Chin. J. Astron. Astrophys. Vol. 3 (2003), No. 4, 341–346
(http://www.chjaa.org or http://chjaa.bao.ac.cn)

Chinese Journal of
Astronomy and
Astrophysics

A Check on the Cardassian Expansion Model with Type-Ia Supernovae Data *

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Received 2003 February 25; accepted 2003 June 9

Abstract We use the magnitude-redshift relation for the type Ia supernova data compiled by Riess et al. to analyze the Cardassian expansion scenario. This scenario assumes the universe to be flat, matter dominated, and accelerating, but contains no vacuum contribution. The best fitting model parameters are $H_0 = 65.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $n = 0.35$ and $\Omega_m = 0.05$. When the highest redshift supernova, SN 1997ck, is excluded, H_0 remains the same, but n becomes 0.20 and Ω_m, 0.15, and the matter density remains unreasonably low. Our result shows that this particular scenario is strongly disfavoured by the SNeIa data.

Key words: supernovae: general — cosmology: theory — distances and redshifts

1 INTRODUCTION

There are four pillars of the standard Big Bang cosmology: the Hubble expansion, the Cosmic Microwave Background Radiation (CMBR), the primordial Big Bang Nucleosynthesis and the structure formation. In recent years, it seems that all these cornerstones combined to point to the expansion of the universe speeding up rather than slowing down (for a recent review see Peebles & Ratra 2003). The main line of evidence comes from the recent, well-known distance measurements of some distant Type Ia supernovae (Perlmutter et al. 1998, 1999; Riess et al. 1998, 2001; Leibundgut 2001). Considering that all known types of matter with positive pressure generate attractive forces and decelerate the expansion of the universe, the cosmological constant (vacuum energy) is naturally chosen to be the accelerating mechanism. However, it suffers from the difficulty in understanding the observed value in the framework of modern quantum field theory (Weinberg 1989; Carroll et al. 1992) and the "coincidence problem", the problem of explaining the initial conditions necessary to yield the near-coincidence of the densities of the matter and the cosmological constant components today. In this case, a dark energy component with generally negative pressure has been invoked. Examples of dark energy include quintessence (Caldwell et al. 1998; Zlatev et al. 1999; Steinhardt et al. 1999), k-essence (Armendariz-Picon et al. 2000), x-matter (Turner & White 1997; Chiba et al. 1997), Chaplygin gas (Kamenshchik et al. 2001; Bento et al. 2002; Bilić et al. 2002) and the frustrated network

* Supported by the National Natural Science Foundation of China.

of topological defects in which $\omega_x = -\frac{l}{3}$, l being the dimension of the defect (Spergel & Pen 1997). Nevertheless, it is still far from reaching a convincing mechanism with solid basis in particle physics for an accelerating universe.

Very recently, Freese & Lewis (2002) proposed a "Cardassian Expansion Scenario" in which the universe is flat, matter dominated and accelerating, but contains no vacuum contribution. The main point of this scenario is to modify the standard Friedman-Robertson-Walker equation to read

$$H^2 = A\rho + B\rho^n, \tag{1}$$

where $H \equiv \dot{R}/R$ is the Hubble parameter (a function of time), R is the scale factor of the universe, B and n are the two parameters of the Cardassian model, and the energy density ρ contains only ordinary matter and radiation (Freese & Lewis 2002). To be consistent with the usual FRW result, one should take $A = 8\pi G/3$. Several authors have explored the agreement of the Cardassian expansion model with the current available observational data (Zhu & Fujimoto 2002, 2003; Sen & Sen 2003; Wang et al. 2003). Until now, only the Supernova Cosmology Project[1] data (Perlmutter et al. 1998, 1999) have been used. In the present work, we use the magnitude-redshift relation for the type Ia supernova data of the High-z Supernova Search Team[2] compiled by Riess et al. (1998). While the Perlmutter sample contains 18 low redshift supernovae and 42 high redshift supernovae, the Riess sample contains 27 low redshift supernovae and 10 high redshift supernovae. The greater number of low redshift supernovae in the latter data would make the determination of the Hubble constant more accurate (see below for details), while the fewer high redshift supernovae would make the determination of the Cardassian model parameters less accurate. However, because the matching of all observed supernovae in one single statistical sample leads to several technical problems (Mézáros 2002; however for an alternative view, see Wang 2000), it is valuable to cross check the results using the Riess et al. (1998) data. Here we obtain the best fitting model with $H_0 = 65.3\,\mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$, $n = 0.35$, $\Omega_m = 0.05$ and $\chi^2 = 44.00$ for 34 degrees of freedom (d.f.). When the highest redshift supernova, SN 1997ck, is excluded, the value H_0 remains the same, but the other s change to $n = 0.20$, $\Omega_m = 0.15$, and $\chi^2 = 44.13$ on 33 d.f. Our analysis thus re-enforces the previously common conclusion: this scenario is disfavoured because it gives rise to an unreasonably low matter density.

