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Strange Stars: Can Their Crust Reach the Neutron Drip Density? *

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Abstract The electrostatic potential of electrons near the surface of static strange stars at zero temperature is studied within the frame of the MIT bag model. We find that for QCD parameters within rather wide ranges, if the nuclear crust on the strange star is at a density leading to neutron drip, then the electrostatic potential will be insufficient to establish an outwardly directed electric field, which is crucial for the survival of such a crust. If a minimum gap width of 200 fm is brought in as a more stringent constraint, then our calculations will completely rule out the possibility of such crusts. Therefore, our results argue against the existence of neutron-drip crusts in nature.

Key words: dense matter — elementary particles — stars: neutron

1 INTRODUCTION

There may exist a more stable state of hadrons than ^{56}Fe, called strange quark matter (SQM), which is a bulk quark phase consisting of roughly equal numbers of u, d, and s quarks plus a smaller number of electrons to guarantee charge neutrality (Witten 1984). This theoretical result is unlikely to be proved or disproved through QCD calculations at least in the foreseeable future. Final adjudication must come from experiments conducted with accelerators *or* from astrophysical tests. One of the important consequences of Witten's hypothesis is the prediction of strange stars (Haensel, Zdunik & Schaeffer 1986; Alcock, Farhi & Olinto 1986, hereafter AFO), i.e., stars made of SQM. The presence of such stars should never be rare, in fact some authors even inferred that all neutron stars might have been converted to strange stars, since the whole Galaxy is likely to be contaminated by stranglets (Glendenning 1990; Madsen & Olesen 1991; Caldwell & Friedman 1991; Medina-Tanco & Horvath 1996). For decades, unremitting efforts have been made to observationally discriminate strange stars from neutron stars, and some well-constrained candidates for strange stars have been forthcoming (Cheng et al. 1998; Li et al. 1999; Xu et al. 2001a, b). However, even now, it is still premature to reach any firm conclusions.

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A strange star can have a normal matter crust. The density at the base of the crust is a crucial parameter, which has been discussed by many authors (AFO; Huang & Lu 1997a, b; Xu & Qiao 1999; Yuan & Zhang 1999; Chen & Zhang 2001; Ma et al. 2002). Recently, Ma et al. (2002) studied the influence on the crust density of such parameters as the strange quark mass (m_s), the bag constant (B), and the strong coupling constant (α_c). However, this study did not derive clear ranges of the parameters that would lead to a crust as dense as neutron drip density. Here we study the problem in more detail. The parameter ranges that support a neutron-drip-density crust will be calculated numerically. This paper is arranged as follows. Section 2 describes the existence of strange star crust. Section 3 gives the detailed procedure of our calculations and numerical results in the $\alpha_c - m_s - B$ space. Finally, we summarize the results in Section 4.

2 THE EXISTENCE OF CRUST

It was pointed out by AFO that a strange star could be covered by a normal material crust. The presence of electrons in SQM is vital to the existence of such crusts. Because s quark's mass is larger than u and d quarks', it is slightly deficient in equilibrium SQM. Electrons are thus called in to make the system electrically charge neutral. As quarks are bounded through strong interaction, they should have a very sharp surface with thickness of the order of 1 fm. By contrast, the electrons, bounded by the Coulomb force, can extend several hundred fermis beyond the quark surface. So, a strong, outwardly directed electric field, $\sim 10^{17}$ V cm^{-1} will be established in a thin layer several hundred fermis thick above the strange matter surface. This field can support a crust, composed of normal nuclear matter, suspended out of contact with the SQM core.

The so-called neutron drip density, $\rho_{\rm drip} \sim 4.3 \times 10^{11}$g cm^{-3} sets an absolute upper limit for the density of the crust, $\rho_{\rm crust}$. The reason is that, if $\rho_{\rm crust}$ reaches $\rho_{\rm drip}$, neutrons will begin to drip out. Being electrically neutral, the neutrons will fall freely into the core, and, by hypothesis, be deconfined to be SQM. As a consequence, $\rho_{\rm crust}$ will keep going down, until it is below $\rho_{\rm drip}$. Conventionally, when the nuclear crust is taken into account, the bottom density was assumed to be $\rho_{\rm drip}$. However, is neutron drip the only limit on the crust density? Could the crust density actually go so far? Huang & Lu (1997a, b) said NO to both questions. By proposing that mechanical balance should be held between electric and gravitational forces on the whole crust, and not only on a single nucleus as modelled by former authors (AFO), they claimed that at a density ($\sim \rho_{\rm drip}/5$) still far lower than the neutron drip density, the crust would begin to break down.

