Chin. J. Astron. Astrophys. Vol. 3 (2003), No. 2, 95–104
(*http://www.chjaa.org* or *http://chjaa.bao.ac.cn*)

Chinese Journal of
Astronomy and
Astrophysics

Rhombic Cell Analysis – A New Way of Probing the Large-Scale Structure of the Universe. I. General Considerations

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Received 2003 February 26; accepted 2003 March 11

Abstract A new way of probing the large-scale structure of the universe is proposed. Space is partitioned into cells the shape of rhombic dodecahedron. The cells are labelled "filled" or "empty" according as they contain galaxies or not. The cell size is so chosen as to have nearly equal numbers of filled and empty cells for the given galaxy sample. Two observables on each cell are definable: the number of its *like* neighbors, n_1, and a two-suffixed topological type τ, the suffixes being the numbers of its like and unlike *neighbor-groups*. The frequency distributions of n_1 and τ in the observed set of filled (empty) cells are then considered as indicators of the morphology of the set. The method is applied to the CfA catalogue of galaxies as an illustration. Despite its limited size, the data offers evidence 1) that the empty cells are more strongly clustered than the filled cells, and 2) that the filled cells, but not the empty cells, have a tendency to occur in sheets. Further directions of development both in theory and application are indicated.

Key words: cosmology: large-scale structure of Universe — cosmology: observations — CfA Catalogue

1 INTRODUCTION

This paper, hopefully, will open up a new line of research into the large scale morphology of the universe by means of *rhombic dodecahedron* cells. In this first paper, I shall outline some of the general principles, and, by way of illustration, apply the analysis to the CfA Catalogue of Galaxies (Davis & Huchra 1982), to address the question of the shape of the filled/overdense and empty/underdense regions of the universe. Further development, both in theory and application, are being actively pursued, and hopefully will appear in print in the near future. A much shorter, first draft of the present paper was published some 10 years ago (Kiang 1993). The journal in which it appeared, however, had a rather limited circulation, particularly so for researchers in China.

2 THE LARGE-SCALE MORPHOLOGY

The classic book on this subject is Hubble's "The Distribution of the Nebulae" (Hubble 1936). In it the author articulated the thesis that the galaxies are the building blocks of the universe and that, at least as a first approximation or a working hypothesis, we should take them as being statistically uniformly distributed in space. But the working hypothesis of an authority has a tendency of persisting as a literal truth in the minds of the later researchers, and it was not until the 1950s that Hubble's idea of uniform distribution was seriously challenged. The challenge came from two directions: on one hand, Neyman and Scott (1952, 1953b), using statistical techniques based on counts-in-cells, showed that the galaxy distribution is statistically *clustered* rather than statistically uniform, and on the other, Abell actually identified from Palomar Sky Survey plates, 2712 operationally well-defined rich *clusters* of galaxies (Abell 1958). (Of course, Abell's catalogue and its Southern extension have since acquired the status of a classic database). The next step in the direction further away from the uniform distribution was taken by myself (Kiang 1965, 1971): I applied the counts-in-cells technique to the Abell clusters and found they themselves were clustered. This led me to the idea that galaxies may be clustered on all scales. About the same time, Peebles began a different way of characterising the galaxy distribution, a way which proved to be highly productive, namely, the calculation of the two-point correlation function (Peebles 1978). The two-point correlation function is in some sense a quantitative expression of my idea of indefinite clustering. As is well-known, the calculation of the two-point correlation, and less often, of the three-point correlation function has become something of an industry in the field of galaxy research. Among the large number of applications, two of the more recent examples may be cited (Zhu 1997a, 1997b).

All the studies mentioned so far share the common belief that the galaxies are the basic constituents of the universe, our exclusive concern when dealing with the question of the large-scale morphology. This outlook seemed so natural. However, a glimpse of an alternative did appear in a paper presented at the 1977 IAU Symposium in Tallinn (Joeveer & Einasto 1978), tellingly entitled "Has the Universe the Cell Structure ?". And it may not be coincidental that, at the same symposium, Zel'dovich gave a paper on his well-known pancake theory of galaxy formation (Zel'dovich 1978), for Zel'dovich's theory would go hand in glove with a cell structure. Since then, large voids or underdense regions and structures like the "Great Wall" have become common knowledge. Thus, the idea dawned that voids may be as a fundamental ingredient of the large-scale structure as are the galaxies.