The plan of the paper is as follows. In Sect. 2 we provide a brief summary of the Cardassian expansion scenario relevant to our work. We then proceed to analyze the type Ia supernova data to assess this scenario in Sect. 3. Section 4 contains our conclusions and a discussion.

2 THE CARDASSIAN EXPANSION MODEL

The "ansatz" Eq.(1) of the Cardassian expansion model may arise as a consequence of embedding our observable universe as a (3+1)-dimensional brane in extra dimensions (see Freese & Lewis 2002 for details). Once the new term dominates, it causes the universe to accelerate. Let us only consider the contribution of ordinary matter with $\rho = \rho_0(R/R_0)^{-3}$, which makes the second term scale as $\sim R^{-3n}$. When this term is much larger than the first, then the scale factor of our universe scale as $R \sim t^{\frac{2}{3n}}$. Then, the expansion of the universe will accelerate if $n < 2/3$. It is also interesting to note that the acceleration will be speeding up, constant, or slowing down according as n is less than, equal to, or greater than $1/3$ (see Freese & Lewis

[1] Supernova Cosmology Project: http://www-supernova.lbl.gov

[2] High-z Supernova Search: http://cfa-www.harvard.edu/cfa/oir/Research/supernova/home.html

2002 for details). One can rewrite Eq.(1) as $H^2 = A\ \rho[1 + (\rho/\rho_{z_{\rm eq}})^{n-1}]$, where $\rho_{z_{\rm eq}}$ is the energy density at which the two terms are equal (Freese & Lewis 2002; Sen & Sen 2003). Once the energy density ρ drops below $\rho_{z_{\rm eq}}$ the universe starts to accelerate. Here $\rho_{z_{\rm eq}}$ is given by $\rho_{z_{\rm eq}} = (A/B)^{\frac{1}{n-1}} = \rho_0(1+z_{\rm eq})^3$, $z_{\rm eq}$ is the redshift at which the second term starts dominating over the first term, and ρ_0 is the present matter density of the universe. Here we ignore the radiation component which is not important for the magnitude-redshift relation for supernovae. Evaluating Eq.(1) for today with $A = 8\pi G/3$, we have $H_0^2 = \frac{8\pi G}{3}\rho_0[1 + (1 + z_{\rm eq})^{3(1-n)}]$, conventionally $\rho_0 = \Omega_m \rho_c$, $\rho_c = 3H_0^2/8\pi G$ being the critical density. Now in the Cardassian model, matter alone makes the universe flat, which means that $\rho_0 = \rho_{c,{\rm Cardassian}}$, the critical density of the universe in the Cardassian expansion model (Freese & Lewis 2002),

$$\rho_{c,{\rm Cardassian}} = \frac{3H_0^2}{8\pi G[1 + (1 + z_{\rm eq})^{3(1-n)}]}\,, \tag{2}$$

and the present matter density of the universe is $\Omega_m = [1 + (1 + z_{\rm eq})^{3(1-n)}]^{-1}$. Parametrizing the model as (Ω_m, n), the redshift dependent Hubble parameter $H(z) = H_0[\Omega_m \times (1 + z) + (1 - \Omega_m) \times (1 + z)^{3n}]^{1/2}$. The luminosity distance as a function of the redshift and the model parameters (Ω_m, n) is given by the integral, $D_L(z) = c(1 + z) \int_0^z dz'/H(z\prime)$. For D_L in units of megaparsecs, the theoretical distance modulus is

$$\mu_t = 5 \log D_L + 25\,, \tag{3}$$

and this will be compared with the observational data of Riess et al.(1998) to constrain the model parameters in next section.

3 SUPERNOVA DATA ANALYSIS

Riess et al. (1998) published their 10 distant SNe Ia (SN 1995ao, 1995ap, 1996E, 1996H, 1996I, 1996J, 1996K, 1996R, 1996T, and 1996U), discovered using the CTIO Blanco 4 m telescope with a prime-focus CCD camera as part of a 3-night program in 1995 Oct-Nov and a 6-night program in 1996 Feb-Mar. Combined with 27 previous nearby SNeIa of Hamuy et al. (1996) and Riess et al.(1999), we have a database totaling 37 SNeIa for analysis. Riess et al. (1998) determined the luminosity distances, as well as the K-corrections, of these SNeIa using two methods, the Multi-Color Light Curve Shape (MLCS) method which employs up to 4-colors of SN Ia photometry to yield excellent distance estimates (precision ≈ 0.15 mag), the uncertainty for each object having been estimated with measurements of the reddening by dust for each event (see Appendix of Riess et al. 1998), and a template fitting method in which the maximum light magnitudes and the initial decline rate parameter $\Delta m_{15}(B)$ for a given SN Ia are derived by comparing the goodness-of-fits of the photometric data to a set of 6-template SN Ia light curves selected to cover the full range of observed decline rates. We will use their distance models obtained by the MLCS method. This data set is plotted in Fig. 1. The solid triangles mark the 10 high-redshift SNeIa from (Riess et al. 1998), while the solid circles, the 27 previous nearby SNeIa from Hamuy et al. (1996) and Riess et al. (1999).

