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The Amplitude of Mass Fluctuations and Mass Density of the Universe Constrained by Strong Gravitational Lensing*

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Abstract We investigate the linear amplitude of mass fluctuations in the universe, σ_8, and the present mass density parameter of the Universe, $\Omega_{\rm m}$, from statistical strong gravitational lensing. We use the two population model of lens halos with fixed cooling mass scale $M_{\rm c} = 3 \times 10^{13} h^{-1} M_\odot$ to match the observed lensing probabilities, and leave σ_8 or $\Omega_{\rm m}$ as a free parameter to be constrained by the data. Another varying parameter, the equation of state of dark energy ω, and its typical values of -1, $-2/3$, $-1/2$ and $-1/3$ are investigated. We find that σ_8 is degenerate with $\Omega_{\rm m}$ in a way similar to that suggested by present day cluster abundance as well as cosmic shear lensing measurements: $\sigma_8 \Omega_{\rm m}^{0.6} \approx 0.33$. However, both $\sigma_8 \leq 0.7$ and $\Omega_{\rm m} \leq 0.2$ can be safely ruled out, the best fit is when $\sigma_8 = 1.0$, $\Omega_{\rm m} = 0.3$ and $\omega = -1$. This result is different from that obtained by Bahcall & Bode, who gave $\sigma_8 = 0.98 \pm 0.1$ and $\Omega_{\rm m} = 0.17 \pm 0.05$. For $\sigma_8 = 1.0$, the higher value of $\Omega_{\rm m} = 0.35$ requires $\omega = -2/3$ and $\Omega_{\rm m} = 0.40$ requires $\omega = -1/2$.

Key words: cosmology: theory — cosmological parameters — gravitational lensing

1 INTRODUCTION

The amplitude of mass fluctuations, denoted as σ_8 when referring to the rms linear density fluctuation in spheres of radius $8h^{-1}$Mpc at $z = 0$, is a fundamental cosmological parameter which describes the normalization of the linear spectrum of mass fluctuations in the early universe. Assuming Gaussian initial fluctuations, the evolution of structure in the universe depends exponentially on this parameter (for an excellent review see Bahcall & Bode 2003).

Recent observations suggest an amplitude that ranges in value from $\sigma_8 \sim 0.7$ to a high value of $\sigma_8 \sim 1.1$. The low amplitude values of $\sigma_8 \sim 0.7$ are suggested by current observations of the cosmic microwave background (CMB) spectrum of fluctuations (Netterfield et al. 2002; Sievers et al. 2003; Bond et al. 2002; Ruhl et al. 2003) and by recent observations of the present cluster abundance as well as cosmic shear lensing measurements (Jarvis et al. 2003; Hamana et al. 2003; Seljak 2002). However, these determinations of σ_8 are degenerate with other parameters like mass density parameter $\Omega_{\rm m}$. The evolution of cluster abundance with time, especially for the most massive clusters, breaks the degeneracy between σ_8 and $\Omega_{\rm m}$ (e.g., Peebles, Daly &

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Juszkiewicz 1989; Eke, Cole & Frenk 1996; Oukbir & Blanchard 1997; Bahcall, Fan & Cen 1997; Carlberg et al. 1997; Bahcall & Fan 1998; Donahue & Voit 1999; Henry 2000). This evolution depends strongly on σ_8, and only weakly on $\Omega_{\rm m}$ or other parameters. Bahcall & Bode (2003) used the abundance of the most massive clusters observed at $z \sim 0.5 - 0.8$ to place a strong limit on σ_8 and found that $\sigma_8 = 0.98 \pm 0.1$, $\Omega_{\rm m} = 0.17 \pm 0.05$, whereas low σ_8 values ($\lesssim 0.7$) are unlikely. In the model of one population of halos (Navarro-Frenk-White, hereafter NFW), combined each galactic halo with a central point mass, the lensing probabilities are shown to be sensitive to σ_8 (Chen 2003a, hereafter Paper I).

