The Universe With Bulk Viscosity

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Abstract   Exact solutions for a model with variable G, Λ and bulk viscosity are obtained. Inflationary solutions with constant (de Sitter-type) and variable energy density are found. An expanding anisotropic universe is found to isotropize during its expansion but a static universe cannot isotropize. The gravitational constant is found to increase with time and the cosmological constant decreases with time as $\Lambda \propto t^{-2}$.

Key words: cosmology: theory — cosmological parameters

1   INTRODUCTION

In a recent paper, Kalligas, Wesson & Everitt (1995) have investigated a flat model with variable gravitational (G) and cosmological (Λ) "constants". Along the same line, Singh et al. (1998) have considered a viscous cosmological model with variable G and Λ. They considered a different energy conservation law from that of Arbab (1997) but, they have found similar solutions as in (Arbab 1997). Very recently, we have studied the Bianchi type I model and have shown that the universe isotropizes in the course of expansion (Arbab 1998). With a similar approach, we wish to study the effect of anisotropy in the universe with the energy conservation advocated by Singh et al. where G and Λ vary with time. We also have shown that the introduction of bulk viscosity has enriched present study of cosmology.

Kalligas et al. have found solutions for a static universe with zero total energy density while G and Λ are allowed to vary with time. However, their solution does not seem to be physically sensible, since one does not expect G to vary with time in an empty universe!

In the present case, we show that the static universe must be empty and must have a vanishing cosmological constant (Λ) and bulk viscosity (η). We have also shown that the presence of viscosity helps an anisotropic universe to isotropize during expansion. We also obtained solutions with constant and variable energy density that correspond to either static or inflationary universe. We remark that these solutions do not hold for our earlier work (Arbab 1998). The gravitational constant is found to increase with time. For a universe in balance (flat), expansion must accelerate in order to overcome future collapse. The present observed acceleration of the universe is justified in the present models.

2   SOLUTIONS FOR THE ISOTROPIC UNIVERSE

In a flat Robertson Walker metric

$$\mathrm{d}s^2 = \mathrm{d}t^2 - R^2(t)(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin\theta^2\mathrm{d}\phi^2). \tag{1}$$

Einstein's field equations with a time-dependent G and Λ read (Weinberg 1971)

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} = 8\pi G(t)T_{\mu\nu} + \Lambda(t)g_{\mu\nu}. \tag{2}$$

Variation of the gravitational constant was first suggested by Dirac (1937) in an attempt to understand the appearance of certain very large numbers, when atomic and cosmic worlds are compared. He postulated that the gravitational constant (G) decreases inversely with cosmic time. On the other hand, Einstein introduced the cosmological constant (Λ) to account for a stable static universe, as appeared to him at the time. When he later knew of the universal expansion he regretted its inclusion in his field equations. Now cosmologists believe that Λ is not identically, but very close to zero. They relate this constant to the vacuum energy that first inflated our universe, causing it to expand. From the point of view of particle physics, a vacuum energy could correspond to a quantum field that is diluted to its present small value. However, other cosmologists dictate a time variation of this constant in order to account for its present smallness. The variation of this constant could resolve some of the standard model problems. Like G, the constant Λ is a gravity coupling and both should therefore be treated on an equal footing. A proper way in which G varies is incorporated in the Brans-Dicke theory (Brans & Dicke 1961). In this theory G is related to a scalar field that shares the long range interaction with gravity. In the literature Λ takes several forms of variation, $\Lambda \propto R^{-2}$, $\Lambda \propto H^2$, $\Lambda \propto 8\pi G\rho$, etc., based on different reasonings (Overdin & Cooperstock 1998 ; Sahni & Starobinsky 1999). In the present work, we allow the variation of G to be cancelled by the variation of Λ (Beesham 1986; Abdel Rahman 1990; Pande 2000; Bonanno & Reuter 2002).

Considering the imperfect-fluid energy momentum tensor

$$T_{\mu\nu} = (\rho + p^*)u_\mu u_\nu - p^* g_{\mu\nu}, \tag{3}$$

Equation (2) yields the two independent equations,

$$3\left(\frac{\ddot{R}}{R}\right) = -4\pi G(3p^* + \rho) + \Lambda \quad, \tag{4}$$

and

$$3\left(\frac{\dot{R}}{R}\right)^2 = 8\pi G\rho + \Lambda \ . \tag{5}$$

Elimination of $\ddot{R}$ between (4) and the differentiated form of Eq.(5) gives

$$3(p^* + \rho)\dot{R} = -\left(\frac{\dot{G}}{G}\rho + \dot{\rho} + \frac{\dot{\Lambda}}{8\pi G}\right)R \ , \tag{6}$$

where a dot denotes differentiation with respect to time t and $p^* = p - 3\eta H$, η being the coefficient of bulk viscosity, H the Hubble constant. The equation of state relates the pressure

