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A Study of Neutron Star Structure in Strong Magnetic Fields that includes Anomalous Magnetic Moments *

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Abstract We study the effect of strong magnetic fields on the structure of neutron star. We find that if the interior field is on the same order as the surface field currently observed, then the influences of the field on the star's mass and radius are negligible; if the field is as large as that estimated from the scalar virial theorem, then considerable effects will be induced. The maximum mass of the star will be increased substantially while the central density is greatly reduced. The radius of a magnetic star can be larger by about 10% $\sim$ 20% than a nonmagnetic star of the same mass.

Key words: stars: neutron stars — stars: magnetic fields — equation of state

1 INTRODUCTION

It is well known that the structure of neutron stars is mainly determined by the nuclear equation of state (EOS) based on the strong interactions. Recent observations have indicated that large magnetic fields are present at the surface of the neutron stars (Michel 1991; Rothschild, Kulkarni & Lingenfelter 1994; Kouveliotou et al. 1998; Woods et al. 1999). The dipole fields inferred from the spin-down rates can be up to 10^{15} G [for a recent review, see Reisenegger (2001)], but the strength of the magnetic field in the interior of the star remains unknown. According to the scalar virial theorem (Shapiro & Teukolsky 1983; Lai & Shapiro 1991) the maximum interior field strength could be as large as $\sim 10^{18}$ G for a star with $R \approx 10\,\mathrm{km}$ and $M \approx 1.4\ M_{\odot}$. Even larger fields can be expected in the core of the stars. Such intense fields will certainly play a role when one evaluates the EOS of neutron star matter and consequently may cause considerable effects on the structure of the star.

The same problem was addressed on the magnetic white dwarfs where a surface magnetic field on the order of $10^5 \sim 10^9$ G and an interior field of $10^9 \sim 10^{13}$ G were estimated. Both the earlier work of Ostriker & Hartwick (1968) and recent calculations of Suh & Mathews (2000)

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predicted an increase of the white dwarf radius in the presence of internal magnetic fields. In the case of a white-dwarf binary system, e.g., LB11146 (PG 0945+245), assumed to consist of one nonmagnetic and one magnetic star of an equal mass of $M = 0.9M_\odot$, an interior field of $B \approx 0.5B_c^e$ (here $B_c^e = 4.414\times10^{13}$ G is the electron critical field) in the latter would cause the radius of the magnetic star to be larger by about 10% than that of the nonmagnetic star (Suh & Mathews 2000).

Theoretical investigations of ideal noninteracting neutron-proton-electron (n-p-e) gas and interacting pure neutron matter under large magnetic fields have been carried out by Suh & Mathews (2001) and Vshivtsev & Serebryakova (1994). Recently, Brückner-Hartree-Fock calculations of spin polarized asymmetric nuclear matter have been performed (Vidana & Bombaci 2002). Based on the meson field theory several authors have incorporated strong magnetic fields into the equation of state of a dense n-p-e system under the beta equilibrium and charge neutrality conditions (Chakrabarty, Bandyopadhyay & Pal 1997, Paper I; Broderick, Prakash & Lattimer 2000, Paper II). Evident changes in the EOS have been found. It was demonstrated that the nucleon anomalous magnetic moments (AMM) play a significant role (Paper II) which may overwhelm the softening of the EOS caused by Landau quantization (Paper I) to one of stiffening. In the meantime, a dramatic increase of the proton fraction with increasing magnetic field was exhibited. However, our previous calculations (Mao et al. 2003) showed that with the AMM of nucleons and electrons taken into account the proton fraction was found to never exceed the field free case. Extremely strong fields would lead to a pure neutron matter rather than a proton-rich matter. In this work we will study the effects of large magnetic fields on the neutron star structure. We will examine the EOS of a dense n-p-e system with the AMM of both nucleons and electrons taken into account. One may argue that the electron self-energy may not change substantially in magnetic fields when high-order terms are taken into account. However, the effect of a systematic incorporation of high-order contributions beyond the AMM term is not yet clear, and this will be the subject of our forthcoming works. Here the effects of magnetic fields on different particles within the considered system are treated on an equal footing. The developed EOS will then be used to investigate the structure of neutron stars with strong internal fields. This paper is organized as follows: a relativistic mean-field theory approach for dense neutron star matter is described in Sect. 2. In Sect. 3 we present our numerical results. A brief summary and outlook will finally be given in Sect. 4.

2 THEORETICAL FRAMEWORK

We consider a neutron-star matter consisting of neutrons, protons and electrons interacting through the exchange of σ, ω and ρ mesons in the presence of a uniform magnetic field B along the z axis. The Lagrangian density can be written as (Serot & Walecka 1986)

$$\begin{aligned}\mathcal{L} \;=\; & \bar{\psi}[i\gamma_\mu\partial^\mu - e\frac{1+\tau_0}{2}\gamma_\mu A^\mu - \frac{1}{4}\kappa_b\mu_N\sigma_{\mu\nu}F^{\mu\nu} - M_N + \mathrm{g}_\sigma\sigma - \mathrm{g}_\omega\gamma_\mu\omega^\mu - \frac{1}{2}\mathrm{g}_\rho\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{R}^\mu]\psi \\ & +\bar{\psi}_e[i\gamma_\mu\partial^\mu - e\gamma_\mu A^\mu - \frac{1}{4}\kappa_e\mu_B\sigma_{\mu\nu}F^{\mu\nu} - m_e]\psi_e + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) \\ & -\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\boldsymbol{R}_{\mu\nu}\cdot\boldsymbol{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{R}_\mu\cdot\boldsymbol{R}^\mu, \end{aligned} \tag{1}$$

and $U(\sigma)$ is the self-interaction part of the scalar field (Boguta & Bodmer 1977)

$$U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}b(\mathrm{g}_\sigma\sigma)^3 + \frac{1}{4}c(\mathrm{g}_\sigma\sigma)^4\,. \tag{2}$$

