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Fibonacci Sequences and the Multiperiodicity of the Variable Star UW Herculis

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Abstract We present an application of the methods recently developed for the study of quasicrystal structures to the analysis of multiperiodicity of semiregular variables. A light curve analysis of UW Her shows frequencies that can be included within the general scheme characterizing the Fourier spectra of Fibonacci quasiperiodic sequences. The analysed data come from the BAA Variable Star Section computerised archive.

Key words: stars: variables

1 INTRODUCTION

Quasicrystals are alloys discovered 20 years ago which exhibit aperiodic ordered structures different from either crystals or disordered materials (Levine & Steinhardt 1986). The structures, which lie somewhere between periodicity and randomness, have been described with the help of word sequences in formal grammars (Escudero 1996, 2000). The Fourier analysis of quasiperiodic structures can be done with standard methods (Allouche & Mendès-France 1995; Escudero 1996). Aperiodic ordered sequences in 1D are the main tools of the study. The scaling factors of the sequences are algebraic integers: roots of monic (the leading coefficient is equal to 1) polynomials with integer coefficients. The rate of periods of some semiregular variables are close to algebraic integers. Kiss et al. (1999) have shown that UW Her (HD 156163, spectral type M5e) has two periods of 172 and 107 days. The period ratio is near the golden number $\sin(2\pi/5)/\sin(\pi/5) = 1.618$, which is the highest root of the equation $x^2 - x - 1 = 0$. The variable ST Cam [HD 30243, spectral type C5, 4 (N5)] has two periods of 372 and 202 days with a ratio approximate to $\sin(2\pi/8)/\sin(\pi/8) = 1.848$, the highest root of the equation $x^4 - 4x^2 + 2 = 0$. V Boo (HD 127335, spectral type M6e) shows a decline in amplitude at its main period and it seems to evolve from Mira-like to semiregular type. Analysis of several international databases of visual observations (AFOEV, VSOLJ HAAVSS, AAVSO) have shown the existence of two periods. The amplitude of the longer cycle (257 days) is decreasing continuously while the amplitude of the shorter cycle (137 days) is distributed within a certain range. In this case the period ratio is a number close to $\sin(2\pi/9)/\sin(\pi/9) = 1.879$, the highest root of the equation $x^3 - 3x - 1 = 0$. In this paper we consider the Fourier analysis of Fibonacci quasiperiodic temporal sequences of sinusoidal fragments and the frequency distribution for the light curve analysis of UW Her obtained from the BAAVSS database.

2 THE FIBONACCI SEQUENCES

The best known example of a self-similar quasiperiodic sequence is the Fibonacci sequence. It is related with the ubiquitous golden number. In the Timaeus, Plato assigned regular solids to the four elements, fire (tetrahedron), earth (cube), air (octahedron), water (icosahedron) and to the cosmos (dodecahedron). The golden number appears in the dodecahedron and the icosahedron which can be found in the geometric structure of quasicrystals (Shechtman et al. 1984; Kramer & Neri 1984). Lindenmayer systems can be used in order to describe the Fibonacci sequences. A 0L-system (Prusinkiewicz & Lindenmayer 1990) is a triple $G = \{\Sigma, r, \omega\}$ where Σ is an alphabet, r is a finite substitution on Σ into the set of subsets of Σ^*, and ω is the axiom. G is called a D0L-system if $\#(r(x)) = 1$, for every $x \in \Sigma$. For the Fibonacci sequence the alphabet is $\{L, S\}$, $r : \{L \longmapsto LS, S \longmapsto L\}$ and the axiom L. The sequence consists in the words $L, LS, LSL, LSLLS, ...$. A 1D quasiperiodic geometric structure can be obtained if L and S represent two segments with a ratio equal to the golden number τ. The sequence is deterministic in the sense that only one word is allowed with a given length. The Fibonacci numbers $F(n)$ can be defined with the help of the recurrence relation $F(n+2) = F(n+1)+F(n)$, $F(0) = F(1) = 1$. By iterating this relation we obtain the sequence 1, 1, 2, 3, 5, 8, 13, 21... The quotient of two successive Fibonacci numbers approach the golden number as n increases.