In this paper, we use the model of two populations of lens halos to calculate the lensing probabilities in flat quintessence cold dark matter (QCDM) cosmology with different cosmic equations of state ω (Chen 2003b, hereafter Paper II), leaving σ_8 or $\Omega_{\rm m}$ as a free parameter to be constrained by the Jodrell-Bank VLA Astrometric Survey (JVAS) and the Cosmic Lens All-Sky Survey (CLASS; Browne et al. 2000; Helbig 2000; Browne et al. 2003; Myers et al. 2002).

2 LENSING PROBABILITIES

When quasars at mean redshift $\langle z_{\rm s}\rangle = 1.27$ are lensed by foreground CDM halos of galaxies and clusters of galaxies, the lensing probability of image separations larger than $\Delta\theta$ and a flux density ratio less than $q_{\rm r}$ is (Schneider et al. 1992),

$$P(>\Delta\theta, <q_{\rm r}) = \int_0^{z_{\rm s}} \frac{dD_{\rm L}(z)}{dz} dz \int_0^{\infty} \bar{n}(M,z)\sigma(M,z)B(M,z)dM, \qquad (1)$$

where $D_{\rm L}(z)$ is the proper distance from the observer to the lens located at redshift z, and $\bar{n}(M,z)$ is the physical number density of virialized dark halos of masses between M and $M + dM$ at redshift z given by Jenkins et al. (2001). The cross section $\sigma(M,z)$ is mass and redshift dependent, and is sensitive to flux density ratio of multiple images $q_{\rm r}$ for SIS (singular isothermal sphere) halos,

$$\sigma(M,z) = \pi\xi_0^2\vartheta(M - M_{\rm min}) \times \begin{cases} y_{\rm cr}^2, & \text{for } \Delta\theta \le \Delta\theta_0; \\ y_{\rm cr}^2 - y_{\Delta\theta}^2, & \text{for } \Delta\theta_0 \le \Delta\theta < \Delta\theta_{y_{\rm cr}}; \\ 0, & \text{for } \Delta\theta \ge \Delta\theta_{y_{\rm cr}}, \end{cases} \qquad (2)$$

where $\vartheta(x)$ is the step function, and $M_{\rm min}$ is the minimum mass of halos above which lenses can produce images with separations greater than $\Delta\theta$. It has been shown (in Paper II) that the contributions from galactic central supermassive black holes can be ignored when $q_{\rm r} \le 10$, so the lensing equation for SIS halos is simply $y = x - |x|/x$, where $x = |\boldsymbol{x}|$ and $y = |\boldsymbol{y}|$, that are related to the position vector in the lens plane and source plane as $\boldsymbol{\xi} = \boldsymbol{x}\xi_0$ and $\boldsymbol{\eta} = \boldsymbol{y}\eta_0$, respectively. The length scales in the lens plane and the source plane are defined to be $\xi_0 = 4\pi(\sigma_v/c)^2(D_{\rm L}^{\rm A}D_{\rm LS}^{\rm A})/D_{\rm S}^{\rm A}$ and $\eta_0 = \xi_0 D_{\rm S}^{\rm A}/D_{\rm L}^{\rm A}$. Since the surface mass density is circularly symmetric, we can conveniently extend both x and y to their opposite values in our actual calculations. From the lensing equation, an image separation at any y can be expressed as $\Delta\theta(y) = \xi_0\Delta x(y)/D_{\rm L}^{\rm A}$, where $\Delta x(y)$ is the image separation in the lens plane for the given y. So in Eq. (2), the source position $y_{\Delta\theta}$, at which a lens can produce the image separation $\Delta\theta$, is the inverse of this expression. Here $\Delta\theta_0 = \Delta\theta(0)$ is the separation of the two images that are just on the Einstein ring; $\Delta\theta_{y_{\rm cr}} = \Delta\theta(y_{\rm cr})$ is the upper-limit of the separation above which the flux ratio of the two images will be greater than $q_{\rm r}$. Note that since $M_{\rm DM}$ is related to $\Delta\theta$

through ξ_0 and $\sigma_v^2 = GM_{\rm DM}/2r_{\rm vir}$, we can formally write $M_{\rm DM} = M_{\rm DM}(\Delta\theta(y))$ and determine $M_{\rm min}$ for galaxy-size lenses by $M_{\rm min} = M_{\rm DM}(\Delta\theta(y_{\rm cr}))$.