In the above expressions ψ and ψ_e are the Dirac spinors of the nucleon and electron; σ, ω_μ, $\boldsymbol{R}_\mu$ represent the scalar meson, vector meson and vector-isovector meson field, respectively. $A^\mu \equiv (0, 0, Bx, 0)$ refers to a constant external magnetic field. Here the field tensors for omega, rho and magnetic field are given in terms of their potentials by

$$\omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \tag{3}$$

$$\boldsymbol{R}_{\mu\nu} = \partial_\mu \boldsymbol{R}_\nu - \partial_\nu \boldsymbol{R}_\mu, \tag{4}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{5}$$

and $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$, $\boldsymbol{\tau}$ is the isospin operator of the nucleon, τ_0 is its third component. M_N and m_e are the free nucleon mass and electron mass and m_σ, m_ω, m_ρ are the masses of the σ-, ω- and ρ-meson, respectively, μ_N and μ_B are the nuclear magneton of nucleons and Bohr magneton of electrons; $\kappa_p = 3.5856$, $\kappa_n = -3.8263$ and $\kappa_e = \alpha/\pi$ are the coefficients of the AMM for protons, neutrons and electrons (Greiner & Reinhardt 1994), respectively. The third set of parameters presented by Glendenning & Moszkowski (1991) is used as the nucleon coupling strengths. It gives $\mathrm{g}_\sigma = 8.7818$, $\mathrm{g}_\omega = 8.7116$, $\mathrm{g}_\rho = 8.4635$, $b\mathrm{g}_\sigma^3 = 27.9060$, $c\mathrm{g}_\sigma^4 = -14.3989$. This yields a binding energy $B/A = -16.3\,\mathrm{MeV}$, saturation density $\rho_0 = 0.153\ \mathrm{fm}^{-3}$ and bulk symmetry energy $a_{\mathrm{sym}} = 32.5\,\mathrm{MeV}$.

Let us first consider the nucleons. The Dirac equation for the nucleons in a uniform magnetic field can be written as

$$\left[i\gamma_\mu \partial^\mu - e\frac{1+\tau_0}{2}\gamma_\mu A^\mu - \frac{1}{4}\kappa_b \mu_N \sigma_{\mu\nu} F^{\mu\nu} - M_N + \mathrm{g}_\sigma \sigma - \mathrm{g}_\omega \gamma_\mu \omega^\mu - \frac{1}{2}\mathrm{g}_\rho \gamma_\mu \tau_0 R_0^\mu \right] \psi = 0. \tag{6}$$

The general solutions of the above equation in free space, i.e., in the absence of meson fields, are discussed in Appendix A. The corresponding solutions in neutron-star matter can be directly written out by replacing the free-case quantities with the effective ones. The positive energy of the protons in the Fermi sea $\left(E^p_{\nu,S}\right)_+$ and the negative energy of the protons in the Dirac sea $\left(E^p_{\nu,S}\right)_-$ read as

$$\left(E^p_{\nu,S}\right)_+ = \left\{ \left[\left(\sqrt{m^{*2} + 2eB\nu} + S\Delta \right)^2 + p_z^2 \right]^{1/2} + \mathrm{g}_\omega \omega_0 + \frac{1}{2}\mathrm{g}_\rho R_{0,0} \right\}, \tag{7}$$

$$\left(E^p_{\nu,S}\right)_- = -\left\{ \left[\left(\sqrt{m^{*2} + 2eB\nu} + S\Delta \right)^2 + p_z^2 \right]^{1/2} - \mathrm{g}_\omega \omega_0 + \frac{1}{2}\mathrm{g}_\rho R_{0,0} \right\}. \tag{8}$$

The positive-energy and negative-energy spectra of the neutrons are

$$(E^n_S)_+ = \left\{ \left[\left(\sqrt{p_x^2 + p_y^2 + m^{*2}} + S\Delta \right)^2 + p_z^2 \right]^{1/2} + \mathrm{g}_\omega \omega_0 - \frac{1}{2}\mathrm{g}_\rho R_{0,0} \right\}, \tag{9}$$

$$(E^n_S)_- = -\left\{ \left[\left(\sqrt{p_x^2 + p_y^2 + m^{*2}} + S\Delta \right)^2 + p_z^2 \right]^{1/2} - \mathrm{g}_\omega \omega_0 - \frac{1}{2}\mathrm{g}_\rho R_{0,0} \right\}. \tag{10}$$

Here $\Delta = -\frac{1}{2}\kappa_b \mu_N B$; $S = \pm 1$ for spin-up and spin-down particles, ν is the quantum number of Landau levels for charged particles (Landau & Lifshitz 1977). The positive energy of the anti-particle is just the negative of the negative energy of the particle, i.e., $\left(\bar{E}^p_{\nu,S}\right)_+ = -\left(E^p_{\nu,S}\right)_-$, $\left(\bar{E}^n_S\right)_+ = -\left(E^n_S\right)_-$ (Mao, Stöcker & Greiner 1999; Mao 2003).

Since neutron stars are cold dense matter, we shall perform numerical calculations at zero temperature. The general chemical equilibrium is realized for valence particles. The chemical potentials of protons and neutrons are defined as

$$\mu_p = \epsilon_f^p + g_\omega \omega_0 + \frac{1}{2} g_\rho R_{0,0}, \tag{11}$$

$$\mu_n = \epsilon_f^n + g_\omega \omega_0 - \frac{1}{2} g_\rho R_{0,0}. \tag{12}$$

They are related to the respective Fermi momenta via the following equations:

$$\left(k_{f,\nu,S}^p\right)^2 = \left(\epsilon_f^p\right)^2 - \left(\sqrt{m^{*2} + 2eB\nu} + S\Delta\right)^2, \tag{13}$$

$$\left(k_{f,S}^n\right)^2 = \left(\epsilon_f^n\right)^2 - (m^* + S\Delta)^2. \tag{14}$$