In order to illustrate the method an artificial light curve is generated by concatenation of simple sinusoids with two different lengths L and S in a golden ratio following the Fibonacci sequence. The curve corresponding to the word $LSLLS$ with $L = F(10) = 89$ days and $S = F(9) = 55$ days can be seen in Fig. 1.

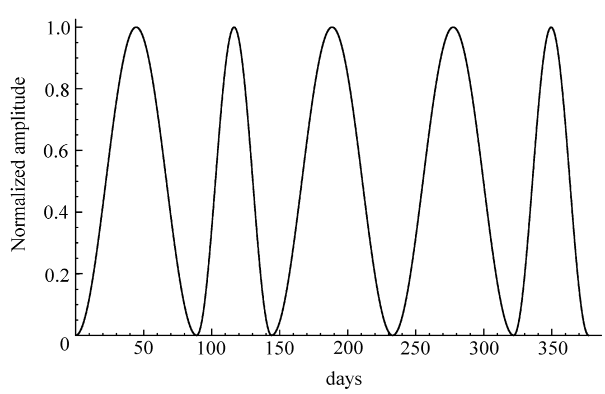

Fig. 1 Artificial data corresponding to the word $LSLLS$.

By iterating nine times the substitution rules to the letter L, a word with length (number of letters) 89 is obtained. The Fourier spectra of the corresponding artificial light curve are shown in Fig. 2. The frequencies of the Fourier spectrum of Fibonacci sequences (Levine & Steinhardt 1986) belong to a set which consists of the linear combinations with integer coefficients of two fundamental frequencies: $f = m_1\omega_1 + m_2\omega_2$, where ω_1 and ω_2 are in a golden ratio and m_1, m_2 are integers. In Fig. 2 it can be seen that the Fourier spectrum is formed of sharp peaks. The periods corresponding to the peaks with the highest amplitudes in Fig. 2 are $P_1 = 123$ days and $P_2 = 76$ days. Observe that there is a factor of 1.38 in relation with the two periods L and S forming the artificial data.

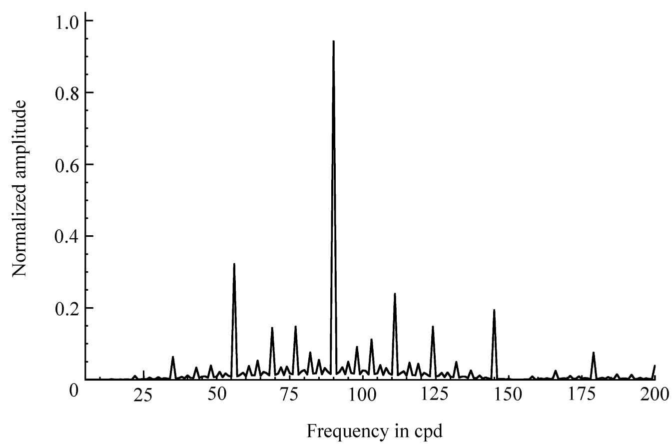

Fig. 2 Fourier spectrum for the artificial data corresponding to the deterministic Fibonacci word with length 89. The amplitudes are normalized and the frequency is given in units of 1/6765 cycles per day (cpd).

A stochastic 0L-system is a 4-ruple $G = \{\Sigma, P, \omega, \pi\}$, where P is a set of productions r_i and $\pi : P \longmapsto (0, 1]$ is a probability distribution. We define a Fibonacci stochastic L-system with $P = \{r_1, r_2\}$ and $r_1 : \{L \longmapsto LS, S \longmapsto L\}, r_2 : \{L \longmapsto SL, S \longmapsto L\}$. The Fourier spectrum of artificial data generated by following the word sequences of this system contains peaks with bandwidths due to the presence of a continuous component.

3 PERIOD ANALYSIS

The period computations are done first by obtaining a continuous version of the light curves generated from the BAAVSS archive with the Built-in Mathematica Object “Interpolation” (Wolfram 1991) which works by finding least-squares fits with polynomial curves. The next step is to obtain from the continuous curve a list of data for evenly spaced points and then the Discrete Fourier Transform of the data is calculated with the same program.