According to the model of two populations of halos, cluster-size halos are modeled with the NFW profile: $\rho_{\rm NFW} = \rho_{\rm s} r_{\rm s}^3/[r(r+r_{\rm s})^2]$, where $\rho_{\rm s}$ and $r_{\rm s}$ are constants. Then we can define the mass of a halo to be the mass within the virial radius of the halo $r_{\rm vir}$: $M_{\rm DM} = 4\pi\rho_{\rm s} r_{\rm s}^3 f(c_1)$, where $f(c_1) = \ln(1+c_1) - c_1/(1+c_1)$, and $c_1 = r_{\rm vir}/r_{\rm s} = 9(1+z)^{-1}(M/1.5\times10^{13}h^{-1}M_{\odot})^{-0.13}$ is the concentration parameter, for which we have used the fitting formula given by Bullock et al. (2001). The lensing equation for NFW lenses is as usual, $y = x - \mu_s g(x)/x$ (Li & Ostriker 2002), where $y = |\boldsymbol{y}|$, $\boldsymbol{\eta} = \boldsymbol{y} D_{\rm S}^{\rm A}/D_{\rm L}^{\rm A}$ is the position vector in the source plane, in which $D_{\rm S}^{\rm A}$ and $D_{\rm L}^{\rm A}$ are angular-diameter distances from the observer to the source and to the lens, respectively, $x = |\boldsymbol{x}|$ and $\boldsymbol{x} = \boldsymbol{\xi}/r_{\rm s}$, $\boldsymbol{\xi}$ is the position vector in the lens plane. The parameter $\mu_{\rm s} = 4\rho_{\rm s} r_{\rm s}/\Sigma_{\rm cr}$ is x independent, in which $\Sigma_{\rm cr} = (c^2/4\pi G)(D_{\rm S}^{\rm A}/D_{\rm L}^{\rm A} D_{\rm LS}^{\rm A})$ is the critical surface mass density, with c the speed of light, G the gravitational constant and $D_{\rm LS}^{\rm A}$ the angular-diameter distance from the lens to the source. The function $g(x)$ has an analytical expression originally given by Bartelmann (1996). The cross section for cluster-size NFW lenses is well studied (Li & Ostriker 2002). The lensing equation is $y = x - \mu_s g(x)/x$ and the multiple images can be produced only if $|y| \le y_{\rm cr}$, where $y_{\rm cr}$ is the maximum value of y when $x < 0$, which is determined by $dy/dx = 0$, and the cross section in the lens plane is simply $\sigma(M, z) = \pi y_{\rm cr}^2 r_{\rm s}^2$.

As for the magnification bias $B(M, z)$, we use the result given by Li & Ostriker (2002) for NFW lenses. For the singular isothermal sphere (SIS) model, the magnification bias is $B_{\rm SIS} \approx 4.76$.

In this paper the spatially flat QCDM cosmology models are considered. The density parameter $\Omega_{\rm m}$ ranges from 0.2 to 0.4 as suggested by all kinds of measurements (e.g., Peebles & Ratra 2003 and the references therein). We investigate the varying parameter σ_8 over its entire observational range from 0.7 to 1.1 (e.g., Bahcall & Bode 2003), the Hubble parameter is fixed at $h = 0.75$. Three negative values of ω in the equation of state $p_{\rm Q} = \omega\rho_{\rm Q}$, along with $\omega = -1$ (cosmological constant), $\omega = -2/3$, $\omega = -1/2$ and $\omega = -1/3$ are examined. We use the conventional form to express the redshift z dependent linear power spectrum for the matter density perturbation and the linear growth suppression factor of the density field in QCDM cosmology, established by Ma et al. (1999), which are needed in Eq. (1).