In the above equations, the effective nucleon mass $m^* = M_N - g_\sigma \sigma$. σ, ω_0 and $R_{0,0}$ are the mean values of the scalar field, the time-like component of the vector field and the time-like isospin 3-component of the vector-isovector field in neutron-star matter, respectively. They are obtained by solving the nonlinear equations of the meson fields,

$$m_\sigma^2 \sigma + b g_\sigma^3 \sigma^2 + c g_\sigma^4 \sigma^3 = g_\sigma \rho_S, \tag{15}$$

$$m_\omega^2 \omega_0 = g_\omega \rho, \tag{16}$$

$$m_\rho^2 R_{0,0} = \frac{1}{2} g_\rho \rho_{0,0}. \tag{17}$$

Here ρ_S, ρ and $\rho_{0,0}$ are the scalar density, the time-like component of the vector density and the time-like isospin 3-component of the vector-isovector density contributed from the valence nucleons, i.e. from the Fermi sea. In principle, there exist additional contributions stemming from the Dirac sea. Here they are neglected according to the *no-sea* approximation since the renormalization of the system under the external magnetic field is a problem to be solved. Thus, the contributed densities are $\rho_S = \rho_S^p + \rho_S^n$, $\rho = \rho_0^p + \rho_0^n$ and $\rho_{0,0} = \rho_0^p - \rho_0^n$, with

$$\rho_S^p = \frac{eBm^*}{2\pi^2} \sum_S \sum_\nu \frac{\sqrt{m^{*2} + 2eB\nu} + S\Delta}{\sqrt{m^{*2} + 2eB\nu}} \ln \left| \frac{k_{f,\nu,S}^p + \epsilon_f^p}{\sqrt{m^{*2} + 2eB\nu} + S\Delta} \right|, \tag{18}$$

$$\rho_S^n = \frac{m^*}{4\pi^2} \sum_S \left[\epsilon_f^n k_{f,S}^n - (m^* + S\Delta)^2 \ln \left| \frac{k_{f,S}^n + \epsilon_f^n}{m^* + S\Delta} \right| \right], \tag{19}$$

$$\rho_0^p = \frac{eB}{2\pi^2} \sum_S \sum_\nu k_{f,\nu,S}^p, \tag{20}$$

$$\rho_0^n = \frac{1}{2\pi^2} \sum_S \left[\frac{1}{3} \left(k_{f,S}^n\right)^3 + \frac{S\Delta}{2} \left((m^* + S\Delta) k_{f,S}^n + \left(\epsilon_f^n\right)^2 (\arcsin \frac{m^* + S\Delta}{\epsilon_f^n} - \frac{\pi}{2}) \right) \right]. \tag{21}$$

The summation of ν runs up to the largest integer for which $\left(k_{f,\nu,S}^p\right)^2$ is positive. For spin-up protons ν starts from 1 while for spin-down protons, 0. It should be pointed out that the so-called spin up and spin down here are just relative notions, since the wave functions are no longer eigenfunctions of 3-component spin operator (see Appendix A), relating mainly to the coupling of the spin to the magnetic field. The contributions of the protons and neutrons to the energy density read as

$$\varepsilon_p = \frac{eB}{4\pi^2}\sum_S\sum_\nu\left[k^p_{f,\nu,S}\epsilon^p_f + \left(\sqrt{m^{*2}+2eB\nu}+S\Delta\right)^2 \ln\left|\frac{k^p_{f,\nu,S}+\epsilon^p_f}{\sqrt{m^{*2}+2eB\nu}+S\Delta}\right|\right], \quad (22)$$

$$\varepsilon_n = \frac{1}{4\pi^2}\sum_S\left\{\frac{1}{2}\left(\epsilon^n_f\right)^3 k^n_{f,S} + \frac{2}{3}S\Delta\left(\epsilon^n_f\right)^3\left[\arcsin\left(\frac{m^*+S\Delta}{\epsilon^n_f}\right)-\frac{\pi}{2}\right]\right.$$
$$\left.+\left[\frac{1}{3}S\Delta - \frac{1}{4}\left(m^*+S\Delta\right)\right]\left[(m^*+S\Delta)k^n_{f,S}\epsilon^n_f + (m^*+S\Delta)^3\ln\left|\frac{k^n_{f,S}+\epsilon^n_f}{m^*+S\Delta}\right|\right]\right\}. \quad (23)$$

In the β-equilibrium system, the electron is assumed to move freely in the strong magnetic field. The wave function of electrons is the same as that of protons in free space with the substitution of the corresponding electron quantities. The energy spectrum of the electrons can be expressed as

$$\left(E^e_{\nu,S}\right)_+ = \left[\left(\sqrt{m_e^2+2eB\nu}+S\Delta\right)^2 + p_z^2\right]^{1/2}, \quad (24)$$

where $\Delta = -\frac{1}{2}\kappa_e\mu_B B$ and $\left(E^e_{\nu,S}\right)_- = -\left(E^e_{\nu,S}\right)_+$. The chemical potential of electrons $\mu_e = \epsilon^e_f$. Its relation to the electron Fermi momentum is

$$\left(k^e_{f,\nu,S}\right)^2 = \left(\epsilon^e_f\right)^2 - \left(\sqrt{m_e^2+2eB\nu}+S\Delta\right)^2. \quad (25)$$

The electron density is defined as

$$\rho^e_0 = \frac{eB}{2\pi^2}\sum_S\sum_\nu k^e_{f,\nu,S}. \quad (26)$$

The contribution of the electrons to the energy density reads as

$$\varepsilon_e = \frac{eB}{4\pi^2}\sum_S\sum_\nu\left[k^e_{f,\nu,S}\epsilon^e_f + \left(\sqrt{m_e^2+2eB\nu}+S\Delta\right)^2 \ln\left|\frac{k^e_{f,\nu,S}+\epsilon^e_f}{\sqrt{m_e^2+2eB\nu}+S\Delta}\right|\right]. \quad (27)$$

Finally, we obtain the energy density contributed by the neutron-star matter

$$\varepsilon_m = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}b\left(\mathrm{g}_\sigma\sigma\right)^3 + \frac{1}{4}c\left(\mathrm{g}_\sigma\sigma\right)^4$$
$$+\frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2 R_{0,0}^2 + \varepsilon_p + \varepsilon_n + \varepsilon_e. \quad (28)$$

The total energy density of the system is given by

$$\varepsilon = \varepsilon_m + \frac{B^2}{8\pi}, \quad (29)$$

where the last term is the contribution from the external electromagnetic field (Landau, Lifshitz & Pitaevskiĭ 1984). In the charge neutral beta-equilibrated matter, the pressure of the system can be expressed as (Paper II)

$$p = \mu_n\rho - \varepsilon_m + \frac{B^2}{8\pi}. \quad (30)$$

The inclusion of the $B^2/8\pi$ term in Eqs. (29) and (30) is equivalent to introducing the electromagnetic-field tensor in the source term of the gravitational equation of general relativity (Bocquet et al. 1995; Bonazzola & Gourgoulhon 1996; Cardall, Prakash & Lattimer 2001). Numerical calculations are performed under the constraints of charge neutrality $\rho^p_0 = \rho^e_0$ and the β-equilibrium $\mu_n = \mu_p + \mu_e$ (Glendenning 1997). These two constraint equations together with the three meson equations are solved self-consistently by an iteration procedure.