(p) and the energy density (ρ) of the cosmic fluid:

$$p = (\gamma - 1)\rho \ , \tag{7}$$

where $\gamma =$ constant. Vanishing of the covariant divergence of the Einstein tensor in Eq.(2) and the usual energy-momentum conservation relation $(T^{\mu\nu}_{;\nu} = 0)$ lead to

$$8\pi \dot{G}\rho + \dot{\Lambda} = 0 \ , \tag{8}$$

and

$$\dot{\rho} + 3(p^* + \rho)H = 0 \ , \tag{9}$$

or

$$\dot{\rho} + 3(p + \rho)H = 9\eta H^2 \ . \tag{10}$$

We see that the bulk viscosity appears as a source term in the energy conservation equation. Hence the RHS of Eq.(10) would correspond to the rate of thermal energy generated due to the viscosity. This may help solve the generation of entropy in the universe associated with the standard model of cosmology.

In this paper we will consider the very special form (Arbab 1997),

$$\Lambda = 3\beta H^2, \quad \beta = \text{const.} \ , \tag{11}$$

and

$$\eta = \eta_0 \rho^n, \quad \eta_0 \geq 0, \ n = \text{const.} \tag{12}$$

We have shown very recently that Eq.(11) is equivalent to writing Λ as $\Lambda = \left(\frac{\beta}{\beta-3}\right) 4\pi G\rho$ for a non-viscous model (Arbab 2002). This form is interesting since it relates the vacuum energy directly to the matter content in the universe. Hence, any change in ρ will immediately imply a change in Λ, i.e., if ρ varies with the cosmic time then Λ also varies with the cosmic time.

In what follows we will discuss the solution of the model equations for an isotropic universe and for an anisotropic universe with the above prescription for Λ and η.

2.1   Solution with Constant Energy Density

One can satisfy Eq.(10) with a constant energy density $(\rho = \text{const})$ with:

(i) $H = 0$, which implies a static universe.

(ii) $\eta = \eta_0 \rho$ (i.e. $n = 1$), $H = \frac{\gamma}{3\eta_0} = \text{const}$. The solution of this equation is of the form $R = \text{const.} \exp(Ht)$. We remark here that the classical inflation with an equation of state $p = -\rho$ is not permitted in this model.

2.2   Solution with Variable Energy Density

Inflationary solution with variable energy density

Consider the bulk viscosity to have the form $\eta = \eta_0 \rho$ (i.e., $n = 1$). With $H = H_0 < \frac{\gamma}{3\eta_0}$, we have $R = \text{const.} \exp(H_0 t)$ so that Eq.(10) yields a decaying mode of the energy density given by

$$\rho = F \exp -3H_0(\gamma - 3\eta_0 H_0)t \ , \quad F = \text{const.} \tag{13}$$

Now consider the case $\beta = 1$. Equations (5), (8) and (11) yield

$$\Lambda = \text{const.} \ , \quad G = 0 \ . \tag{14}$$

We remark that this solution is not possible within the framework of the conventional inflationary models. It is however remarked by Abdel Rahman (1990) that a possible interpretation of Eq.(14) is that the universe came to being just prior to the onset of gravity at $t = 0$ as a result of a vacuum fluctuation propelled by the repulsive effect of the positive cosmological constant.

3   SOLUTIONS FOR AN ANISOTROPIC UNIVERSE

For the Bianchi type I metric

$$\mathrm{d}s^2 = \mathrm{d}t^2 - R_1^2 \mathrm{d}x^2 - R_2^2 \mathrm{d}y^2 - R_3^2 \mathrm{d}z^2 \ , \tag{15}$$

with an imperfect-fluid energy momentum tensor, Einstein's field equations yield (Arbab 1998)

$$\frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} + \frac{\dot{R}_1 \dot{R}_3}{R_1 R_3} + \frac{\dot{R}_2 \dot{R}_3}{R_2 R_3} = -8\pi G\rho - \Lambda \ , \tag{16}$$

$$\frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} + \frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} = 8\pi G p^* - \Lambda \ \ , \tag{17}$$

$$\frac{\dot{R}_1 \dot{R}_3}{R_1 R_3} + \frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_3}{R_3} = 8\pi G p^* - \Lambda \ \ , \tag{18}$$

$$\frac{\dot{R}_2 \dot{R}_3}{R_2 R_3} + \frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3} = 8\pi G p^* - \Lambda \ \ , \tag{19}$$

and

$$8\pi \dot{G}\rho + \dot{\Lambda} + 8\pi G \left[\dot{\rho} + (\rho + p^*) \left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3} \right) \right] = 0 \ . \tag{20}$$

From Eqs.(16)–(20) one obtains

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3} = 4\pi G(\rho + 3p^*) - \Lambda \ , \tag{21}$$

where $p^* = p - 3\eta H$. The energy conservation ($T^{\mu\nu}_{;\nu} = 0$) implies that

$$\dot{\rho} + 3H(\rho + p^*) = 0 \ , \tag{22}$$

or

$$\dot{\rho} + 3H(\rho + p) - 9\eta H^2 = 0 \ . \tag{23}$$

Here we define the average scale factor R by $R \equiv (R_1 R_2 R_3)^{1/3}$ so that

$$H = \frac{\dot{R}}{R} = \frac{1}{3} \left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3} \right) \ . \tag{24}$$