The first paper that treated the underdense and overdense regions on an equal footing was by Gott et al. (1986). These authors found (i) that the two regions are each a sponge-like, connected entity, and (ii) that they are equivalent. I think that statement (i) is entirely plausible, but to give statement (ii) any quantitative interpretation should perhaps await a more penetrating analysis than what they did: they partitioned space into cubic cells and examined the interface between the two regions. It is my belief that partitioning space into what I call *rhombic* cells would provide a much more powerful means of analysis, and that the analysis should not be confined to the interface, but should extend to all the constituent cells. I hope the present paper will offer a first glimpse of the great richness that is inherent in the rhombic cell analysis.

3 RHOMBIC CELL ANALYSIS

I imagine space is partitioned into cells the shape of *rhombic dodecahedron.* The latter can be imagined to form in the following way: 1. Space is partitioned into identical cubes, and the cubes are painted alternately black and white into a three-dimensional chessboard. 2. Each white cube is cut into six identical pyramids with the faces as the bases and the centre as the common vertex. 3. To a given black cube, we stick on the six adjoining white pyramids. The result: a solid with 12 identical white rhombic faces generated out of a black cube. I shall call the black cube the *generating cube* of the dodecahedron. The size (volume) of the dodecahedron is, of course, twice the size of its generating cube. The identical rhombic face has its semi-minor diameter, semi-major diameter and sides in the ratios of 1: $\sqrt{2}$: $\sqrt{3}$. More of the geometry of the rhombic dodecahedron is given in the Appendix at the end of this paper. The minor diameters of the 12 faces are just the 12 edges of the generating cube;—this one-to-one correspondence was made use of in my construction of the Table 1 below.

Thus, space is partitioned into such 12-rhombic-faced cells. From now on, these cells will simply be referred to as "rhombic cells". An analysis of the CfA Catalog of Galaxies (Davis & Huchra 1982) now follows, both for its own sake and as an illustration of the method: all the definitions and arguments will equally apply to any other galaxy sample. If a rhombic cell contains no CfA galaxies, then we call it an *empty* cell; otherwise, it is a *filled* cell. The ensemble of the empty cells forms the *empty region*, and similarly the *filled region*. So the question we are concerned with here is "Do the filled and empty regions have the same morphology ?", or more precisely, "granted that they are each a connected entity, do they have the same mix of 1-dimensional string-like, 2-dimensional sheet-like, and 3-dimensional clump-like contents ?"

3.1 Optimal Cell Size

Obviously, the question is at its most meaningful when the two regions have the same size, for the morphology of a region clearly depends on its overall size. The size (volume) of the individual cell when this happens will be called the optimal cell size, $v_{\rm optm}$. It is easily shown that $v_{\rm optm}$ always exists for any reasonably spread-out distribution of the galaxies. Suppose we have a sample of N galaxies distributed throughout some volume of space V. We enclose each galaxy in a small sphere, the result: N tiny spheres in a huge empty sea: the two ensembles differ completely in morphology and size. Now let us keep on increasing the size of the spheres. The spheres will begin and keep on coalescing, but independently of any merging, provided the galaxies are reasonably spread out to start with, the total volume of the filled region, $V_{\rm filled\,region}$, will keep on growing at the expense of the total volume of the empty region, $V_{\rm empty\,region}$, and we just stop the process when the two become equal. This demonstrates the "mathematical" existence of $v_{\rm optm}$. Its practical evaluation can always be done by trial and error, but a good starting value could be $v_{\rm optm} = \ln 2.\ (V/N)$ — the value that would result from a Poisson distribution of the number of galaxies per cell.

3.2 Test Statistics

Here we come to the heart of the present analysis. The issue is, given an ensemble of filled or empty cells, what observables or functions of observables of the individual cells can we identify whose number distributions in the given ensemble can serve as probes of the morphology of that ensemble ?