3 NUMERICAL RESULTS

Since there is no information directly available on the interior magnetic field of the star, we assume that the field varies from the surface to the center and adopt the following parametrization (Bandyopadhyay, Chakrabarty & Pal 1997)

$$B(\rho/\rho_0) = B_{\rm surf} + B_{\rm cent}\left[1 - \exp\left(-\beta(\rho/\rho_0)^{\gamma}\right)\right], \tag{31}$$

where the parameters are chosen to be $\beta = 0.01$ and $\gamma = 3$. We further set $B_{\rm surf} = \alpha B_c^e$, $B_{\rm cent} = \alpha \times 10^4 B_c^e$ and take α (which should not be confused with the fine structure constant) as a free parameter to check the effect of different fields. In this configuration the magnetic field decreases from the center to the surface of a star. We further take the surface magnetic field to be in the range of $10^{13} \sim 10^{15}$ G, in accordance with the values inferred from observations. Therefore, the applied magnetic field does not influence the stationary configuration of the neutron star considered to be spherical here (note that the stability of an axially symmetric neutron star in large average magnetic fields has been discussed recently (Broderick, Prakash & Lattimer 2002; Martinez, Rojas & Cuesta 2003)). The EOS of neutron-star matter under strong magnetic fields is shown in Fig. 1. It can be seen that the equation of state becomes stiffer than in the field free case at the main part of the density range. A stronger field leads to a stiffer EOS, but in the very low density part the situation is just the other way round, where a softer EOS is obtained when the magnetic field is incorporated; this can be more clearly seen from the upper panel of the figure.

In Fig. 2 we separate the cases with and without the inclusion of the AMM effects. Here we have dropped the contributions from the electromagnetic field to the pressure and energy density and present the EOS for the matter part only. A uniform magnetic field from the surface to the center of neutron stars is assumed. The dashed line denotes the case of $B = 50B_c^e$ and neglecting the AMM effects. One can see that the EOS turns out to be softer than the field free case at small energy densities and approaches to it in the main domain. The inclusion of the AMM effects here practically leads to indistinguishable results since at such low field the anomalous magnetic moments do not play any role. The whole situation changes substantially when a strong magnetic field is present, as indicated in the figure by the dash-dotted and dash-dot-dotted lines for $B = 5 \times 10^5 B_c^e$. In accordance with the findings in Paper I & Paper II the EOS becomes softer compared to the case of $B = 0$ when the AMM is neglected and stiffer when it is included. Therefore, the softening of the EOS at small energy densities exhibited in Fig. 1 is mainly due to the fact that at the surface region we have taken a relatively small magnetic field where the effects of the AMM are negligible. Under the employed parametrization the field increases with increasing density and the AMM terms gradually become operative. This leads to a stiffening of the EOS as shown in the region of large energy densities.

The structure of neutron stars can be obtained by applying the developed EOS to solve the Tolman-Oppenheimer-Volkoff (TOV) equation for a relativistic, spherical and static star (Shapiro & Teukolsky 1983; Glendenning 1997). We obtain stable solutions for the TOV equation based on the EOS with strong applied magnetic fields of Eq. (31). Figure 3 displays the gravitational mass of the neutron star as a function of the central density, for a set of values of the interior magnetic field. Rather evident effects are induced by strong magnetic fields. The maximum mass is increased drastically in the presence of strong fields. For the case of $\alpha = 50$, $M_{\rm max}$ is increased by about 40% over the field free case; in the latter case, it is known that the effects of the larger maximum mass can be balanced if the hyperon degrees of freedom are taken into account (Glendenning & Moszkowski 1991). In the mean time the central density is greatly decreased with increasing fields. This can be seen from Fig. 1 since the EOS stiffens when a strong magnetic field is embedded. Note an enhancement of the maximum mass around 13% – 30% has been reported (Bocquet et al. 1995; Bonazzola & Gourgoulhon 1996; Cardall, Prakash & Lattimer 2001) where the authors accomplished calculations of the structure of axisymmetric relativistic stars. The effects of strong magnetic fields were included in the stress-energy tensor of the gravitational equation while neglected in the input equations of state.

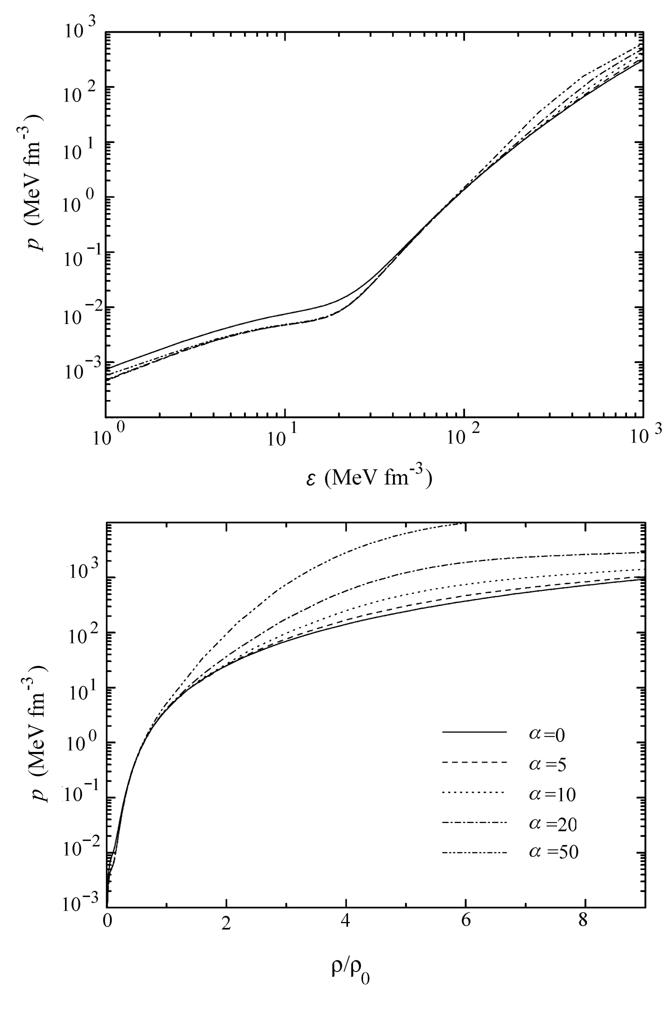

Fig. 1 Equation of state of neutron-star matter with strong magnetic fields. The upper panel shows the pressure as a function of the total energy density and the lower panel, as a function of the central density. Different curves correspond to the different magnetic field strengths indicated.

