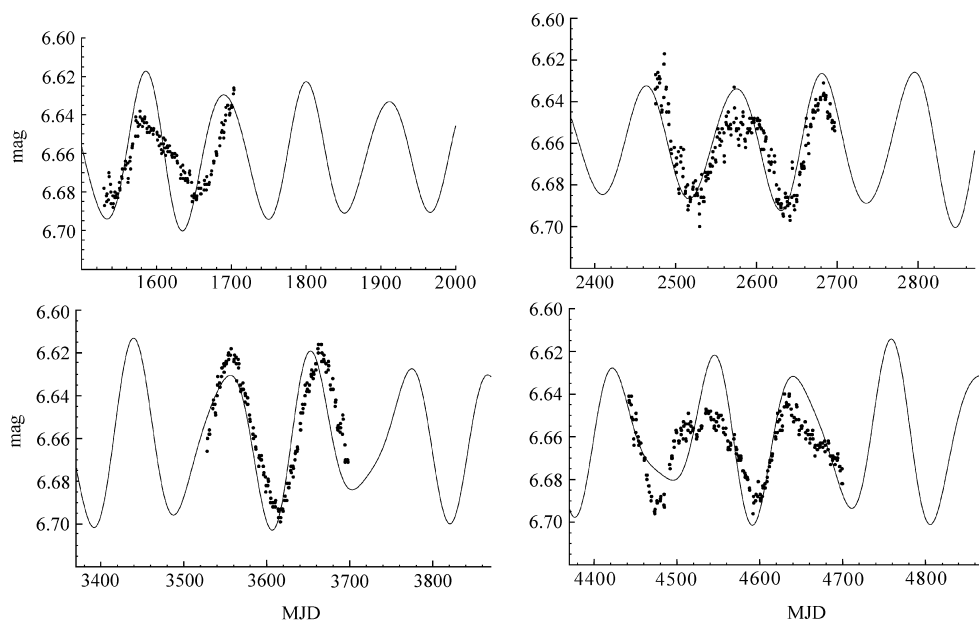

Fig. 3   Observed individual light curves of V784 Cas and the artificial light curve component with four harmonics.
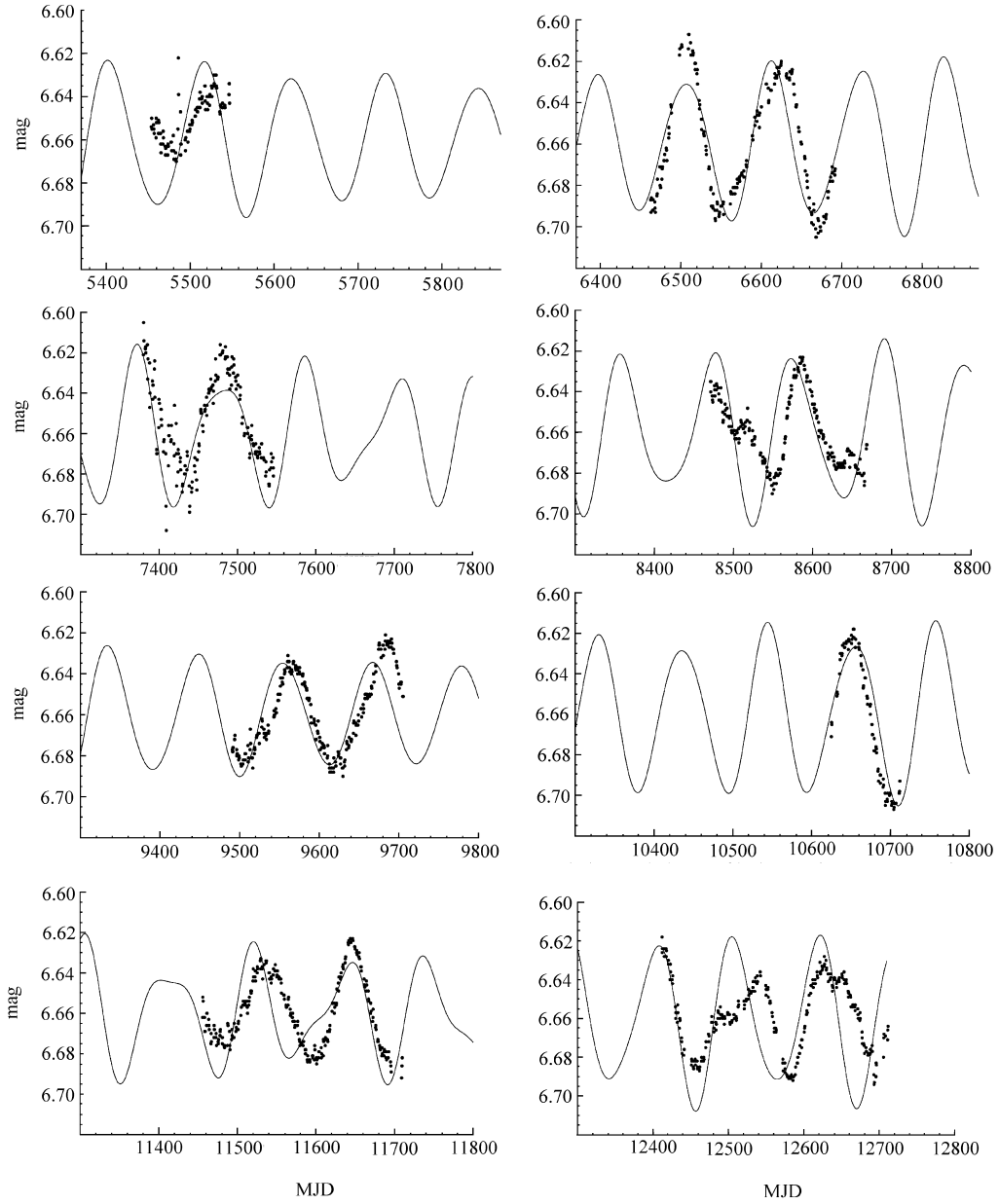
Fig. 3   Continued.

14.5 cpd have normalized amplitudes $1, 0.25, 0.17, 0.18$ and phases $2.15, 2.58, 2.96, -1.56$ rad, respectively. The dominant period is in good agreement with the one listed in the Hipparcos catalog and in Kiss et al. (2002). The other frequencies are not very different in spite of the fact that the analyzed interval is not the same. The curve obtained with this set of four frequencies together with the observed individual light curves can be seen in Fig. 3.

4   CONCLUSIONS

In this work we have studied the extension to the study of multi-periodic δ Scuti stars of the methods introduced in Escudero (2002, 2003) in the context of semiregular variables. In the analysis of the V784 Cas multimode pulsation some time intervals are without any available observational data. The ratio of the number of long and short fragments has been chosen to be the golden number but the ordering inside these intervals is arbitrary. In these gaps we have tried to use some general properties of Fibonacci sequences. Although several word sequences have been considered with similar results for the main frequencies, different criteria can be used and grammars with other production rules must be analyzed. In the model presented in this work all the sinusoidal amplitudes are the same. In Escudero (2003) it is shown, by introducing amplitude changes according to the empirical data, how the results can be improved. Observations with better time sampling and spanning longer periods of time are needed. This is suggested also by the fact that most noticeable variations are situated on the first and last segments of the data which corresponds to the letters not belonging to the word w generated by the context sensitive grammar.

An example of a δ Scuti-type variable has been studied in this work. The model is related with the golden number which is an algebraic integer appearing in nature in different contexts. Other δ Scuti stars must be considered in the future (see Rodriguez et al. 2000; Rodriguez & Breger 2001 for recent catalogues of δ Scuti stars) by means of different types of formal grammars and algebraic integers. The effect of the approach presented in this paper in the determination of the interior structure of the star requires further study, but only two independent pulsation modes seem to appear on its oscillations.

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