In order to compare with other established results, an analysis of the V Boo periodicities has been done, using data of the BAAVSS archive from JD 2419000 to JD 2451600. The spectrum shows two peaks with periods of 257.1 and 136.7 days. They are very similar to the ones obtained in (Kiss et al. 1999) for observations that spanned the interval from JD 2419000 to JD 2451000 and conserve their ratio. Another test was done by analysing the BAAVSS database for UW Her in the interval MJD 350–8950 (MJD=JD−2441799). The averaging procedure consists in taking 10-day bins and calculating the mean value from the individual points. The peaks with the highest intensities correspond to the periods of 107, 172 and 1012 days which are in good agreement with those found by Kiss et al. (107, 172 and 1000 days) in spite of the fact that the databases are different.

Let us illustrate the method by considering the time interval MJD 3876–9798 for UW Her [see Escudero (2002) for an analysis of a different interval]. The spectrum for that interval is shown in Fig. 3. The most intense peaks correspond to the periods of 108, 172 and 1077 days.

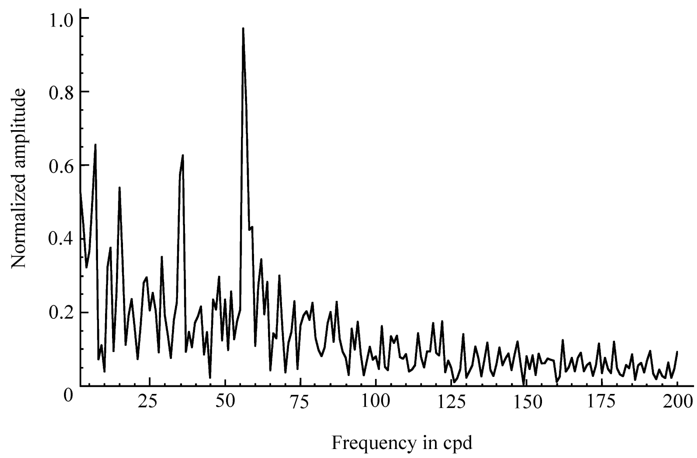

Fig. 3 Fourier spectrum for UW Her (for MJD 3876–9798; frequency in units of 1/5922 cpd).

In what follows a model is introduced in order to obtain a description of the variability as simple as possible. The model consists of a sequence of sinusoidal fragments with lengths of $L = 126$ and $S = 78$ days. The following nondeterministic derivation is the basis for the generation of the artificial light curve $L \longmapsto LS \longmapsto LSL \longmapsto LSLLS \longmapsto LSLLSLSL \longmapsto LSLSLSLLSLLLS \longmapsto SLLSLLSLLLSLSLLSLSSLL \longmapsto LLSLSLSLLSLSLSLS$ $LLSLLSLLSLLSLLSLLS \longmapsto LSLSLLSLLSLLSLSLLSLLSLLSLLSLSLLSLSL$ $LSLSLLSLSLLSLSLLSLSL$. This sequence has been obtained by matching with the observed light curves (see Fig. 4). If the amplitudes of the sinusoidal fragments are in a golden ratio then the Fourier spectrum (see Fig. 5) shows two intense peaks at $P_1 = 174$ and $P_2 = 108$ days.

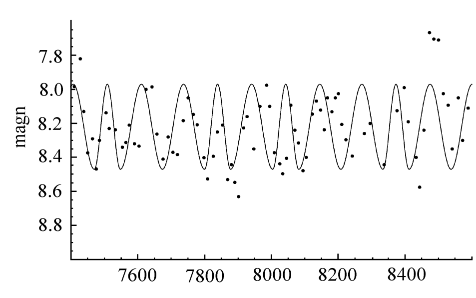

Fig. 4 BAAVSS averaged light curve and a segment of the sequence.