3.2.1 The Number of Like Neighbors n_1

If two neighboring cells are both filled or both empty, then their common face will be called an *inner wall*; otherwise, an *outer wall*. Let, for a given cell, n_1 be the number of its inner walls ($n_1 = 0, 1, 2, \cdots, 12$). Now consider the frequency distribution or number distribution of n_1 for the whole filled region, $N_\oplus(n_1)$. Since we know the galaxies are not distributed in space purely at random, we can expect $N_\oplus(n_1)$ to depart from the well-known binomial form.

And the same is true for the similarly defined $N_{\bigcirc}(n_1)$ for the empty region. Each of the two distributions of n_1, then, reflects some non-random features of its own region, and it may be reasonable to expect that any difference between their morphologies are in turn reflected in some difference between the two distributions. However, it should be noted here that the difference between the mean n_1- values of the two regions, $\langle n_1\rangle_{\oplus} - \langle n_1\rangle_{\bigcirc}$, tells us nothing about the intrinsic morphologies, because any non-zero difference in the mean value will be entirely due to accidental contributions from the "boundary" walls and to a departure from exact equality between the numbers of filled and empty cells. (This statement can be easily proved starting from the fact that an outer wall for a filled cell is also an outer wall for an empty cell, and that the numbers of outer and inner walls of any cell must add up to 12). Thus, only differences between $N_{\oplus}(n_1)$ and $N_{\bigcirc}(n_1)$ in respect of some parameters other than the mean value are useful to us.

3.2.2 The Topological Type τ_{m_1,m_2}

Consider a cell with n_1 inner walls and $n_2(= 12 - n_1)$ outer walls. Adjacent inner walls are said to form an *inner wall group*; similarly, an *outer wall group.* A cell with m_1 inner wall groups and m_2 outer wall groups is then said to be of topological type τ_{m_1,m_2}.

A single cell surrounded by 12 unlike neighbors is of type $\tau_{0,1}$. For further correlations, it is convenient to adopt the following nomenclature. An agglomeration of like cells which is at least 3-cells thick in at least one dimension will be called a *thick clump.* A string of single cells is called a *single strand,* a sheet one cell thick, a *mono-layer.* A 2-cell-thick string is a *2-ply*, and a 2-cell-thick sheet, a *double-layer.* The following statements are then obvious: only the inside cells of thick clumps are of type $\tau_{1,0}$; only the cells of single strands are of type $\tau_{2,1}$; only the cells of mono-layers are of type $\tau_{1,2}$; while the boundary cells of thick clumps, and the cells of double-layers and of 2-plys are all $\tau_{1,1}$. Thus, the number distribution of τ-types can only give a rough indication of the mix of various types of objects. However, when the data is of limited size, the univariate τ-distribution may be all that we can work with.

3.2.3 The Reference Bivariate (n_1, τ) *Distribution*

As already mentioned, in the case of pure random distribution of filled and empty cells (ie, when a given cell has an equal probability of on-half of being filled or empty), the univariate n_1-distribution is the binomial distribution with parameter one-half. The univariate τ-distribution in the random case is, however, unknown and must be evaluated *ab initio.* While attempting to do this, I realised that I could as easily evaluate the *bivariate* (n_1, τ)-distribution at the same time. And the latter may in any case be required when larger samples become available. Distributions for the random case will be used as reference and will be labelled as such; this particular bivariate distribution will be denoted by $N_{\mathrm{ref}}(n_1, \tau)$.

