Chin. J. Astron. Astrophys. Vol. 3 (2003), No. 5, 410–414
(http://www.chjaa.org or http://chjaa.bao.ac.cn)

Chinese Journal of
Astronomy and
Astrophysics

Relativistic Corrections to the Thermal Sunyaev-Zel'dovich Power Spectrum *

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Received 2003 March 12; accepted 2003 April 7

Abstract We present a quantitative estimate of the relativistic corrections to the thermal SZ power spectrum produced by the energetic electrons in massive clusters. The corrections are well within 10% for current experiments with working frequencies below $\nu < 100\,\text{GHz}$, but become non-negligible at high frequencies $\nu > 350\,\text{GHz}$. Moreover, the corrections appear to be slightly smaller at higher ℓ or smaller angular scales. We conclude that there is no need to include the relativistic corrections in the theoretical study of the SZ power spectrum especially at low frequencies unless the SZ power spectrum is used for precision cosmology.

Key words: cosmic microwave background — cosmology: theory — galaxies: clusters: general

1 INTRODUCTION

As the largest gravitationally bound objects in the universe, clusters of galaxies serve as a fountain of hot plasma with temperature of $10^7 - 10^8$ K. This arises primarily from gravitationally driven shocks and compression when baryons fall into the gravitational potential well of clusters dominated by dark matter. The energetic electrons contained in clusters will interact the passing cosmic microwave background (CMB) photons through the inverse Compton Scattering known as the thermal Sunyaev-Zel'dovich (SZ) effect, giving rise to a subtle change in the CMB spectrum (Sunyaev & Zel'dovich 1972). Direct SZ detections of known clusters at high signal-to-noise are now routine (e.g. Carlstrom et al. 2000) and several dedicated interferometric arrays for non-targeted SZ surveys are under construction (e.g. Kneissl 2000; Holder et al. 2000; Udomprasert, Mason & Readhead 2000; Fan & Chiueh 2001). On the other hand, one is able to measure the weak thermal SZ signals using a statistical approach without resolving individual clusters, namely, the SZ power spectrum. Indeed, the first statistical signature of CMB anisotropy resulting from the thermal SZ effect of clusters has been reported recently (Dawson et al. 2002; Bond et al. 2002), and more data at arcminute scales will become available in the next few years.

* Supported by the National Natural Science Foundation of China.

Nevertheless, it has been noted that the original thermal SZ effect was derived in the non-relativistic approximation. This holds for low-mass clusters but may introduce a systematic error in the predictions of both SZ counts and power spectrum of massive clusters of $kT > 10\,\mathrm{keV}$. Many efforts have thus been made in recent years towards the inclusion of the relativistic correction in the study of the thermal SZ effect based on the Kompaneets equation (e.g. Rephaeli 1995; Challinor & Lasenby 1998; Itoh, Kohyama & Nozawa 1998; Nozawa et al. 2000; Diego, Hansen & Silk 2002). In particular, the relativistic corrections depend critically on the observing frequencies. It is desirable to understand what extent the relativistic corrections to the thermal SZ effect of hot clusters would be for ongoing and future high-frequency experiments such as *WMAP* and *Planck*. In a recent investigation, Diego et al. (2002) have explored the impact of the relativistic corrections on the *Planck* measurements of the kinematic SZ effect of clusters, the SZ cluster counts and the CMB power spectrum. It is concluded that the CMB power spectrum down to $\ell \approx 2000$ observed at the Planck channel 353 GHz is not very much affected by the thermal SZ effect when the relativistic corrections are taken into account. In this paper we concentrate on the relativistic correction to the thermal SZ power spectrum at higher multipoles, having been motivated by the recent discovery of the excess power of CMB power spectrum relative to primary CMB component at $\ell \approx 2000 - 10^4$ (Dawson et al. 2002; Bond et al. 2002). We will illustrate the relativistic corrections at various frequencies up to 900 GHz regardless of dust contamination. Throughout this paper we adopt a flat cosmological model (Λ CDM) of $\Omega_M = 0.35$ and $\Omega_\Lambda = 0.65$, and assume a baryon density $\Omega_b h^2 = 0.02$, the present dimensionless Hubble constant $h = 0.65$, and the normalization parameter $\sigma_8 = 0.9$.

2 FORMALISM

2.1 Thermal SZ Power Spectrum

Following the conventional treatment (e.g. Cole & Kaiser 1986), we separate the angular power spectrum of the SZ sky into a Poisson term C_ℓ^P and a clustering term C_ℓ^C:

$$C_\ell^P = \int dz \frac{dV}{dz d\Omega} \int dM \frac{d^2N(M,z)}{dMdV} |f(\nu, M) y_l(M,z)|^2, \tag{1}$$

and

$$C_\ell^C = \int dz \frac{dV}{dz d\Omega} P(\ell/D_0, z) \left[\int dM \frac{d^2N(M,z)}{dMdV} b(M,z) f(\nu, M) y_\ell(M,z) \right]^2, \tag{2}$$

where D_0 is the comoving distance to the halo of mass M at z, and y_ℓ is the Fourier transforms of the Compton y-parameter, $b(M,z)$ is the bias parameter, for which we use the analytic prescription of Mo & White (1996), and $d^2M/dMdV$ denotes the comoving number density of dark halos.