3 DISCUSSION AND CONCLUSIONS

Since the lensing rate is sensitive to the source redshift $z_{\rm s}$, and the results change considerably when a redshift distribution is included in the calculations (e.g., Sarbu, Rusin & Ma 2001). However, because the distribution in the JVAS/CLASS survey is still poorly understood, we take the estimated mean value of $\langle z_{\rm s}\rangle = 1.27$ (Marlow et al. 2000; Chae et al. 2002; Oguri 2003; Huterer & Ma 2004, Paper I) in the present calculation.

For comparison with Paper I, here in Fig. 1 we plot the lensing probability versus image separation angle for the same sets of cosmological and lens halo model parameters as made explicit in the right panel of the Fig. 1 of Paper I, except the mean quasar redshift $\langle z_{\rm s}\rangle$, the amplitude of mass fluctuations σ_8 and the mass density parameter $\Omega_{\rm m}$. This is a model of one population of halos (NFW), with each halo combined with a central point mass ($M_{\rm eff}$). A slightly higher value of $\sigma_8 = 1.0$ is used here (in Paper I, we used $\sigma_8 = 0.95$). The histogram shows the results of JVAS/CLASS; the solid, dash-dotted, dashed and dotted lines (from top down) stand for, respectively, $(q_{\rm r}, M_{\rm eff}/M_\bullet) = (10, 200), (100, 100), (1000, 50), (10000, 30)$, $M_\bullet$

being the mass of the galactic central black hole. Five values of Ω_m ranging from 0.2 to 0.4 (as indicated in each panel) are chosen to test the effect of this parameter on the lensing probability. We have found that the lensing probability is sensitive to Ω_m. However, a good fit can be obtained only when $\omega = -1$ and $\Omega_m = 0.4$; this result is different from the result given in Paper I. The reason is that the lensing probabilities are quite sensitive to the mean redshift of quasars $\langle z_s \rangle$, and the higher redshift will produce a larger value of lensing probability. This means that the NFW+point-mass model for galaxy-size lens halos indeed reduces the probabilities considerably when a smaller image flux density ratio is taken into account; this can be confirmed when a comparison is made with the SIS model, and in the following discussion.