My evaluation of $N_{\mathrm{ref}}(n_1, \tau)$ was laborious. First, recalling the $(1, 1)$-correspondence between the cell faces and the edges of the generating cube, any mix of inner and outer walls of a cell is equivalent to the same mix of two types of edges of a cube (type-1 for the inner, type-2 for the outer, say). Now, we must distinguish between a *complexion* and a *configuration.* A complexion is any assignment of either 1 or 0 to each of the 12 edges (regarded as distinct or labelled) of the cube. There are altogether 4096 $(= 2^{12})$ complexions. A *configuration*, on the other hand, is any *relative* arrangement of the two types of edges on the cube. A configuration generally corresponds to f complexions, with f taking the values, 4, 6, 8, 12, 16, 24, 48, 72, 96, with very different frequencies. Now, each configuration has a unique value of n_1, and a unique τ-type. So the given configuration adds f to the frequency in the (n_1, τ)-box. Going through all the configurations results in the entire distribution $N_{\text{ref}}(n_1, \tau)$. And that is all there is to it;—in principle. In practice, it is often difficult to be sure (1) that indeed *all* configurations have been examined, and (2) that the configurations examined contained no duplications. Another source of error lurks in the evaluation of f: all symmetries in the given configuration must be noted and allowed for, otherwise f will be grossly overestimated. I found the Schlegel diagram representation of the cube (a plane graph which preserves only the topology of the cube) convenient for determining the τ-type of a given configuration, and the "unfolded cube" (where the faces are opened out along common edges onto the same plane) sometimes useful for checking whether a given configuration has not already been examined and for recognizing any symmetry it might possess. Having now detailed the difficulties, I now mention a welcome simplification: we need only evaluate for values of n_1 between 0 and 6. This can be shown as follows. Consider a (n_1, τ_{m_1,m_2}) -configuration. By interchanging the inner and outer walls (or the type-1 and type-2 edges) we obtain a $(12 - n_1, \tau_{m_2,m_1})$ -configuration. The two configurations have one and the same f-value, so this pair of "mirror configurations" make the same contribution f to their respective (n_1, τ)-boxes. But all the configurations can be so paired off, hence we have, generally,

$$N_{\text{ref}}(n_1, \tau_{m_1,m_2}) = N_{\text{ref}}(12 - n_1, \tau_{m_2,m_1}).$$

The result of my evaluation some 10 years ago is given in Table 1. I have recently repeated the evaluation and confirmed the results.

I suggested to Mr Y. F. Wu (Wu Yongfeng) of the Center of Astrophysics, University of Science and Technology of China, Hefei, that he might make an independent evaluation on the computer. He wrote a program using the C language and duly verified the entire table. We can now be confident that all the numbers given in Table 1 are correct.

Table 1 The Reference Bivariate Distribution $N_{\text{ref}}(n_1, \tau)$

τ_{--}	n_1													$N_{\text{ref}}\ [\tau]$
$\{m_1, m_2\}$	0	1	2	3	4	5	6	7	8	9	10	11	12	
{1,1}		12	24	56	126	252	340	252	126	56	24	12		1280
{2,1}			42	120	252	336	160	24						934
{1,2}						24	160	336	252	120	42			934
{2,2}					12	72	240	72	12					408
{3,1}				44	96	72								212
{1,3}								72	96	44				212
{3,2}						36	12							48
{2,3}							12	36						48
{4,1}					6									6
{1,4}									6					6
{4,2}					3									3
{2,4}									3					3
{0,1}	1													1
{1,0}													1	1
$N_{\text{ref}}\ [n_1] =$	1	12	66	220	495	792	924	792	495	220	66	12	1	4096

3.3 The Distance Effects

There are two distance effects due to properties inherent in astronomical surveys. First, since every astronomical survey covers some region on the sky, we make the most efficient use of the data when we line up our rhombic cells in fixed solid angles. This means that, in order that all the cells have the same size (volume), their radial dimension must be inversely proportional to the square of their distance. If we set the generating cube of the cell at some far, fiducial distance to be more or less a cube, ie, with all its sides more or less equal, then as we move closer, the generating cube will depart further and further from being a cube, its two transverse dimensions each decreasing as the distance r, and its radial dimension increasing as r^{-2}.