AFO proposed a model to describe the gap between the SQM core and the nuclear crust within the framework of the Thomas-Fermi model. The electric field should be described with the following Poisson's equation:

$$\frac{d^2V}{dz^2} = \begin{cases} 4\alpha(V^3 - V_{\rm q}^3)/[3\pi(\hbar c)^2], & z \le 0, \\ 4\alpha V^3/[3\pi(\hbar c)^2], & 0 < z \le z_{\rm G}, \\ 4\alpha(V^3 - V_{\rm c}^3)/[3\pi(\hbar c)^2], & z_{\rm G} < z, \end{cases} \qquad (1)$$

where z is a space coordinate measuring the height above the quark surface, α is the fine-structure constant, $V_{\rm q}^3/3\pi^2\hbar^3$ is the quark charge density inside the quark matter, V is the electrostatic potential of electrons, $V_{\rm c}$ is the electron Fermi momentum deep in the crust, which represents the positive charge density of the ions within the crust, and $z_{\rm G}$ is the width of the gap between the SQM surface and the base of the crust. In fact, $V_{\rm q}$ and $V_{\rm c}$ are the boundary values of the above equation: $V \rightarrow V_{\rm q}$ as $z \rightarrow -\infty$, $V \rightarrow V_{\rm c}$ as $z \rightarrow +\infty$. Meaningful solutions to the above equations exist only if $V_{\rm c} < V_{\rm q}$. Actually in a core-crust system, V_q turns out to be the electron electrostatic potential at R_m. Here R_m represents the maximum radius below which electrical charge neutrality is locally satisfied in the SQM core. Certainly R_m is smaller than R, the radius of the core.

Since the electron chemical potential (we will prove that it equals V later) at which neutron drip occurs is ∼26 MeV (Baym, Pethick & Sutherland 1971), to keep a crust at a density of $\rho_{\rm drip}$ suspended, the electrostatic potential of the electrons near the edge of the SQM core, V_q, must at least be larger than 26 MeV. In this work, we calculated V_q for static SQM cores at zero temperature. Because of the uncertainties inherent in the critical QCD-related parameters, the s quark mass, m_s, strong interaction coupling constant, α_c, and bag constant, B, our calculations were actually carried out for a certain region in the space of those three parameters. We found that for parameters within rather wide ranges, it is not possible to support a crust at a density leading to neutron drip. Especially, for the conventional choice of the parameters, i.e., m_s=200 MeV, α_c=0.3, and $B^{1/4}$=145 MeV, our calculations indicate $V_q \simeq$20 MeV, well below 26 MeV.

It is interesting to mention that the properties of charm-quark stars (viz. strange-quark stars with an additional charm-quark population) have been studied by Kettner et al. (1995). But unfortunately, they found charm-quark stars are unstable against radial oscillations, i.e., no such stars can exist in nature. So our work will only pivot around strange-quark stars.

3 ELECTROSTATIC POTENTIAL OF ELECTRONS IN SQM CORES

3.1 Governing Equations

The property of SQM is generally described using the phenomenological MIT bag model (Chodos et al. 1974), which simplifies the dynamics of confinement by introducing an approximation that the quarks are separated from the vacuum by a phase boundary and the region in which quarks live is endowed with a constant universal energy density B. Since the description of SQM has been introduced elsewhere (Farhi & Jaffe 1984; Kettner et al. 1995), we will go straight to the governing equations and describe the procedure of our calculations. Our goal is to determine the electrostatic potential of electrons at R_m, i.e., V_q.

We assume that charge neutrality is locally satisfied. This is popularly held to be true: Due to the rearrangement of electron charge inside and outside of the surface of an SQM core, the redundant positive charge of the quarks will be balanced locally by e^- only up to radial distances $r \leq R_m$. However, R_m in fact is only minutely smaller (several hundred fermis, see AFO; Kettner et al. 1995) than R, the core's radius.

Basing upon the realization that a core-crust system's temperature is generally much smaller than the typical chemical potentials of the constituents (u, d, s, e^-), we further assume the core is at zero temperature. And we include first-order α_c effects in our calculations.