The spectral dependence is represented by $f(\nu, M)$. In the non-relativistic approximation, $f(\nu, M)$ has the well-known analytic form

$$f(\nu, M) = x \coth \frac{x}{2} - 4, \tag{3}$$

where $x = h_\mathrm{p}\nu/k_B T_\mathrm{CMB}$, and $T_\mathrm{CMB} = 2.728\,\mathrm{K}$ is the temperature of CMB. Many analytic fitting formulae of $f(\nu, M)$ have been proposed depending on where the relativistic corrections are included. In this study we adopt the analytic expression up to $O(\theta_e^5)$ given by Itoh et al. (1998),

$$f(\nu, M) = \sum_{i=0}^{i=4} Y_i(x)\theta_e^i, \tag{4}$$

where $\theta_e^i = k_B T/m_e c^2$.

2.2 Dark Halos

We take the universal density profile suggested by numerical simulations (Navarro, Frenk & White 1997; NFW)

$$\rho_{\rm DM}(r) = \frac{\delta_{\rm ch}\rho_{\rm crit}}{(r/r_{\rm s})(1+r/r_{\rm s})^2}\,, \tag{5}$$

where $\delta_{\rm ch}$ and $r_{\rm s}$ are the characteristic density and length of the halo, respectively, and $\rho_{\rm crit}$ is the critical density of the universe. For a given halo of mass M at redshift z, we can fix the two free parameters through an empirical fitting formula (Bullock et al. 2001)

$$c = \frac{10}{1+z}\left(\frac{M}{2.1\times 10^{13} M_\odot}\right)^{-0.14}, \tag{6}$$

in which $c = r_{\rm vir}/r_{\rm s}$ reflects the concentration of the halo.

For the spatial distribution and cosmic evolution of dark halos we adopt the Press-Schechter (1974) mass function

$$\frac{d^2N}{dMdV} = -\sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{M}\frac{\delta_{\rm c}(z)}{\sigma^2(M)}\frac{d\sigma(M)}{dM}\exp\left(-\frac{\delta_c^2(z)}{2\sigma^2(M)}\right), \tag{7}$$

in which $\delta_c \approx 1.686$ is the linear collapsing over-density, and $\sigma^2(M)$ is the variance of the mass density fluctuation in sphere of mass M

$$\sigma^2(M) = \frac{1}{2\pi^2}\int_0^\infty k^2 P(k)|W(kR)|^2 dk, \tag{8}$$

and $W(x) = 3(\sin x - x\cos x)/x^3$ is the Fourier representation of the top-hat window function. The matter power spectrum, $P(k) \propto k^n T^2(k)$, is normalized by the rms fluctuation on an $8h^{-1}$ Mpc scale, σ_8, and we take the transfer function $T(k)$ from an adiabatic CDM model given by Bardeen et al. (1986) for the Harrison-Zel'dovich case $n = 1$.

2.3 Gaseous Halos

The thermal SZ effect is rather insensitive to the underlying nongravitational process of the hot gas in very massive halos, although preheating, radiative cooling or energy feedback may play a prominent role in the cosmic evolution of low-mass gaseous halos. Consequently, we may simply take the gas-traces-mass assumption and admit the universality of the baryonic (gas) mass fraction f_b for the most massive halos like clusters. Under these assumptions, the electron number density profile reads

$$n_e = \frac{f_b}{\mu_e m_p}\rho_{\rm DM}, \tag{9}$$

in which $\mu_e = 1.13$ is the mean electron weight. Furthermore, the gas is assumed to be isothermal with temperature (e.g. Bryan & Norman 1998)

$$k_B T = 1.39\ {\rm keV}\ f_T\left(\frac{M}{10^{15}\ M_\odot}\right)^{2/3}\left(h^2E^2(z)\Delta_{\rm c}\right)^{1/3}, \tag{10}$$

where Δ_c is the overdensity of dark matter with respect to the critical value ρ_{crit}, $E^2 = \Omega_M(1+z)^3 + \Omega_\Lambda$, and the normalization factor is taken to be $f_T = 0.8$. The modification of the predicted SZ effect using a varying temperature profile derived from the equation of hydrostatic equilibrium is only minor.