We show in Fig. 4 the radius of the neutron star as a function of its interior magnetic field strength for three fixed star masses at $M/M_{\odot}$ = 1.4, 1.6, and 1.8. If the field is weak, as is the normal case, the heavier stars have smaller radii due to the effects of the gravitational force. However, the radius increases greatly with increasing magnetic field strength. When strong enough fields are present, the heavier stars can even have larger radii, because stars with different internal magnetic fields belong to different sequences. In general, the radius of a magnetic star can be enhanced by about 10% $\sim$ 20% depending on the star mass, over the radius of a nonmagnetic star of the same mass.

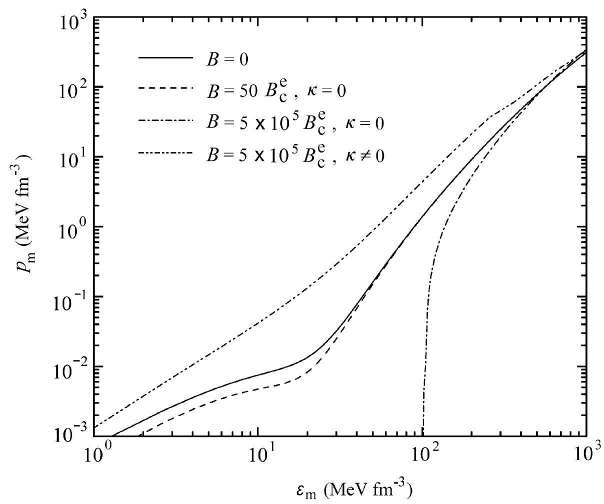

Fig. 2 Equation of state of neutron-star matter with a uniform magnetic field. Contributions of the magnetic field to the pressure and energy density are neglected. Different curves refer to different cases of magnetic fields B and with or without the anomalous magnetic moments κ as indicated in the figure.

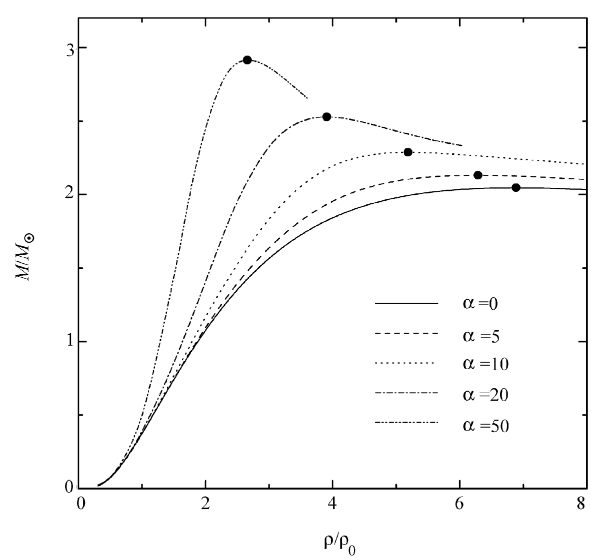

Fig. 3 Gravitational mass of the neutron star as a function of the central density, for the different interior magnetic fields indicated

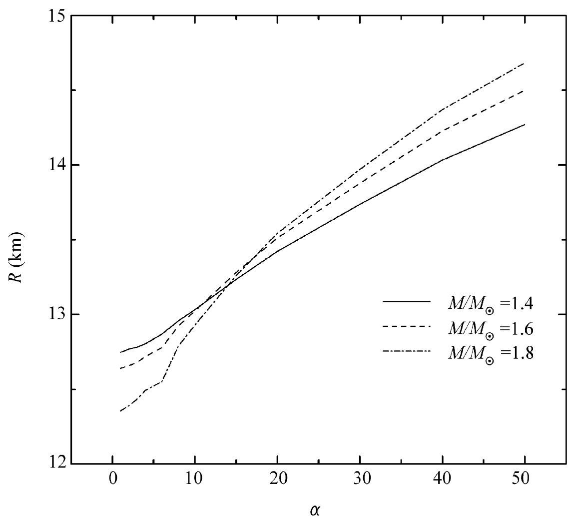

Fig. 4 Radii of neutron stars with equal gravitational masses as a function of interior magnetic field strength. Different curves correspond to different groups of star masses.

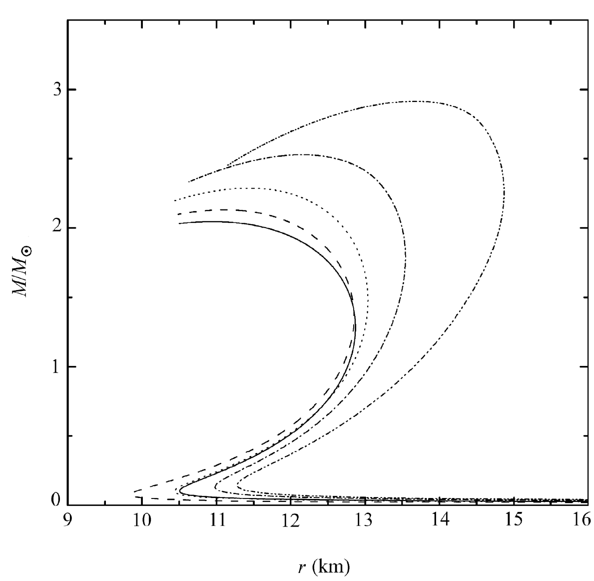

Fig. 5 Mass-radius relation of neutron stars for different interior magnetic fields as indicated by the different curves. For key, see Fig. 3.

We have also investigated the neutron star structure under a constant field strength from surface to center. For $B = 50B_c^e$ the relationship between the mass and the central density is indistinguishable from that of the field free case. The enhancement of the star radii is found to be less than 2%. It seems difficult to observe any effects of the magnetic field by measuring the star mass and radius if the magnitude of the interior field is of the same order as the surface field, though the influence of the surface field itself on the star properties should be pursued more closely in models of matter below the neutron drip (Baym, Pethick & Sutherland 1971; Lai & Shapiro 1991). Finally, in Fig. 5 we display the mass-radius relation at different magnetic fields. In order to describe the surface region the model for non-uniform matter at low densities (Shen 2002) should be used, this will be done in our future investigations.