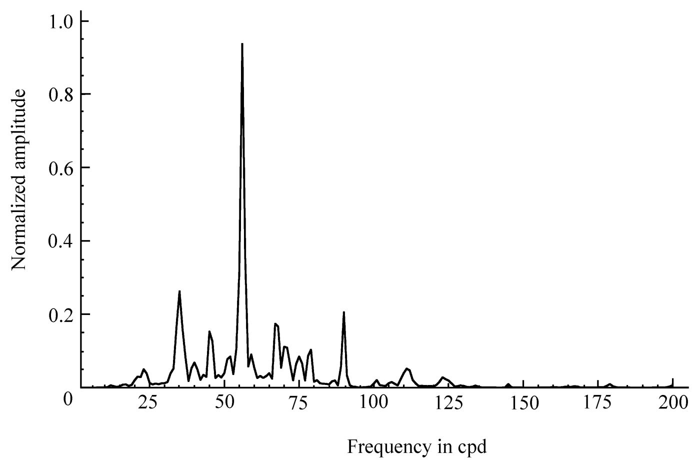

Fig. 5 Fourier spectrum for the model with 55 sinusoidal fragments following a substitutional sequence (frequency in units of 1/5922 cpd).

In Fig. 4 we can see that the amplitudes of certain sinusoids are higher. This fact suggests to include in the model amplitude changes. When the amplitudes of the sinusoids situated at positions number 7, 8, 16, 30, 32, 43, 52 and 54 are increased by a factor of 3, then a third peak appears corresponding to a period of $P_3 = 1077$ days (Fig. 6).

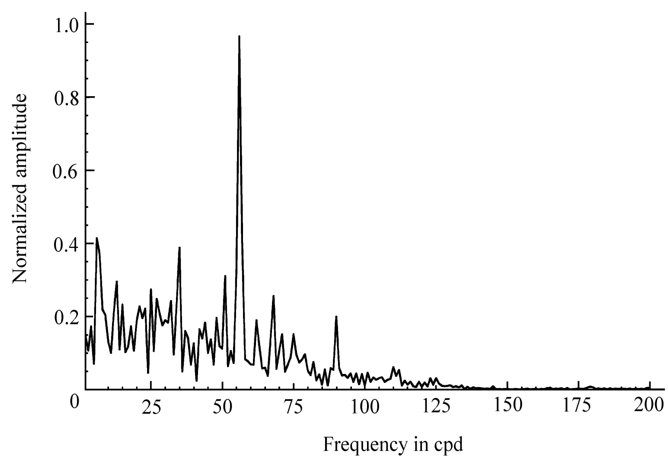

Fig. 6 Spectrum for the model with sudden amplitude changes.

4 CONCLUDING REMARKS

This paper is part of a project on variable stars with two or more periods and the regularity of the periodicities they have. The periodicities of UW Her have been examined by using the

BAAVSS archive. A simple model for the variation has been proposed and applied to the interval MJD 3876–9798. The building blocks are two sinusoidal fragments with lengths of 126 and 78 days arranged in a quasiperiodic sequence. By introducing sudden amplitude changes at certain positions in the artificial curve according with the observations, the Fourier analysis shows that the peaks correspond approximately to the ones obtained in the analysis of the observed data. In order to improve the results, other amplitudes for the sinusoids, local fluctuations between the two basic blocks in the sequences, and the analysis of a larger period, should be considered. The model studied in this work is valid only for UW Her. For stars with periods not close to the golden number different types of substitutional sequences must be used. The relationship with stellar physics models must also be considered in the future.

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References

Allouche J. P., Mendès-France M., 1995, In: Axel F., Gratias D., eds., Beyond Quasicrystals, EDP-Springer Series of the Les Houches Center for Physics., Vol.3, p.293

Escudero J. G., 1996., J. Phys. Soc. Japan, 65, 3776

Escudero J. G., 2000, Mat. Sci. Eng. A, 294, 388

Escudero J. G., 2002, In: C. Aerts, et al. eds., ASP. Conf. Ser. Vol.259, Radial and Nonradial Pulsations as Probes of Stellar Physics, San Francisco: ASP, p.568

Kiss L. L, Szatmáry K., Cadmus R. R., Mattei J. A., 1999, A&A, 346, 542

Kramer P., Neri R., 1984, Acta Crystallogr A, 40, 667

Levine D., Steinhardt P. J., 1986, Phys. Rev. B, 34, 596

Prusinkiewicz P., Lindenmayer A., 1990, The Algorithmic Beauty of Plants, Springer

Shechtman D., Blech I., Gratias D., Cahn J. W., 1984, Phys. Rev. Lett., 53, 1951

Wolfram S., 1991, Mathematica, Addison-Wesley Publishing Co.