The second effect is due to the fact that, in respect of *completeness*, we can at best have surveys *complete to some apparent magnitude m*. For the CfA Catalogue, $m = 14.5$. Then suppose we consider only galaxies brighter than absolute magnitude –15.5 (which comprises 96% of all the galaxies in the CfA catalogue), then the catalogue is complete only up to distance $r = 10\,\mathrm{Mpc}$ and is increasingly incomplete after that. The completeness factor at distance r, ($r > 10\,\mathrm{Mpc}$), is

$$s(r) = \frac{\int^{M(r)} \Phi(M)\mathrm{d}M}{\int^{-15.5} \Phi(M)\mathrm{d}M} \;, \tag{1}$$

where

$$M(r) = 14.5 - 5\log(r/\mathrm{Mpc}) - 25, \tag{2}$$

and $\Phi(M)$ is the adopted luminosity function of galaxies. Now, the number of CfA galaxies lying within 10 Mpc is very small: to obtain results of any statistical significance we must include those lying beyond, but in doing so, we must increase the cell size by $s(r)^{-1}$ so as to maintain the same mathematical expectation of the number of galaxies per cell. The two distance effects together mean that the radial separation between the cell centres should vary as $1/(r^2\,\mathrm{s})$. And there is some advantage in taking 10 Mpc as the fiducial distance, thus:

$$\Delta r = \Delta r(r) = \Delta r_{10\,\mathrm{Mpc}}/[(r/10\,\mathrm{Mpc})^2 s(r)] \tag{3}$$

where $\Delta r_{10\,\mathrm{Mpc}}$ is the value of Δr at 10 Mpc, and is set equal to $(\Delta v_{\mathrm{optm}}/2)^{1/3}$ and $s(r) = 1$ for $r < 10\,\mathrm{Mpc}$, and is given by (1) for $r \geq 10\,\mathrm{Mpc}$. The advantage is that this way we get the greatest number of least distorted cells.

In practice, of course, we may cut off our sample at some reasonable value of s, 0.2 or 0.1, say.

3.3.1 The Boundary Effect

Because of the shape of the rhombic cell, we have cells with the property that, while their centres lie inside the region covered by the data, parts of them lie outside. Such cells will be called “boundary cells”. If a boundary cell is a filled cell, then we can be sure that it *is* a filled cell, but if it is an empty cell, then we cannot be sure of its empty status without knowing whether there are any galaxies in its outlying part. In either case we do not know the inner-wall/outer-wall status of any “boundary faces” (faces that adjoin outside cells). It might be suggested that we should simply discard all empty boundary cells, but this is not sufficient, for the inner/outer status of the walls of any cell next to any one of them is still uncertain. So we discard all cells next to any empty boundary cells as well. The result would be a very jagged working region. This is generally undesirable, so we discard all the boundary cells, and all the cells in the “next shell”: the consequent wastage might then prove unacceptable.

There is a simple way out. Imagine the entire observed sample of galaxies is repeated along the two transverse coordinate directions. Then, all the boundary cells will have a sure empty/filled status, and all the boundary walls, a sure inner/outer status. Of course, some spurious correlation will be introduced, but unless the region surveyed is exceptionally narrow, the effect of any such correlation can be expected to be unimportant.

4 APPLICATION TO THE CfA CATALOGUE

For the luminosity function in Equation (1) I followed Davis & Huchra (1982) and took it to be the Schechter function with $\alpha = -1.3$ and $M_* = -19.4$. Then after a few trials I found that if we set $\Delta r_{10\,\mathrm{Mpc}} = 2.2\,\mathrm{Mpc}$ (the Hubble constant is set to be $100\,\mathrm{km\ s^{-1}\ Mpc^{-1}}$ throughout this paper), then the northern sample of the CfA will give 262 empty and 248 filled cells, and the southern sample, 55 empty and 65 filled cells. The combined sample would then give 317 empty and 313 filled cells;—probably as equal a pair as we could ever get.

Incidentally, the ratio in solid angle between the two samples is $0.83/1.83 = 0.45$ (Davis & Huchra 1982), but the ratio in the number of usable cells is only $(55+65)/(262+248) = 0.24$. This is because the southern sample occupies an elongated strip in the sky and its poor return is in spite of the fact that I had moved the origin of its equal-area projection to somewhere near the middle of the strip.

Because of the limited size of the sample, results will be given only for the two marginal distributions. Figure 1 displays the observed n_1-distributions (the filled and empty ellipses) against the reference n_1-distribution (the histogram), and Fig. 2 displays similarly the observed and reference τ-distributions.