The thermodynamic potentials (per unit volume) as functions of the chemical potentials of the constituents read (Farhi & Jaffe 1984):

$$\Omega_f(\mu_f) \;=\; -\frac{{\mu_f}^4}{4\pi^2(\hbar c)^3}\Big(1-\frac{2\alpha_c}{\pi}\Big), \quad f=u,d, \tag{2}$$

$$\begin{aligned}\Omega_s(\mu_s) &= -\frac{1}{4\pi^2(\hbar c)^3}\{\mu_s({\mu_s}^2-{m_s}^2c^4)^{1/2}({\mu_s}^2-\frac{5}{2}{m_s}^2c^4)+\frac{3}{2}{m_s}^4c^8\ln(\frac{\mu_s+({\mu_s}^2-{m_s}^2c^4)^{1/2}}{m_sc^2}) \\ &\quad -\frac{2\alpha_c}{\pi}[3(\mu_s({\mu_s}^2-{m_s}^2c^4)^{1/2}-{m_s}^2c^4\ln(\frac{\mu_s+({\mu_s}^2-{m_s}^2c^4)^{1/2}}{m_sc^2}))^2-2({\mu_s}^2-{m_s}^2c^4)^2 \\ &\quad -3{m_s}^4c^8\ln^2(\frac{m_sc^2}{\mu_s})+6\ln(\frac{\rho_R}{\mu_s})(\mu_s{m_s}^2c^4({\mu_s}^2-{m_s}^2c^4)^{1/2} \\ &\quad -{m_s}^4c^8\ln(\frac{\mu_s+({\mu_s}^2-{m_s}^2c^4)^{1/2}}{m_sc^2}))]\}\,, \qquad (3)\\ \Omega_e(\mu_e) &= -\frac{{\mu_e}^4}{12\pi^2(\hbar c)^3}\,. \qquad (4)\end{aligned}$$

The renormalization point for the SQM, ρ_R, which appears in Eq. (3), is chosen to be 313 MeV in this work, because of the reasons pointed out by AFO. The number densities for every constituent can be expressed in terms of μ_i $(i = u, d, s, e)$ from

$$n_i(\mu_i) = -\frac{\partial\Omega_i}{\partial\mu_i}\,, \qquad (5)$$

and charge neutrality requires

$$\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e = 0\,. \qquad (6)$$

In fact, Eq. (6) could be rendered into an equation of μ_i $(i = u, d, s, e^-)$ if we substitute Eqs. (2)–(5) into it. Later in this section, we will demonstrate that all μ_i at a given energy density can be determined when combined with the chemical equilibrium conditions and the energy density equation, and we will point out that V_q is equal to μ_e.

Chemical equilibrium between the three quark flavors and the electrons is maintained by weak interactions (i.e., β-stable SQM)

$$d \rightarrow u + e + \overline{\nu}_e, \qquad (7)$$

$$u + e \rightarrow d + \nu_e, \qquad (8)$$

$$s \rightarrow u + e + \overline{\nu}_e, \qquad (9)$$

$$u + e \rightarrow s + \nu_e\,, \qquad (10)$$

and

$$s + u \leftrightarrow d + u\,. \qquad (11)$$

The loss of neutrinos by the star implies that their chemical potential is equal to zero (we ignore the effect due to the finite mass of neutrinos). Hence, at equilibrium the chemical potentials should obey:

$$\mu_d = \mu_s \equiv \mu\,, \qquad (12)$$

$$\mu_u + \mu_e = \mu\,. \qquad (13)$$

Combined with the condition of charge neutrality (Eq. (6)), these equations leave us with only one independent chemical potential, which we have denoted by μ (see Eq. (12)). Therefore, we still need one equation to close the system.

The total energy density ρ is given by

$$\rho = \sum(\Omega_i(\mu_i) + \mu_i n_i(\mu_i)) + B, \quad i = u, d, s, e^-. \qquad (14)$$

Since all μ_i could be expressed in terms of μ, solving Eq. (14) can determine μ if given the value of ρ, and hence, μ_i.