4 SUMMARY AND OUTLOOK

Within a relativistic field theory approach we have studied the effects of strong magnetic fields on the equation of state of beta-equilibrium and charge neutrality matter relevant to neutron stars. Anomalous magnetic moments of both nucleons and electrons are covariantly incorporated in the model. We present analytical expressions of the Dirac spinors under a uniform magnetic field. The numerical results show that if the magnitude of the magnetic field is on the order of the surface field of neutron stars, then the effects of the AMM are negligible. The EOS deviates from the field free case only in very low density regions. If a much larger field is considered, the AMM plays a significant role so that the EOS becomes quite stiffer compared to the nonmagnetic case. Thus, if the interior magnetic field of a neutron star is at the same level as the surface field inferred from pulsars, it may not cause evident effects on the star mass and radius, and on the maximum mass of a star sequence. But if ultra-strong fields do exist in neutron stars, then considerable effects can be observed. We assume the magnetic field varies from the surface to the center. For the surface field we take the values inferred from pulsars. Strong fields up to $10^{18} \sim 10^{19}$ G have been considered for the center of neutron stars. With the adopted parametrization of the field changing with the density, the maximum mass could be enhanced by about 40%. The central density of the star is in turn reduced dramatically. For two equal-mass magnetic and nonmagnetic stars, the radius of the magnetic star can be larger by about $10\% \sim 20\%$ than in the nonmagnetic star.

It would be interesting to check whether the enhancement of the maximum mass induced by strong internal fields can be balanced by the hyperon degrees of freedom which was known to decrease the maximum mass. Since the effects of star rotations increase the star radii, it is necessary to study the effects of rotations and magnetic fields simultaneously. Theoretically one should solve the coupled Einstein-Maxwell equations for axisymmetric configuration with the effects of magnetic fields taken into account both in the source term of the gravitational equation of general relativity and in the nuclear equation of state. In future astronomical observations if we can catch two pulsars with similar rotation periods and masses but quite different radii, a possible explanation is that the star having larger radius may contain a strong interior magnetic field.

Now let us discuss several issues involved in the description of an electron in intense magnetic fields. It has long been recognized that the ground-state energy of an electron may be shifted after taking into account the AMM term. In previous calculations (O'Connell 1968; Chiu & Canuto 1968) it was thought that the Landau levels always start from $\nu = 0$. Thus, the ground-state energy of an electron in the presence of magnetic field can be written as

$$E_0 = m_e - \frac{\alpha}{2\pi}\mu_B B. \tag{32}$$

The vacuum becomes unstable with respect to the electron-positron pair creations when $B > 7.6 \times 10^{16}$ G, which may have dramatic astrophysical consequences. As pointed out in the

Appendix, the Landau levels start from $\nu = 1$ for spin-up particles since the wave functions vanish at $\nu = 0$. It causes the ground-state energy to change to the form

$$E_0 = \sqrt{m_e^2 + 2eB} - \frac{\alpha}{2\pi}\mu_B B. \tag{33}$$

The critical field for pair creations now turns out to be 2.6×10^{20} G. However, Eq. (33) is valid at the level of the anomalous magnetic moment term (Jancovici 1969). Higher-order terms, e.g. radiative correction due to vacuum polarization effect (Schwinger 1951; 1973), may become effective at such large field. The incorporation of these terms in the present model is a problem to be addressed in future studies.

APPENDIX A

In this Appendix we derive the Dirac spinors and energy spectra of nucleons in the presence of a constant uniform magnetic field along the z axis. The Dirac equation for a free nucleon which has an anomalous magnetic moment μ_N in an external magnetic field can be written as

$$\left[i\gamma_\mu \partial^\mu - e\frac{1+\tau_0}{2}\gamma_\mu A^\mu - \frac{1}{4}\kappa_b \mu_N \sigma_{\mu\nu} F^{\mu\nu} - M_N\right]\psi = 0. \tag{A1}$$

If one drops the term involving the AMM, the solutions for neutrons are just the conventional Dirac spinors (Greiner 1990; Weinberg 1995). The Dirac theory for free electrons in a homogeneous magnetic field was first investigated by Rabi (1928). The wave functions for charged particles without the inclusion of the AMM have been studied by several authors (Kobayashi & Sakamoto 1983; Das & Hott 1996). Johnson & Lippmann (1950) considered the inclusion of the AMM in the Dirac equation within a noncovariant description. A covariant energy spectrum was discussed by Vshivtsev & Serebryakova (1994). Recently, Broderick, Prakash & Lattimer (2000, Paper II) derived the spinors and energy spectra for baryons in the Dirac representation. However, their formulae are quite complicated which leads to a lack of analytical expressions for the wave functions. Here we resolve the problem in the chiral representation. The employed γ-matrices then become (Itzykson & Zuber 1980).

$$\gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}, \qquad \boldsymbol{\alpha} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & -\boldsymbol{\sigma} \end{pmatrix}, \tag{A2}$$

$$\boldsymbol{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \tag{A3}$$

where $\boldsymbol{\sigma}$ is the Pauli matrix. Equation (A1) can be rewritten as

$$i\frac{\partial}{\partial t}\psi = \left[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} - e\frac{1+\tau_0}{2}\alpha_2 Bx + \beta M_N + \frac{1}{4}\beta\kappa_b\mu_N\sigma_{\mu\nu}F^{\mu\nu}\right]\psi. \tag{A4}$$

In the following we define $\Delta = -\frac{1}{2}\kappa_b \mu_N B$ and consider the cases of protons and neutrons separately.

