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Size-Flux Relation in Solar Active Regions *

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Abstract We present a study of the relationship between integral area and corresponding total magnetic flux for solar active regions. It is shown that some of these relationships are satisfied to simple power laws. Fractal examination showed that some of these power laws can not be justified inside the simple models of stationary magnetic flux tube aggregation. All magnetic fluxes and corresponding areas were calculated using the data measured with the Solar Magnetic Field Telescope of the Huairou Solar Observing Station in Beijing.

Key words: Sun: activity — Sun: magnetic fields

1 INTRODUCTION

Are total sunspot area and total magnetic flux proportional to each other in solar active regions?

The question of a connection between total sunspot area and total magnetic flux in solar active regions (ARs) has not been intensively studied because of its seeming evidence. It is generally supposed that these two characteristics are connected by a simple linear ratio (c.f. Bruzek & Durrant 1977; Bray & Loughhead 1964). However, our quantitative estimations on the basis of the magnetograms for several ARs, show that it is not a trivial question.

First it was revealed that the time variations of the total (daily average) sunspot area Sa and of the integral (daily average) magnetic flux Ha in similar solar ARs can have practically arbitrary time shifts relative to each other. It means that different situations may be realized at different evolutionary phases of a solar AR. For example: a decrease in the total sunspot area may accompany an increase in the integral flux. See, for instance, Figure 1 where is shown the time variations of the normalized sunspot area (Sa/MSa) and of the normalized total magnetic flux (Ha/MHa) for NOAA 5395. (MSa is average total sunspot area over the whole time of observation, MHa is the same for the total magnetic flux). However, there are also ARs where the total sunspot area and integral flux vary quite synchronously (for instance, Figure 2).

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The corrected sunspot areas of ARs were taken from the section 'Sunspot groups' of the *Solar Geophysical Data*. The total magnetic fluxes were derived from magnetograms of the Huairou Solar Observing Station. (For the reducing algorithm, see below).

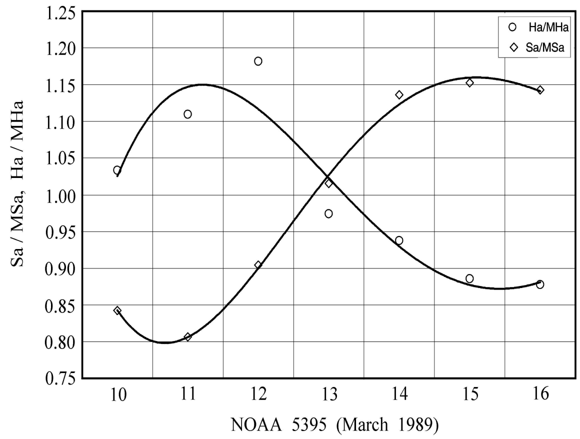

Fig. 1 Time variations of normalised total sunspot area (Sa/MSa) and normalised total magnetic flux (Ha/MHa) in NOAA 5395.

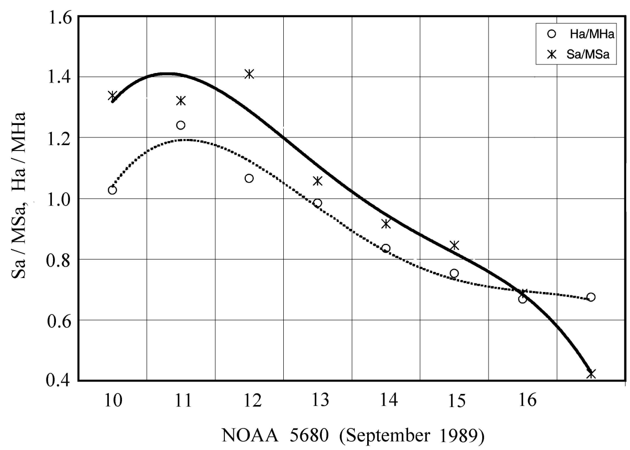

Fig. 2 Time variations of normalised total sunspot area (Sa/MSa) and normalised total magnetic flux (Ha/MHa) in NOAA 5680.