Figure 1 shows that the two observed n_1-distributions depart greatly from the binomial form. This is not surprising since we know the galaxy distribution in space is clustered rather than statistically uniform. But what is perhaps surprising is that the degree of clustering seems to be higher in the empty region than in the filled region. This statement is based on the following features not involving the mean value (cf. the *caveat* in 3.2.1): 1) The modal n_1 is 8 for the empty region as against 7 for the filled region. 2) The fraction of cells with $n_1 = 10$ for the empty region is 8%, higher than the 5% for the filled region, while the fraction of cells with $n_1 = 5$ for the empty region is 10%, lower than the 14% for the filled region.

Figure 2 shows the following notable features. 1) The observed numbers of $\tau_{1,1}$-type cells in both the empty and filled regions greatly exceed the random expectation, while the observed numbers of $\tau_{2,2}$-, $\tau_{3,1}$-, $\tau_{1,3}$-type cells in both regions fall short of the random expectations. Both these features are consistent with a tendency for likes to stick together for both filled and empty cells. 2) That the observed number of $\tau_{1,1}$-type cells is higher in the empty than the filled region is probable evidence for a greater clustering tendency in the former. However, in view of the analysis of the $\tau_{1,1}$-type cell in 3.2.2, a firmer and more precise conclusion must await an analysis of the *array* or *conditional* distribution $N(n_1|\tau_{1,1})$. 3) Both regions tend to avoid single strands ($\tau_{2,1}$-type cells). 4) The filled region, but not the empty region, seems to have more $\tau_{1,2}$-type cells than random expectation: the galaxies, but not the voids, seem to have a tendency of occurring in very thin sheets (mono-layers). This is a rather remarkable result, and should certainly be checked further with larger size data.

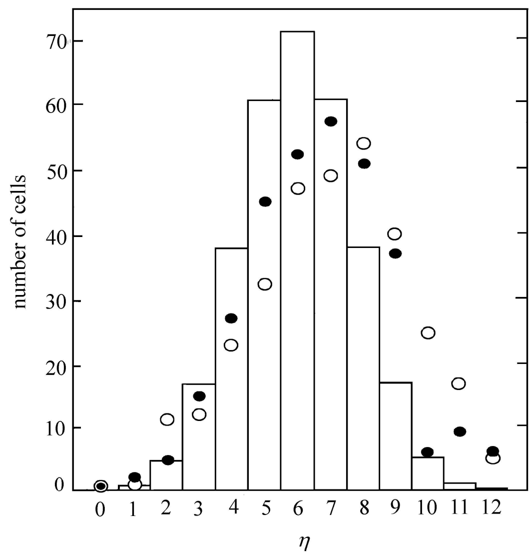

Fig. 1 Frequency distributions of the number of like neighbours n_1 for filled cells (filled ellipses) and empty cells (larger, unfilled ellipses). The histogram is the reference (binomial) n_1-distribution of Table 1, normalized to the mean observed total, 315.

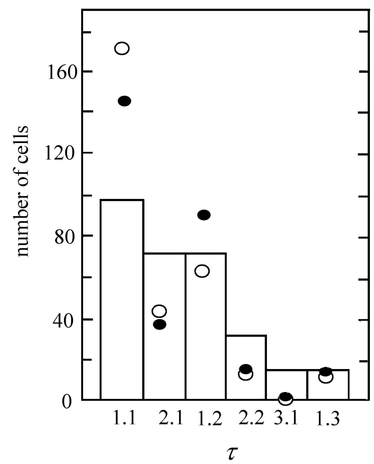

Fig. 2 Marginal distributions of the topological type τ (part only). Symbols have the same meaning as in Fig. 1 and the histogram is the reference τ-distribution of Table 1 normalized to a total of 315.

5 PROSPECTS OF FURTHER DEVELOPMENT

There is great scope for development both in the theory of rhombic cell analysis and in its applications.

Much larger survey results than the CfA Catalogue are now available. To each of these, we can, of course, apply the present method of analysis. But more than that, we can refine the analysis in two ways:

1. Recall that, because of the small size of the CfA Catalogue, I had to include the more distant regions where the catalogue is incomplete, by the device of increasing proportionately the size of the cells (Section 3.1). But this procedure implicitly assumes that the morphology in the near space on a certain standard scale (the optimal cell size) is the same as the morphology in the far space at larger and varying scales. When much larger datasets become available, we might be able to investigate the near and far regions separately, thus instead of *making* the assumption that the morphology is scale invariant, we actually *investigate* it.