Protons

Let us specify the wave functions of protons as

$$\psi(X) = e^{-iEt + ip_y y + ip_z z}\phi_p(p_y, p_z, x). \tag{A5}$$

In the static system we obtain the eigenequation

$$\begin{pmatrix} p_z - E & \xi_+ & -M_N + \Delta & 0 \\ \xi_- & -p_z - E & 0 & -M_N - \Delta \\ -M_N + \Delta & 0 & -p_z - E & -\xi_+ \\ 0 & -M_N - \Delta & -\xi_- & p_z - E \end{pmatrix} \begin{pmatrix} \phi_p^{(1)} \\ \phi_p^{(2)} \\ \phi_p^{(3)} \\ \phi_p^{(4)} \end{pmatrix} = 0. \tag{A6}$$

Here we have defined $\xi_\pm \equiv -i\partial_x \mp i(p_y - eBx)$. Provided $eB > 0$, one can introduce the eigenfunction of ξ_+, ξ_-

$$\begin{aligned} I_{\nu,p_y}(x) \;=\; & \left(\frac{eB}{\pi}\right)^{1/4} \exp\left[-\frac{1}{2}eB\left(x - \frac{p_y}{eB}\right)^2\right] \frac{1}{\sqrt{\nu!}} H_\nu\left[\sqrt{2eB}\left(x - \frac{p_y}{eB}\right)\right], \\ & (\nu = 0, 1, 2, ...) \end{aligned} \tag{A7}$$

where $H_\nu(x)$ is the Hermite polynomial defined by

$$H_\nu(x) = (-1)^\nu \exp\left(\frac{x^2}{2}\right) \frac{d^\nu}{dx^\nu} \exp\left(-\frac{x^2}{2}\right). \tag{A8}$$

$I_{\nu,p_y}(x)$ is normalized as

$$\int dx I_{\nu,p_y}(x) I_{\mu,p_y}(x) = \delta_{\nu\mu}, \tag{A9}$$

$$\sum_{\nu=0}^{\infty} I_{\nu,p_y}(x) I_{\nu,p_y}(x') = \delta(x - x'). \tag{A10}$$

It satisfies the following relations:

$$\xi_- I_{\nu,p_y}(x) = -i\sqrt{2eB\nu}\, I_{\nu-1,p_y}(x), \qquad (I_{-1,p_y}(x) = 0) \tag{A11}$$

$$\xi_+ I_{\nu,p_y}(x) = i\sqrt{2eB(\nu+1)}\, I_{\nu+1,p_y}(x). \tag{A12}$$

The eigenvalues and eigenfunctions of Eq. (A6) can be deduced in a standard way by performing matrix calculations. The energy spectra of protons are

$$\left(E^p_{\nu,S}\right)_+ = E^\nu_S, \qquad \left(E^p_{\nu,S}\right)_- = -E^\nu_S, \tag{A13}$$

with

$$E^\nu_S = \left[\left(\sqrt{M_N^2 + 2eB\nu} + S\Delta\right)^2 + p_z^2\right]^{1/2}, \tag{A14}$$

here $S = \pm 1$ for the spin-up and spin-down particles. The respective eigenfunctions are as follows:

$$\psi_1(X) \;=\; e^{-iE^\nu_{+1}t + ip_y y + ip_z z} \sqrt{\frac{E^\nu_{+1} + p_z}{2E^\nu_{+1}}} \left[1 + \frac{2eB\nu}{\left(\sqrt{M_N^2 + 2eB\nu} + M_N\right)^2}\right]^{-1/2}$$

$$\times \begin{pmatrix} -\dfrac{i\sqrt{2eB\nu}}{\sqrt{M_N^2+2eB\nu}+M_N} I_{\nu,p_y}(x) \\ -\dfrac{\sqrt{M_N^2+2eB\nu}+\Delta}{E_{+1}^{\nu}+p_z} I_{\nu-1,p_y}(x) \\ -\dfrac{i\sqrt{2eB\nu}\left(\sqrt{M_N^2+2eB\nu}+\Delta\right)}{\left(E_{+1}^{\nu}+p_z\right)\left(\sqrt{M_N^2+2eB\nu}+M_N\right)} I_{\nu,p_y}(x) \\ I_{\nu-1,p_y}(x) \end{pmatrix}, \tag{A15}$$

$$\psi_2(X) = e^{-iE_{-1}^{\nu}t+ip_y y+ip_z z}\sqrt{\frac{E_{-1}^{\nu}+p_z}{2E_{-1}^{\nu}}}\left[1+\frac{2eB\nu}{\left(\sqrt{M_N^2+2eB\nu}-M_N\right)^2}\right]^{-1/2}$$

$$\times \begin{pmatrix} \dfrac{i\sqrt{2eB\nu}}{\sqrt{M_N^2+2eB\nu}-M_N} I_{\nu,p_y}(x) \\ \dfrac{\sqrt{M_N^2+2eB\nu}-\Delta}{E_{-1}^{\nu}+p_z} I_{\nu-1,p_y}(x) \\ -\dfrac{i\sqrt{2eB\nu}\left(\sqrt{M_N^2+2eB\nu}-\Delta\right)}{\left(E_{-1}^{\nu}+p_z\right)\left(\sqrt{M_N^2+2eB\nu}-M_N\right)} I_{\nu,p_y}(x) \\ I_{\nu-1,p_y}(x) \end{pmatrix}, \tag{A16}$$

$$\psi_3(X) = e^{-i(-E_{+1}^{\nu})t+ip_y y+ip_z z}\sqrt{\frac{E_{+1}^{\nu}+p_z}{2E_{+1}^{\nu}}}\left[1+\frac{2eB\nu}{\left(\sqrt{M_N^2+2eB\nu}+M_N\right)^2}\right]^{-1/2}$$

$$\times \begin{pmatrix} -\dfrac{i\sqrt{2eB\nu}\left(\sqrt{M_N^2+2eB\nu}+\Delta\right)}{\left(E_{+1}^{\nu}+p_z\right)\left(\sqrt{M_N^2+2eB\nu}+M_N\right)} I_{\nu,p_y}(x) \\ I_{\nu-1,p_y}(x) \\ \dfrac{i\sqrt{2eB\nu}}{\sqrt{M_N^2+2eB\nu}+M_N} I_{\nu,p_y}(x) \\ \dfrac{\sqrt{M_N^2+2eB\nu}+\Delta}{E_{+1}^{\nu}+p_z} I_{\nu-1,p_y}(x) \end{pmatrix}, \tag{A17}$$

$$\psi_4(X) = e^{-i(-E_{-1}^{\nu})t+ip_y y+ip_z z}\sqrt{\frac{E_{-1}^{\nu}+p_z}{2E_{-1}^{\nu}}}\left[1+\frac{2eB\nu}{\left(\sqrt{M_N^2+2eB\nu}-M_N\right)^2}\right]^{-1/2}$$

$$\times\begin{pmatrix} \frac{i\sqrt{2eB\nu}\left(\sqrt{M_N^2+2eB\nu}-\Delta\right)}{\left(E_{-1}^{\nu}+p_z\right)\left(\sqrt{M_N^2+2eB\nu}-M_N\right)} I_{\nu,p_y}(x) \\ \\ -I_{\nu-1,p_y}(x) \\ \\ \frac{i\sqrt{2eB\nu}}{\sqrt{M_N^2+2eB\nu}-M_N} I_{\nu,p_y}(x) \\ \\ \frac{\sqrt{M_N^2+2eB\nu}-\Delta}{E_{-1}^{\nu}+p_z} I_{\nu-1,p_y}(x) \end{pmatrix}. \tag{A18}$$