A χ^2 minimization method is used to determine the model parameters n and Ω_m. Both n and Ω_m span the interval [0, 1] in step of 0.01.

$$\chi^2(n, \Omega_m) = \sum_i \frac{[\mu_{t,i}(z_i; H_0; n, \Omega_m) - \mu_{0,i}]^2}{\sigma^2_{\mu_{0,i}} + \sigma_v^2}, \tag{4}$$

where σ_v is the dispersion in the galaxy redshift (in units of distance moduli) due to peculiar velocities. This term also includes the uncertainty in galaxy redshift. However, σ_v is much less than the uncertainty of σ_{μ_0} and has not be taken into account in our analysis. The summation is over all of the observational data points. We have calculated this χ^2 statistic for a wide range of the parameters n and Ω_m. We do not consider the unphysical region of the parameter space with $\Omega_m < 0$. Table 1 summarizes our χ^2 results for typical parameter values as well as our best fit values. The best fit, with $H_0 = 65.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $n = 0.35$, $\Omega_m = 0.05$ and $\chi^2 = 44.00$ on 34 degrees of freedom (d.f.), is depicted in Fig. 1 as a solid line. For comparison, three other curves with model parameters n and Ω_m, taken from the table 1 of Freese & Lewis (2002), are also shown.

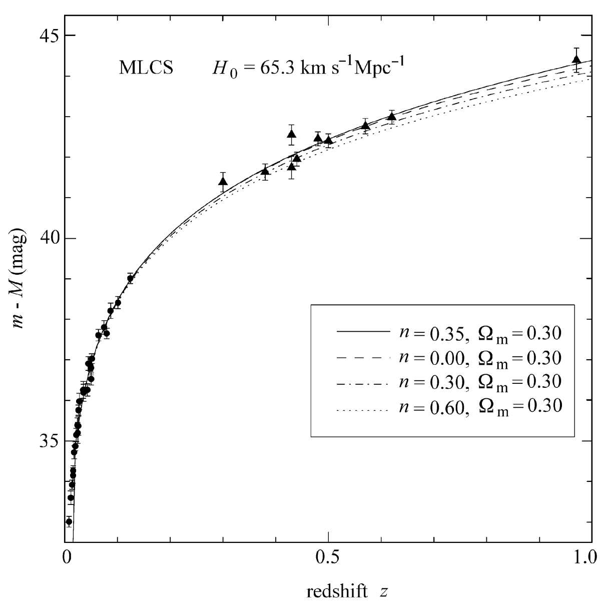

Fig. 1 Hubble diagram for 10 high-redshift SNeIa [solid triangles (Riess et al. 1998)] and 27 low-redshift SNeIa (filled circles) compiled by Hamuy et al. (1996) and Riess et al. (1999). The solid curve corresponds to our best fit, with $H_0 = 65.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $n = 0.35$, $\Omega_m = 0.05$. The values of (n, Ω_m) for the other three curves are taken from the table 1 of Freese & Lewis (2002).

One glance at Figure 1 reveals immediately that there is a big gap between the supernova SN 1997ck with the highest redshift in the sample and the rest, which may lead one to think the possibility of it being an outlier. We therefore reanalyze the sample with SN 1997ck excluded. Now the best fit happens at $H_0 = 65.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $n = 0.20$, $\Omega_m = 0.15$ and $\chi^2 = 44.13$ on 33 d.f. The Hubble constant is exact the same as before, which indicates that the Hubble constant can be determined to a very high precision because of the greater number of low redshift supernovae in the Riess et al. (1998) sample, as stated in Sect. 1. Although the matter density of the universe has gone up to 0.15, it is still unreasonably low.

In Figure 2, we show the 68.3% and 95.4% confidence regions of the fitting results in the (n, Ω_m)-plane for both the whole Riess et al. (1998) sample [Figure 2(a)] and the subsample with SN 1997ck excluded [Figure 2(b)]. As it shows, although our best fits within the framework of Cardassian expansion give a universe with an unreasonably low matter density, one should keep in mind that some Cardassian models with $\Omega_m \sim 0.3$ can also match the data well within 1σ level. The few number of high redshift supernovae (only 10) makes the constraint on the parameters less stringent.