When the Thomas-Fermi model is invoked to describe the electrons associated with the quarks, we can easily arrive at the conclusion that $V = \mu_e$, i.e., the electrostatic potential is equal to the chemical potential for the electrons (see, e.g., AFO; Kettner et al. 1995). The derivation is as follows. The number density of electrons is given by the local Fermi momentum P_e,

$$n_e = {P_e}^3/(3\pi^2\hbar^3). \tag{15}$$

On the other hand,

$$n_e = -\partial\Omega_e/\partial\mu_e = {\mu_e}^3/(3\pi^2\hbar^3 c^3). \tag{16}$$

From the above two equations, we obtain $\mu_e = P_e c$. Since the electrons are confined within a sphere of infinite radius, their total energy $-V + Pc$ should obey, $-V + Pc \le -V(\infty) = 0$, i.e., $-V + P_e c = 0$, and so,

$$V = P_e c = \mu_e. \tag{17}$$

As μ_e at R_m can be calculated following the procedure stated above, V_q is readily found.

3.2 Numerical Results

Since μ_e decreases with the density (Kettner et al. 1995), which reflects the fact that less electrons are needed in denser SQM, the electrostatic potential of electrons increases monotonically from the center toward the surface of the strange star. Because when $r < R_m$ the electrical charge neutrality is locally satisfied, only the electron electrostatic potential at R_m, corresponding to V_q in Eq. (1), is responsible for supporting the nuclear crust. Owing to the equation of state $P = (\rho - 4B)/3$ (Witten 1984)[1], the energy density at the surface of SQM cores is universally equal to $4B$, independent of the star's mass, or in other words, of the central density. Hence, although in our calculations we fixed the energy density at $4B$, our results will stand for the complete equilibrium sequences of compact SQM core-crust systems as determined by Glendenning, Kettner & Weber (1995).

There exist large uncertainties in the three QCD-related parameters: the s quark mass, m_s, the strong interaction coupling constant, α_c, and the bag constant, B. Our calculations are actually carried out over a range in the space of those parameters. Their exact values are unknown but are probably constrained within: 50 MeV $\le m_s \le$ 340 MeV, $0 \le \alpha_c \le 0.6$, and 135 MeV $\le B^{1/4} \le$ 165 MeV.

The V_q contour diagrams in the $\alpha_c - m_s$ plane for $B^{1/4}$= 135, 145, 155, 165 MeV are respectively presented in Fig. 1(a)–(d). The solid and dashed curves refer to V_q=26 MeV (the electron chemical potential at which neutron drip occurs) and V_q=0 MeV, respectively. The gray regions show where the energy per baryon of the SQM, μ_n ($\mu_n \equiv \mu_u + \mu_d + \mu_s$), exceeds the lowest energy per baryon found in nuclei, which is 930 MeV for ^{56}Fe. The black regions are regions of no physical solutions.

[1] Although this EOS is derived in the limit $m_s \to 0, \alpha_c \to 0$, for intermediate values of m_s this equation differs by less than 4% from the full expression (AFO).