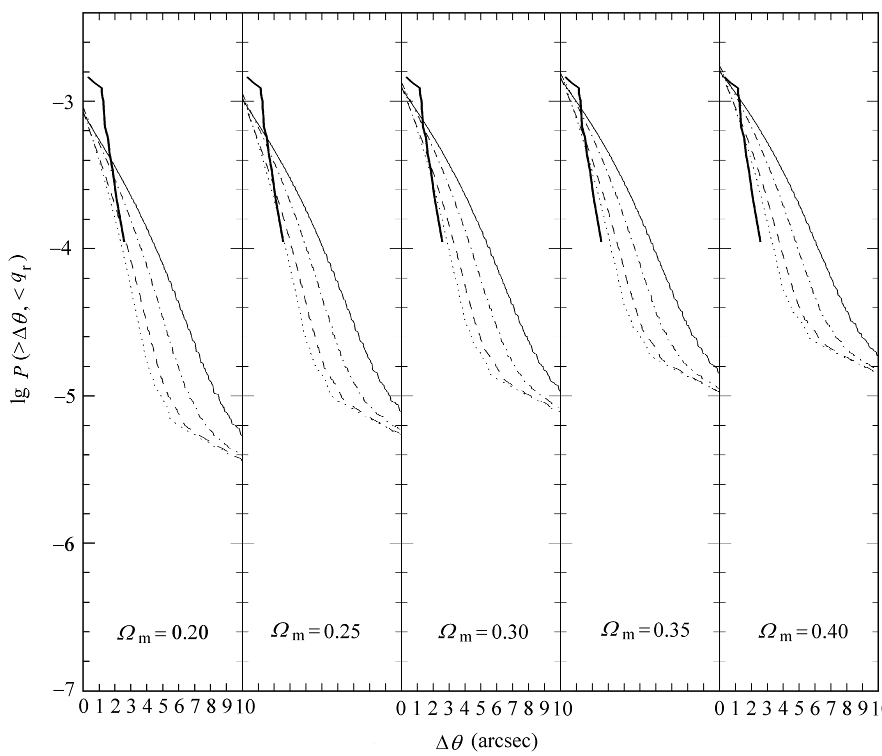

Fig. 1 Predicted lensing probability with image separations $>\Delta\theta$ and flux density ratios $<q_r$ in ΛCDM cosmology. The cluster-size lens halos are modelled by the NFW profile, and galaxy-size lens halos by NFW+BULGE. Instead of SIS, we treat the bulge as a point mass, its value $M_{\rm eff}$ is so selected for each q_r that the predicted lensing probability can match the results of JVAS/CLASS represented by histogram. In each panel, the solid, dash-dotted, dashed and dotted lines (from top downwards) stand for, respectively, the matched values of the pair $(q_r, M_{\rm eff}/M_\bullet)$ ($M_\bullet$ is a galactic central black hole mass) of $(10, 200)$, $(100, 100)$, $(1000, 50)$ and $(10000, 30)$. Here $\langle z_s \rangle = 1.27$ and $\sigma_8 = 1.0$ are for all panels.

As just mentioned, when we use the NFW+point-mass to model galaxy-size lens halos, the predicted lensing probabilities can match observations only when a higher value of $\langle z_s \rangle$ is used. We have argued in Paper II that a two population model of lens halos with mass distributions NFW ($M_{\rm DM} > M_c$) and SIS ($M_{\rm DM} < M_c$), can match the observations better, even when a reasonable lower value of $\langle z_s \rangle$ and of M_c are used. We choose the cooling mass scale to be $M_c = 3.0 \times 10^{13} h^{-1} M_\odot$ in this paper rather than $M_c = 5.0 \times 10^{13} h^{-1} M_\odot$ used in Paper I.

So it would be interesting to investigate both the $\Omega_{\rm m}$ and σ_8 dependent lensing probability with the combined SIS+NFW model. In each panel of Fig. 2, the parameters are: $q_{\rm r} = 10$, $\sigma_8 = 1.0$, while $\Omega_{\rm m}$ takes the five values indicated in Fig. 1. We find that lensing probability is also sensitive to $\Omega_{\rm m}$ and clealy $\Omega_{\rm m} = 0.2$ can be safely ruled out. For $\omega = -1$ (cosmological constant), the best fit value of mass density parameter is $\Omega_{\rm m} = 0.3$. Higher value of $\Omega_{\rm m} = 0.35$ requires $\omega = -2/3$ and $\Omega_{\rm m} = 0.40$ requires $\omega = -1/2$. This result is different from those obtained with other methods (see Bahcall & Bode 2003).