Finally, given the electron number density and temperature profiles, the thermal SZ effect can be evaluated by

$$\frac{\Delta T(\theta)}{T_{\rm CMB}} = f(x, M)\, y(\theta), \quad (11)$$

$$y(\theta) = \int n_e \sigma_T \left(\frac{k_B T_e}{m_e c^2}\right) d\chi. \quad (12)$$

3 RESULTS

We calculate the SZ power spectra from all halos out to $z = 1000$ for four frequencies ranging from 30–900 GHz. Major contributions to the SZ power spectra arises from rich clusters within redshift $z = 3$. We display in Fig. 1 the results with and without relativistic correlations. To facilitate a quantitative comparison, we list in Table 1 the relative correlations for a set of multipoles ranging from $\ell = 100$ to $\ell = 10^4$. It turns out that $|\Delta C_\ell|/C_\ell$ increases dramatically with observing frequency, and the corrections become non-negligible at frequencies higher than $\nu \approx 400$ GHz, for which dust contamination may preclude effective measurement of the CMB power spectrum for the *Planck* mission. There is also a trend that $|\Delta C_\ell|/C_\ell$ decreases slightly with multipoles. In the range $10^3 < \ell < 10^4$, where the thermal SZ effect is believed to dominate over the primary CMB anisotropies, the typical corrections at low frequency $\nu = 30$ GHz are only 2%–3%, which is indeed negligible. Even at a higher frequency of up to $\nu = 350$ GHz, the corrections at $\ell > 1000$ is less than $\sim 10\%$, consistent with the recent findings of Diego et al. (2002).

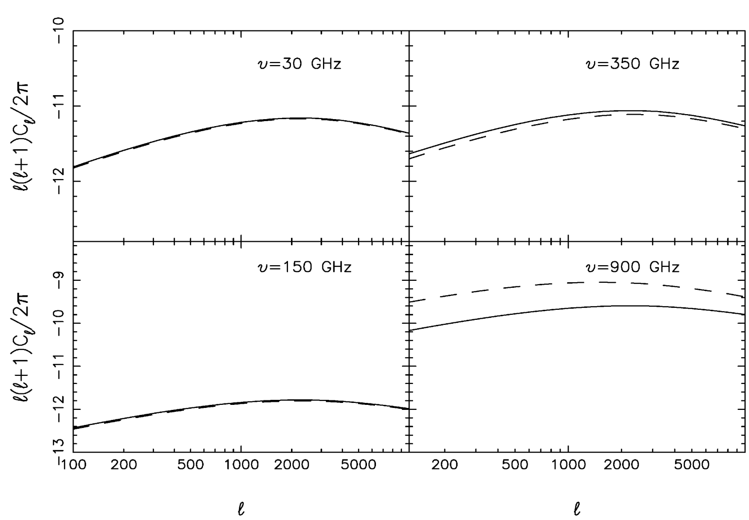

Fig. 1 Angular power spectra of thermal SZ effect with (solid lines) and without (dashed lines) the relativistic correlations at different frequencies.

Table 1 Relative Correlations for a Set of Multipoles Range

ν	30 GHz	150 GHz	350 GHz	900 GHz
ℓ		$\vert\Delta C_\ell\vert/C_\ell$		
100	3.4%	6.3%	14.1%	358%
500	3.2%	5.9%	13.2%	322%
1000	3.0%	5.5%	12.4%	289%
5000	2.3%	4.3%	9.6%	191%
10000	2.0%	3.7%	8.4%	157%

4 CONCLUSIONS

We have calculated the relativistic corrections to the thermal SZ power spectrum produced primarily by massive clusters. These corrections at frequencies up to $\nu = 350\,\text{GHz}$ are well within $\sim$ 10%, which is very small for experiments like *WMAP* and *Planck*. At very high frequencies beyond $\nu \sim 350\,\text{GHz}$, the relativistic corrections become very significant. However, it is unlikely that one can actually use signals at these frequencies for the measurement of SZ power spectrum because of dust contamination. On the other hand, at the new era of precision cosmology, an uncertainty of $\sim$ 10% in the SZ power spectrum should also be taken into account if the SZ power spectrum is used for cosmological purposes.

Acknowledgements This work was supported by the National Science Foundation of China, under Grant No. 10233040, and the Ministry of Science and Technology of China, under Grant No. NKBRSF G19990754.

References

Bardeen J. M., Bond J. R., Kaiser N., Szalay A. S., 1986, ApJ, 304, 15
Baugh C. M., Efstathiou G., 1993, MNRAS, 265, 145
Bryan G. L., Norman M. L., 1998, ApJ, 495, 80
Bullock J. S., et al. 2001, MNRAS, 321, 559
Bond J. R., et al. 2002, ApJ, in press (astro-ph/0205386)
Cooray A., Hu W., Tegmark M., 2000, ApJ, 540, 1
Mo H.-J., White S. D. M., 1996, MNRAS, 315, 356
Navarro J. F., Frenk C. S., White S. D. M., 1995, MNRAS, 275, 720
Press W. H., Schechter P., 1974, ApJ, 187, 425
Seljak U., Burwell J., Pen U.-L., 2000, Phys. Rev. D, 63, 063001
Soltan A. M., Freyberg M., Hasinger G., 2002, A&A, submitted (astro-ph/0209262)
Song Y.-S, Cooray A., Knox L., Zaldarriaga M., 2002, ApJ, in press (astro-ph/0209001)