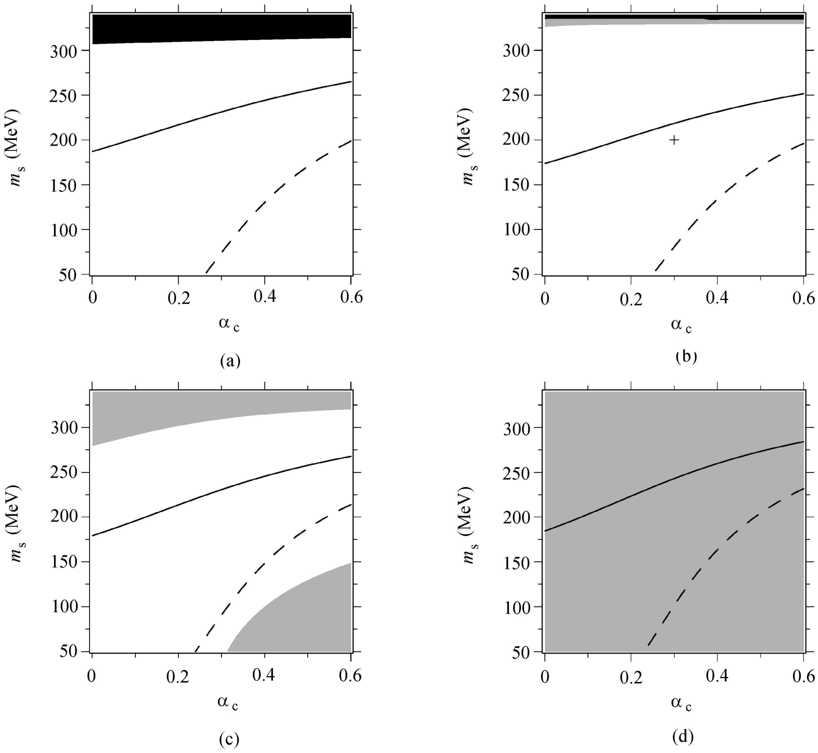

Fig. 1 Contours of fixed V_q in the $\alpha_c - m_s$ plane for $B^{1/4} = 135, 145, 155, 165$ MeV. The cross near the center of (b) marks the position of conventional choice of QCD parameters, i.e., m_s=200 MeV, α_c=0.3, and $B^{1/4}$=145 MeV. The solid and dashed curves refer to V_q=26 MeV (the electron chemical potential at which neutron drip occurs) and V_q=0 MeV, respectively. The gray regions show where the energy per baryon of the SQM exceeds the lowest energy per baryon found in nuclei, which is 930 MeV for iron. The black regions are regions of no physical solutions. Obviously, for QCD parameters within rather wide ranges (including the conventional choice), SQM cores are incapable of supporting crusts at neutron drip density (see text for details). These results are independent of the stellar mass, or in other words, of the central density, because in the MIT bag model the energy density at zero pressure (corresponding to the surface of SQM cores) is universally equal to $4B$.

To support a crust at a density leading to neutron drip, V_q must, at least, be larger than 26 MeV (in order to establish an outwardly directed field, or in other words, to obtain meaningful solutions to Eq. (1)). So, only for parameters in the regions above the solid curves are such crusts possible, and, of course, they should avoid the gray and black regions. Conventionally, the three parameters are chosen to be m_s=200 MeV, α_c=0.3, and $B^{1/4}$=145 MeV. The cross in Fig. 1(b) represents this set. Obviously, it lies outside the permitted region, and actually V_q equals roughly 20 MeV. We underline that, here, $V_q < 26\,\mathrm{MeV}$.

Figure 1 clearly shows that for QCD parameters within rather wide ranges, owing to the insufficient electrostatic potential of electrons, an SQM core is not capable of carrying a suspended crust at ρ_{drip}.

Another factor we should take into account is the gap width, z_G. Note that the lattice spacing in the crust is ∼200 fm, on the same order of z_G, while the model (i.e., Eq. (1))

assumed a smooth distribution of the ionic charges in the crust. In order to make their analysis self-consistent, AFO assumed that the width of the gap should be larger than 200 fm. As z_G is entirely determined by V_c and V_q in AFO's model, we thus can also determine V_q when the values of z_G and V_c are known. For z_G= 200 fm and V_c= 26 MeV (viz. a neutron-drip crust), V_q was found to be ∼125 MeV after solving Eq. (1) with the associated boundary conditions (i.e., $V \to V_{\rm q}$ as $z \to -\infty$, $V \to V_{\rm c}$ as $z \to +\infty$, and $V, dV/dz$ both continuous at $z = 0$ and at $z = z_G$) as well as the approximation $dV/dz = 0$ for $z \geq z_G$ (viz. the gravitation forces are neglected[2]). Therefore, if a gap width larger than 200 fm is really a necessary qualification for the stability of a core-crust system, the assumed presence of crusts at neutron drip density would have been completely ruled out, since no QCD parameters can make V_q so high, at least within the ranges in which our calculations were done.

Although our calculations were limited to SQM at zero temperature, we can state unambiguously that a finite temperature can only make such dense crusts' situation even worse. According to Kettner et al. (1995)'s calculations (see fig. 12 in their paper), V_q decreases with temperature of the SQM. It thus becomes even more unlikely that a neutron-drip crust can exist above strange stars at higher temperatures.

4 CONCLUSIONS

We have studied the electrostatic potential of electrons near the surface of SQM cores for a region in the parameter space of m_s, α_c and B, to see whether crusts at neutron drip density can exist on strange stars. Our numerical results indicate:

1. For QCD parameters within rather large ranges (Fig. 1), the electrons in an SQM core are incapable of establishing an outwardly directed electric field to carry a suspended neutron-drip crust.

2. If it is a sound criterion that the gap width z_G should be larger than 200 fm, then the possibility of neutron-drip crust on strange stars is completely ruled out by our calculations.

Therefore, our results argue against the presence of such crusts in nature.

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[2] Though this approximation has been criticized by Huang & Lu (1997), it is accurate enough for our purpose here.

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