Table 1 Fitting Results for the Cardassian Model from Distant Type Ia Supernova *

N	n	Ω_m	χ^2
37	0.60	0.30	60.73
37	0.50	0.30	55.79
37	0.40	0.30	51.84
37	0.30	0.30	48.83
37	0.20	0.30	46.69
37	0.10	0.30	45.34
37	0.00	0.30	44.72
37			
37	0.35	0.05	44.00
36	0.20	0.15	44.13

* Data compiled by Riess et al. (1998) with and without SN 1997ck. The best fitting result is shown in the last row. The other values of $(n,\ \Omega_m)$ are taken from table 1 of Freese & Lewis (2002).

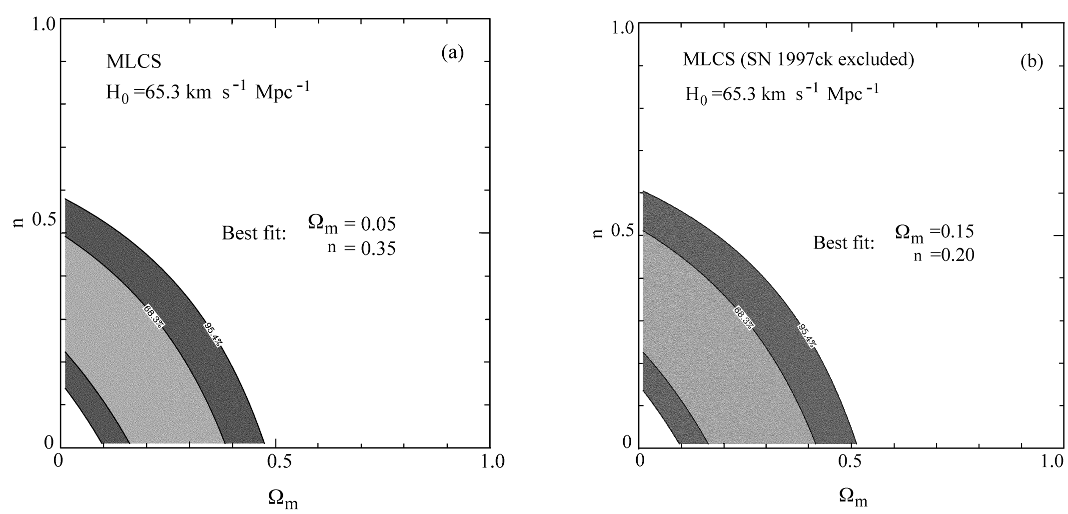

Fig. 2 Confidence region plot for the parameters n and Ω_m of the Cardassian model for the type Ia supernova data compiled by Riess et al. (1998). (a) the result for the whole sample; (b) with SN 1997ck excluded.

4 CONCLUSIONS AND DISCUSSION

In this paper we have considered the cosmological consequences of the Cardassian expansion scenario of Freese & Lewis (2002), which was proposed as an alternative explanation for the current acceleration of the universe. We have shown that this scenario is strongly disfavored by the distant SNeIa data, re-enforcing the previous conclusions (Zhu & Fujimoto 2002, 2003; Sen & Sen 2003; Wang et al. 2003). In order to be consistent with the supernova data, one would need a very low matter density $\Omega_m \sim 0.05$, which is inconsistent with other available observational data (see, e.g., Primack 2002; Turner 2002). There seems to be a tendency for a model that excludes the dark energy component to dispel also dark matter (see Avelino &

Martins 2002 for another analysis). However, it is worth keeping in mind that a universe with a low matter density $\Omega_m \sim 0.1$ can also fit the data of Perlmutter et al. (1999) surprisingly well (Mészáros 2002).

Riess et al. (1998) and Perlmutter et al. (1999) have carefully studied the main uncertainties in the cosmological parameter extraction from high-redshift type Ia supernovae samples caused by progenitor and metallicity evolution, extinction, sample selection bias, local perturbations in the expansion rate, gravitational lensing and sample contamination. It was found that none of these factors can seriously change the result; and that includes our result that the matter density predicted by the Cardassian scenario is much less than 0.1. Thus, the Cardassian expansion scenario is strongly disfavoured by the result we obtained from the type Ia supernovae data compiled by Riess et al. (1998).

Acknowledgements The author would like to thank Prof. Shuji Deguchi and Prof. Yoshikazu Nakada for their hospitality and help at NAOJ, the anonymous referee for valuable comments and useful suggestions, which improved this work very much. This work was supported by the National Natural Science Foundation of China (NSFC) and the Special Funds for Major State Basic Research Projects of China.

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