Using Eqs.(22) and (24), Eq.(20) yields

$$\dot{\Lambda} + 8\pi \dot{G}\rho = 0 \ \ . \tag{25}$$

Let us now assume that the energy density is given by the power law

$$\rho = At^m, \quad A = \text{const.} \ , \quad m = \ \text{const.} \ , \tag{26}$$

and the average scale factor is

$$R = Bt^\alpha, \quad \alpha = \text{const.}, \quad B = \text{const.} \tag{27}$$

Substituting Eqs.(26) and (27) in (23), (11) and (25) one obtains

$$\rho = At^{-1/(1-n)}, \tag{28}$$

$$\Lambda = 3\alpha\beta t^{-2} \tag{29}$$

$$G = \frac{3\beta\alpha^2(1-n)}{4\pi A(2n-1)}t^{(2n-1)/(1-n)}, \quad n \neq \frac{1}{2}, 1 \tag{30}$$

and the condition $m = \frac{-1}{1-n}$. For physical significance $G > 0$, so $n > \frac{1}{2}$. This implies that the gravitational constant is an ever-increasing function of time. Consequently, for a flat (balanced) universe the expansion must increase (accelerate) in order that the universe can remain in balance. Thus, the present observed acceleration of the universe may be attributed to this ever growing gravity instead of invoking any exotic matter.

We now consider the anisotropy energy (σ) defined by

$$8\pi G\sigma = \left(\frac{\dot{R}_1}{R_1} - \frac{\dot{R}_2}{R_2}\right)^2 + \left(\frac{\dot{R}_1}{R_1} - \frac{\dot{R}_3}{R_3}\right)^2 + \left(\frac{\dot{R}_2}{R_2} - \frac{\dot{R}_3}{R_3}\right)^2. \tag{31}$$

Using Eqs.(5), (16) and (24), the above equation becomes

$$8\pi G\sigma = 18\left(\frac{\dot{R}}{R}\right)^2 + 48\pi G\rho + 6\Lambda. \tag{32}$$

We see that the anisotropy energy (σ) becomes

$$8\pi G\sigma = Dt^{-2}, \quad D = \text{const.}, \tag{33}$$

a result that has been obtained by Arbab (1998).

3.1 Constant Energy Solution

Using Eq.(12), Eq.(23) can be written in the form

$$\dot{\rho} + 3\gamma H\rho = 9\eta_0\rho^n H^2. \tag{34}$$

Now consider the following two cases:

(i) *Static universe* ($H = 0$, $R = \text{const.}$)

The above equation yields $\rho = \text{const.}$, and Eqs.(11) and (25) yield $\Lambda = 0$ and $G = \text{const.}$ It follows from Eqs. (5) and (32) that the anisotropy energy $\sigma = 0$. It is shown by Kalligas et al. (1995) that a static universe can only be isotropic, i.e., $\sigma = 0$. However, a static universe with a constant energy density can not exist unless $G = 0$, as is evident from Eq.(5). Hence, only an empty static universe can exist. We remark here that the claim made by Kalligas et al. that a universe with a vanishing total energy density has G and Λ vary with time is physically nonsensical.

(ii) *Inflationary universe with constant energy density* ($\rho = \text{const.}$, $H = \text{const.}$, $n = 1$)

$H = H_0 = \text{const.}$ implies $R = \text{const.} \exp(H_0 t)$. Equation (34) gives $H_0 = \frac{\gamma}{3\eta_0}$.

Hence, Eqs.(11), (25) and (32) give $\Lambda = \text{const.}$, $G = \text{const.}$ and $\sigma = \left(\frac{9H_0^2}{2\pi G}\right) = \left(\frac{\gamma^2}{2\pi\eta_0^2 G}\right)$, whether $\Lambda = 0$ or not.

4   CONCLUSIONS

In this paper, we have studied both isotropic and anisotropic models with variable G, Λ and bulk viscosity. We found that energy conservation is guaranteed provided the three scalars, G, Λ and η conspire to satisfy it. We also found that a constant energy density would lead to either a static or inflationary universe. We argued that the classical inflation of an equation of state $p = -\rho$ is not permitted. Inflationary solutions with constant and variable energy density are found. These solutions are influenced by the presence of the bulk viscosity. Such a solution is triggered by the presence of the bulk viscosity alone. An initially expanding anisotropic universe is found to isotropize as it evolves. However, a static empty universe must be isotropic. This may explain why the present universe is highly isotropic. Finally, we have found that the gravitational constant increases with time in radiation-dominated ($\gamma = \frac{4}{3}$) and matter-dominated ($\gamma = 1$) epochs. The present observed acceleration of the universe may be interpreted as due to a balance between expansion and gravity. Moreover, since gravity increases expansion must accelerate!

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