2. Again, because of the limited sample size, this paper only examined the two *marginal* distributions n_1 and τ_{m_1,m_2}. When the sample is sufficiently large, we can examine the conditional distributions of n_1 at a given τ. The conditional distribution of n_1 at $\tau_{1,1}$ is of particular interest: it is by far the largest of all such conditional distributions, hence most amenable to a finer analysis, and by its analysis we may be able to estimate what fraction of the large number of $\tau_{1,1}$-type cells comes from what source, thick clumps, double-layers or 2-plys. It would be most interesting if the double-layer turns out to be a favorite form for the filled region, since we already seem to have evidence that the mono-layer is so (see end of Section 4).

As regards theoretical development, there is a completely new dimension to be explored. The present analysis is of cells considered as individuals. Now, for each cell there is a natural definition of its (like) neighbors of ranks 1, 2, 3, etc. And there should be many powerful test statistics based on the properties of entire neighborhoods of various ranks. But even within the field of individual cells, we can (i) raise the threshold for the filled cell to 2 or more galaxies, so that the resulting filled and empty regions will be closer to being underdense and overdense regions, and (ii) search for new test statistics besides n_1 and τ_{m_1,m_2}.

Of course, any one of the theoretical developments could be applied to any one of the new datasets, and conversely each new application could suggest new additions to the store of test statistics. The scope for development is very great indeed.

Acknowledgements I thank Dr X.-Y. Xia (Xia Xiao-yang) for providing me with an electronic copy of the CfA Catalogue in the early days of this work. I also thank Mr. Y. F. Wu (Wu Yongfeng) for checking the contents of Table 1 with a computer program.

APPENDIX

Geometry of the Rhombic Dodecahedron

This Appendix lists some of the regular and semi-regular properties of the rhombic dodecahedron. The dodecahedron has 12 identical rhombic faces. Figure A1 shows the linear dimensions of the rhombic face, in units of its semi-minor diameter. The 12 minor diameters of the dodecahedron coincide with the 12 edges of its generating cube. The dodecahedron has six 4-vertices (where 4 faces meet) corresponding to the six face-centres of the generating cube, and eight 3-vertices (the eight corners of the generating cube).

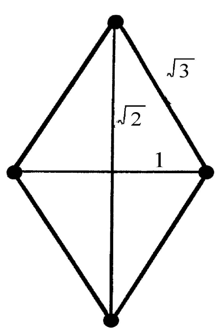

Fig. A1

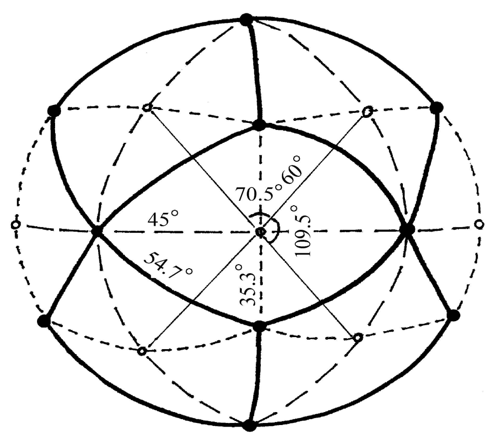

Fig. A2

Figure A2 is the apparent view of the dodecahedron seen from its centre. The 12 faces each subtends a solid angle of $\pi/3$, each has four sides of 54.7°, a major diameter of 90°and a minor diameter of 70.5°. The arrangement of the faces can be described as follows. Choose any pair of opposite 4-vertices as north and south poles, then four faces meet side-by-side at the north pole, four at the south pole and four lie length-wise along the equator. A given face has four neighbors. With respect to the centre of a given face, the centres of its four neighbors are all 60°away, but they are not isotropically placed: the directions to two neighbors sharing a 3-vertex with the given face include an angle of 70.5°, while the directions to two neighbors sharing a 4-vertex, one of 109.5°.

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