2 WHAT IS THE CORRELATION BETWEEN TOTAL MAGNETIC FLUX AND TOTAL AREA IN SOLAR ACTIVE REGIONS?

The total magnetic flux of north (N) polarity, Hn, was derived from:

$$\mathrm{Hn} = \sum_{i=1}^{\mathrm{Nn}} Bn_i \ ,$$

$$|Bn_i| > B_d.$$

Here Nn is the number of pixels of N polarity, Bn_i, the field strength in the i-th pixel, and B_d is the adopted threshold field. In this paper we used the value 20 G: it was considered sufficient for excluding noise and the week network fields. The total magnetic flux of south (S) polarity, Hs, was derived similarly.

The area of N or S polarity is defined here as the total number of pixels for in which the field strength (in absolute value) exceeds the threshold value B_d.

It was found that there was no correlation between the time variations of the area and the magnetic flux for either one or the other polarity.

A typical example is shown in Figure 3 which displays the time variations in NOAA 5669, of the normalized area of north polarity (Nhn/MNhn) and the corresponding normalized magnetic flux (Hn/MHn). Here Nhn and Hn are the daily averages, and MNhn and MHn are the averages over the whole period of observation.

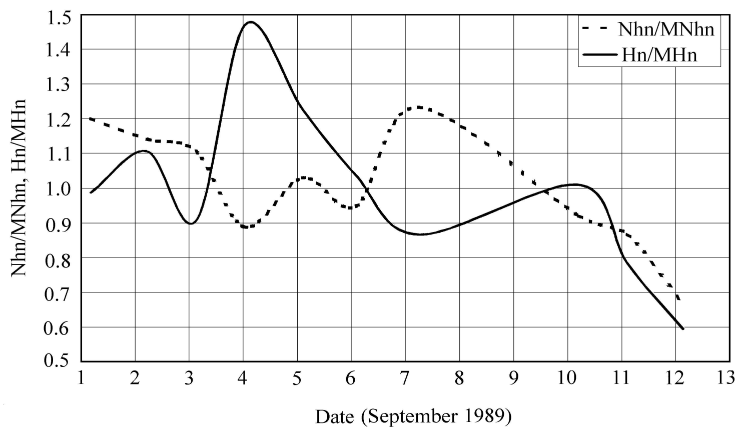

Fig. 3 Time variations of normalized area and magnetic flux of north polarity in NOAA 5669.

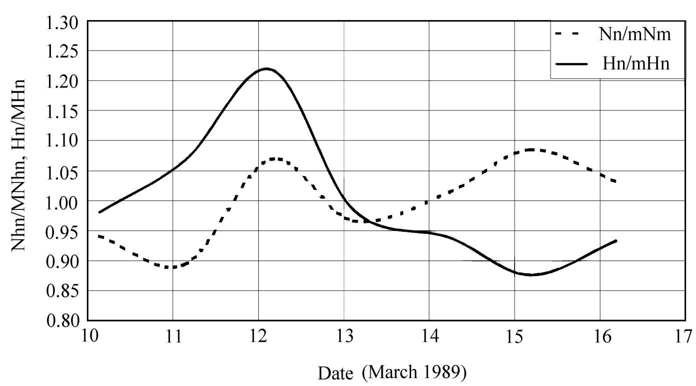

Fig. 4 Variations of normalized area and flux of north polarity in NOAA 5395.

In particular Figure 3 shows anomalous behavior of the area and the flux on 1989 September 3 — an abrupt growth in magnetic flux accompanied by a simultaneous decrease in area. A similar occurrence took place in NOAA 5395 on 1989 March 10, when a rather fast flux growth was accompanied by a slight decrease in the area (Figure 4). There are other such examples.

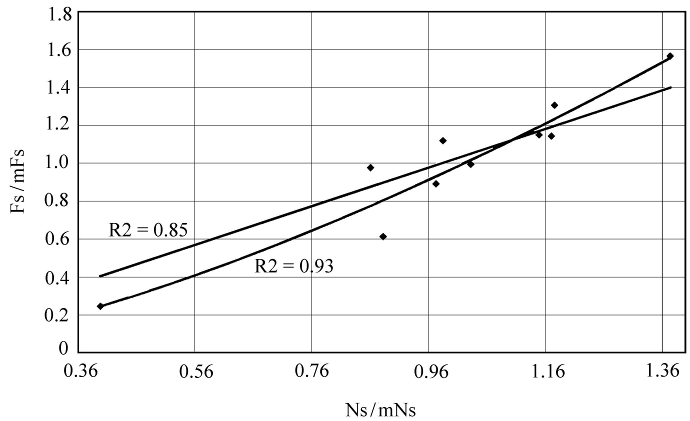

Fig. 5 Area - Flux correlation of south polarity for ten ARs.