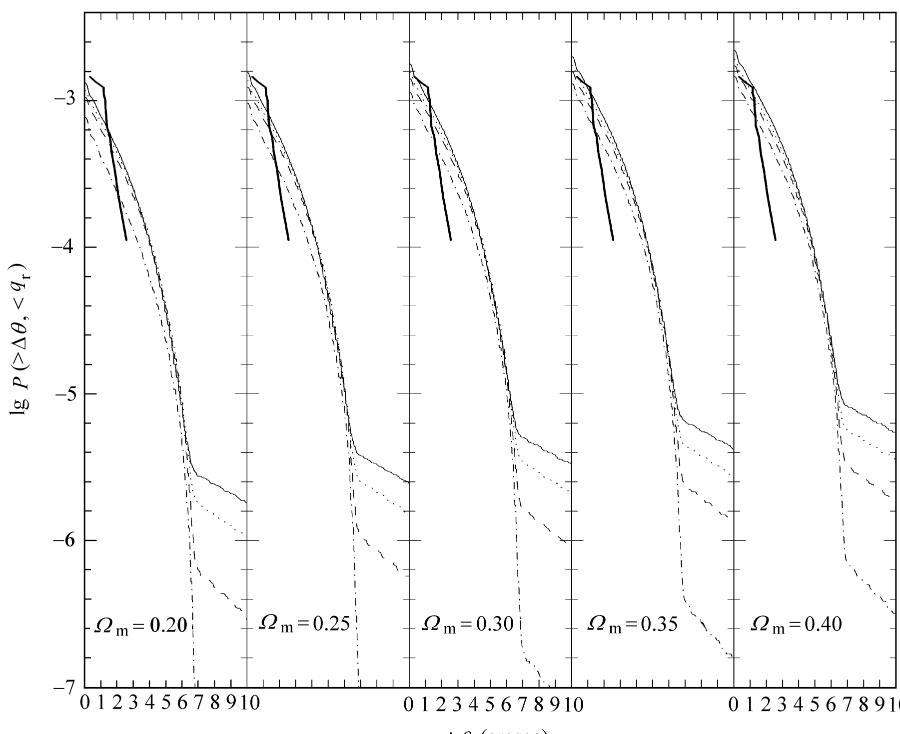

Fig. 2 Integral lensing probabilities with image separations larger than $\Delta\theta$ and flux density ratio less than $q_{\rm r}$, for quasars at mean redshift $\langle z_{\rm s} \rangle = 1.27$ lensed by NFW ($M_{\rm DM} > M_{\rm c}$) and SIS ($M_{\rm DM} < M_{\rm c}$) halos. In each panel, $q_{\rm r} = 10.0$ and $\sigma_8 = 1.0$. The solid, dashed, dash-dotted and dotted lines stand for $\omega = -1$, $-2/3$, $-1/2$ and $-1/3$, respectively.

Our model prefers a higher value of $\Omega_{\rm m} \geq 0.3$. In Fig. 2, we have already used higher values of σ_8 ($= 1.0$) and $M_{\rm c}$ ($= 3.0 \times 10^{13} h^{-1} M_\odot$). Lower values of $\Omega_{\rm m}$ require still higher values of these two parameters, which would be out of the range suggested by other measurements. In order to see the effect of σ_8 on lensing probability, we fix the value of $\Omega_{\rm m}$ at 0.3 and 0.35, respectively, and vary σ_8 from 0.7 to 1.1 in each case. The results are shown in Fig. 3 and Fig. 4. It has been found if we take $\Omega_{\rm m} = 0.3$, the lensing probabilities are only slightly sensitive to σ_8 at small image separations ($0.3'' < \Delta\theta < 3''$), where JVAS/CLASS survey has a well-defined sample suitable for the analysis of lens statistics. The lensing probabilities are sensitive to σ_8 at larger image separations, but there is no suitable sample for analysis in this range. Although $\sigma_8 \leq 0.7$ seems unlikely, all the values in the range $0.8 \leq \sigma_8 \leq 1.1$ are possible. For a larger value of $\Omega_{\rm m} = 0.35$, as shown in Fig. 4, the lensing probabilities are more sensitive to σ_8 than in Fig. 3. In this case, even $\sigma_8 = 0.7$ will be quite acceptable (it predicts 12.5 lenses with image