One can easily check that $\psi_i (i = 1, 4)$ forms a complete orthogonal set. Note that the Landau levels start at $\nu = 0$ for spin-down particles and $\nu = 1$ for spin-up particles since ψ_1 and ψ_3 vanish at $\nu = 0$. As pointed out in Sect. 2, here the so-called spin up and spin down are just relative notions for convenience of description.

Neutrons

The wave functions of neutrons can be specified as

$$\psi(X) = e^{-iEt+i\mathbf{p}\cdot\mathbf{x}}\phi_n(\mathbf{p}). \tag{A19}$$

Inserting it into Eq. (A4) we have the following eigenequation:

$$\begin{pmatrix} p_z - E & p_x - ip_y & -M_N + \Delta & 0 \\ p_x + ip_y & -p_z - E & 0 & -M_N - \Delta \\ -M_N + \Delta & 0 & -p_z - E & -(p_x - ip_y) \\ 0 & -M_N - \Delta & -(p_x + ip_y) & p_z - E \end{pmatrix} \begin{pmatrix} \phi_n^{(1)} \\ \phi_n^{(2)} \\ \phi_n^{(3)} \\ \phi_n^{(4)} \end{pmatrix} = 0. \tag{A20}$$

Through solving the above matrix equation we obtain the energy spectra of neutrons as

$$(E_S^n)_+ = E_S, \qquad (E_S^n)_- = -E_S, \tag{A21}$$

with

$$E_S = \left[\left(\sqrt{p_x^2 + p_y^2 + M_N^2} + S\Delta\right)^2 + p_z^2\right]^{1/2}. \tag{A22}$$

The corresponding eigenfunctions read as

$$\psi_1(X) = e^{-iE_{+1}t+i\mathbf{p}\cdot\mathbf{x}}\sqrt{\frac{E_{+1}+p_z}{2E_{+1}}}\left[1 + \frac{p_x^2+p_y^2}{\left(\sqrt{M_N^2+p_x^2+p_y^2}+M_N\right)^2}\right]^{-1/2}$$
$$\times\begin{pmatrix} -\frac{p_x-ip_y}{\sqrt{M_N^2+p_x^2+p_y^2}+M_N} \\ \\ -\frac{\sqrt{M_N^2+p_x^2+p_y^2}+\Delta}{E_{+1}+p_z} \\ \\ -\frac{(p_x-ip_y)\left(\sqrt{M_N^2+p_x^2+p_y^2}+\Delta\right)}{(E_{+1}+p_z)\left(\sqrt{M_N^2+p_x^2+p_y^2}+M_N\right)} \\ \\ 1 \end{pmatrix}, \tag{A23}$$

$$\psi_2(X) = e^{-iE_{-1}t+i\mathbf{p}\cdot\mathbf{x}}\sqrt{\frac{E_{-1}+p_z}{2E_{-1}}}\left[1+\frac{p_x^2+p_y^2}{\left(\sqrt{M_N^2+p_x^2+p_y^2}-M_N\right)^2}\right]^{-1/2} \times \begin{pmatrix} \frac{p_x-ip_y}{\sqrt{M_N^2+p_x^2+p_y^2}-M_N} \\ \frac{\sqrt{M_N^2+p_x^2+p_y^2}-\Delta}{E_{-1}+p_z} \\ -\frac{(p_x-ip_y)\left(\sqrt{M_N^2+p_x^2+p_y^2}-\Delta\right)}{(E_{-1}+p_z)\left(\sqrt{M_N^2+p_x^2+p_y^2}-M_N\right)} \\ 1 \end{pmatrix}, \tag{A24}$$

$$\psi_3(X) = e^{-i(-E_{+1})t+i\mathbf{p}\cdot\mathbf{x}}\sqrt{\frac{E_{+1}+p_z}{2E_{+1}}}\left[1+\frac{p_x^2+p_y^2}{\left(\sqrt{M_N^2+p_x^2+p_y^2}+M_N\right)^2}\right]^{-1/2} \times \begin{pmatrix} -\frac{(p_x-ip_y)\left(\sqrt{M_N^2+p_x^2+p_y^2}+\Delta\right)}{(E_{+1}+p_z)\left(\sqrt{M_N^2+p_x^2+p_y^2}+M_N\right)} \\ 1 \\ \frac{p_x-ip_y}{\sqrt{M_N^2+p_x^2+p_y^2}+M_N} \\ \frac{\sqrt{M_N^2+p_x^2+p_y^2}+\Delta}{E_{+1}+p_z} \end{pmatrix}, \tag{A25}$$

$$\psi_4(X) = e^{-i(-E_{-1})t+i\mathbf{p}\cdot\mathbf{x}}\sqrt{\frac{E_{-1}+p_z}{2E_{-1}}}\left[1+\frac{p_x^2+p_y^2}{\left(\sqrt{M_N^2+p_x^2+p_y^2}-M_N\right)^2}\right]^{-1/2} \times \begin{pmatrix} \frac{(p_x-ip_y)\left(\sqrt{M_N^2+p_x^2+p_y^2}-\Delta\right)}{(E_{-1}+p_z)\left(\sqrt{M_N^2+p_x^2+p_y^2}-M_N\right)} \\ -1 \\ \frac{p_x-ip_y}{\sqrt{M_N^2+p_x^2+p_y^2}-M_N} \\ \frac{\sqrt{M_N^2+p_x^2+p_y^2}-\Delta}{E_{-1}+p_z} \end{pmatrix}. \tag{A26}$$

Again, $\psi_i (i = 1, 4)$ is orthonormalized.

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