Figure 5 shows the correlation between averaged (over the period of observation) areas and fluxes of south polarity for 10 ARs. From this figure one can see that the claim that the total magnetic flux in an AR is proportional to its area, seems to be rather arbitrary. We might consider such a proportionality as a statistical tendency. The trustworthiness, R^2, of approximating the area-flux correlation by a linear law is 0.85, while by a power law it is 0.93. So the power low appears to be preferable.

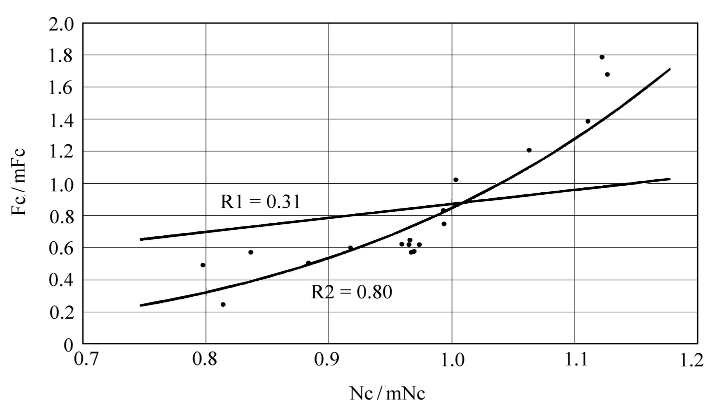

Fig. 6 Area - Flux correlation for NOAA 5629.

We can see the advantage of power law expressed more clearly in some ARs. Figure 6 shows the correlation in NOAA 5629 where we find that $R^2 = 0.80$ for the by power-law approximation, greatly exceeding the $R^2 = 0.31$ for the linear approximation. Attempts to represent the area-flux correlation by other laws gave, as a rule, lower values of trustworthiness.

3 SUMMARY OF RESULTS

In Table 1 the data concerning the area-flux correlation are listed for all 10 observed regions. The first column is a serial number for this paper, the second one gives the BAO (Beijing astronomical Observatory) serial number, the third column, the NOAA serial number. The fourth column gives the number of magnetograms. The fifth one gives the exponent q of the power law approximation to the area-flux correlation irrespective of polarity, the sixth column shows the trustworthiness of the approximation. Columns 7 and 8 show the analogous quantities for north-polarity only, and columns 9 and 10, for south-polarity only. Weighted average values of q and R^2 are given in the last line of the table, with weights based on the size of the observations and the value of R^2.

Table 1 Summary of Results

No.	HSOS	NOAA	DtNm	Common flux		N polarity		S polarity	
				q	R^2	q	R^2	q	R^2
1	36	5354	7	10.4	0.70	–1.8	0.44	1.77	0.84
2	65	5395	49	2.86	0.39	1.24	0.56	1.02	0.70
3	83	5441	25	6.75	0.54	1.57	0.17	0.43	0.21
4	192	5629	18	4.32	0.80	1.28	0.37	1.28	0.43
5	199	5634	25	1.55	0.54	1.08	0.38	1.14	0.65
6	202	5643	53	7.14	0.57	1.05	0.19	2.19	0.56
7	220	5669	58	2.12	0.79	1.76	0.70	0.86	0.34
8	225	5680	88	5.03	0.87	3.15	0.90	0.41	0.21
9	226	5686	10	2.80	0.74	–0.4	0.29	2.07	0.88
10	258	5747	37	5.54	0.92	1.27	0.95	2.0	0.85
			Aver	4.47	0.70	1.60	0.59	1.14	0.48

4 WHAT EXPONENTS ARE THE MOST PROBABLE?

The columns "Common flux" for both polarities exhibit large values of the exponent and high levels of trustworthiness of the power-law approximation. We should note the large scatter of these values and also the fact that all of the exponents exceed unity. The columns of "N polarity", on the contrary, show small and even negative exponents and low values of R^2. The negative exponents of two ARs are rather faulty, caused by large scatters in the data and small numbers of observations. Comparing the pairs of S and N polarity columns one can see that the exponents and the R^2 are higher for N polarity than for S polarity. It is also notable that *power laws with exponents exceeding unity exhibit as a rule higher levels of trustworthiness.* The last line of the table justifies this conclusion.