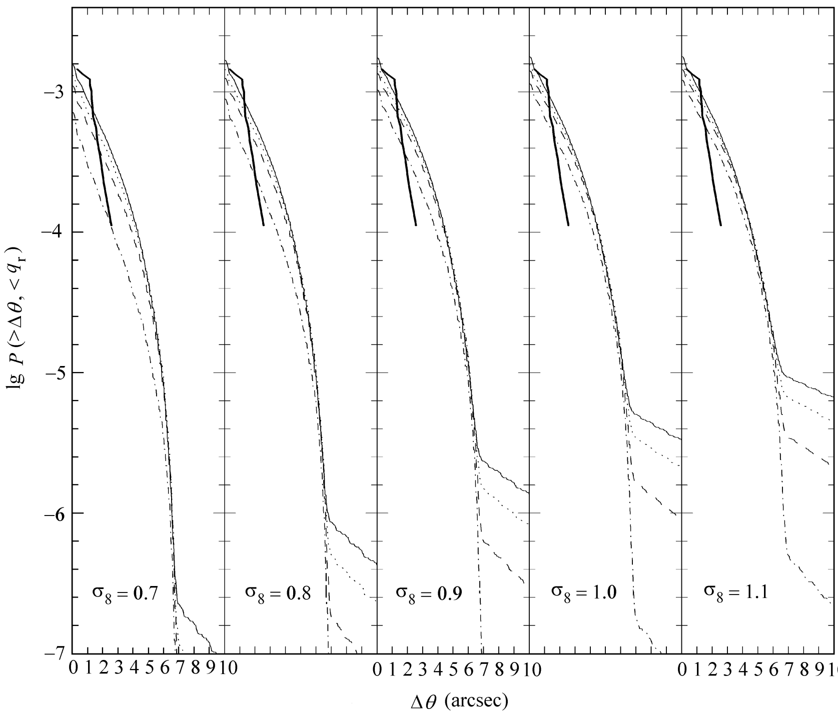

Fig. 3 Same as Fig. 2, except the value of σ_8. In each panel, $\Omega_{\rm m} = 0.3$.

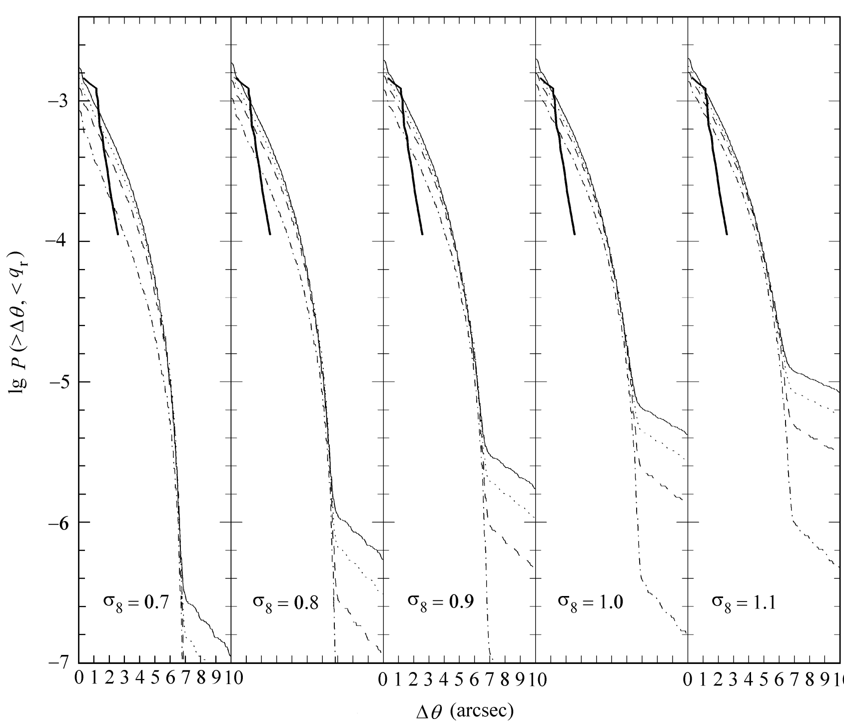

Fig. 4 Same as Fig. 3, except that $\Omega_{\rm m} = 0.35$.

separation $\geq 0.3''$, while the observed value is 13), and $\sigma_8 = 1.1$ matches a value of $\omega = -2/3$.

Note that, from a likelihood analysis of the lens statistics, Chae et al. (2002)'s main results on the cosmological parameters are $\Omega_{\rm m} = 0.31^{+0.27}_{-0.14}$ and $\omega = 0.55^{+0.18}_{-0.11}$, both at 68% confidence level. Our results, although not quite precise, are in agreement with theirs. However, since Chae et al. (2002) used the Schechter luminosity function rather than Press-Schechter mass function to account for the mass distribution, they did not refer to σ_8. More precise results using the same model presented in this paper from a likelihood analysis will be given in another paper.

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