5 WHETHER A COMMON POWER-LAW OF AREA-FLUX CORRELATION EXISTS FOR ALL OF THE ACTIVE REGIONS?

We should pay attention to a fact which we can conclude from an analysis of the table, namely, there is apparently *no single power-law for the area-flux correlation common to all the* ARs. Moreover, this is true for both the magnetic and velocity fields. Undoubtedly the weight-averaged values of q and R^2 characterize adequately our sample, and in some sense we can assign them to all solar ARs. However, deviations from these mean values, seemingly, cannot be interpreted within the standard law of random deviation. We certainly cannot assert the statistical trustworthiness of these mean values because of the small size of our samples. Therefore, *the obtained results do not exclude the possibility of the existence of separate values of the exponent in the area-flux correlation.* However, such an assumption is not fully confirmed here and needs to be checked further using a larger sample.

6 CAN SOLAR ACTIVE REGIONS BE FRACTAL CLUSTERS OF MAGNETIC FLUX TUBES?

Since the beginning of the 1980's the question has been discussed on correlation between the chaotic component of total magnetic field in upper layers of the Sun and its structured part in the form of magnetic flux tubes (MFT). According to different estimations, a part of total magnetic flux between 50% and 95% is contained in MFT. Local magnetic fields, including the ARs fields, are considered here as being hierarchical aggregates of MFT. In addition, it is assumed that MFT are sufficiently stable objects but their basic characteristics weakly change in time.

As the last line of Table 1 shows, the most probable observed values of the exponent in the area-flux correlation exceed unity. Thus, as the area of an AR grows, its integral magnetic flux increases in a non-linear way. This implies that with increasing area, the average magnetic field also increases. The average field increase can be realized in principle in three ways: by extension of the MFT concentration, by growth of the MFT field itself, or by the combination of the two. However, for a stationary MFT model the last two possibilities can not be realized by definition. We will show that the first possibility contradicts the observational data.

We suppose that the magnetic field in the AR is stipulated by some hierarchy of stationary MFT, and the average field variations are due to variations of MFT concentration. Fractal degree c of such a structure can be determined through of the empirical rule:

$$c = \lg(P)/\lg(a)\,, \tag{1}$$

where P is the number of the components, on which the element of i-th hierarchy level is divided when it passes to the $(i-1)$ level; $a = r_i/r_{i-1}$, where r_i is linear size of the element of i-th level, r_{i-1} is correspondingly that of the $(i-1)$-th; $a > 1$ (Mandelbrot 1975).

We take R as the median AR size according to magnetometric data, r_0 the cross section size of MFT. Then we have $R = a^k r_0$, where k is a hierarchy level. Number n of MFT in the AR is equal to: $n = P^k$. Taking into account that $P = a^c$, we have $n = (R/r_0)c$.

Furthermore, we denote the magnetic flux of a MFT as F_0, then the total magnetic flux of the AR is $F = nF_0$. However, $n = (R/r_0)^c = (S/s_0)^{c/2}$, where S is the AR area according to magnetometric data, and s_0 is the cross section area of the MFT, then $F = (F_0/{s_0}^{c/2})S^{c/2}$.

Thus, for the considered AR model there is a power dependence between area and flux, where the exponent is equal to one-half of the fractal degree c. Comparing this power dependence with the power laws, found from observations, we conclude that $c = 2q$. All of the most probable values of q exceed unity (see Table 1), therefore $c > 2$. Although, it follows from definition (1), that for the considered two-dimensional model, the value of c can not exceed 2 (the filling factor can not exceed 1). Hence, *some difficulties can appear if one regards an* AR *as hierarchical association of the stationary extended* MFT.

7 CONCLUSIONS

We recall the principal results of the present work:

1. It is found that the time variations of magnetic flux and of the area corresponding to the flux, are weakly connected with each other. Synchronous changes of the two is more an exception than a rule.

2. It is shown that the connection between area and flux in AR is non-linear. The trustworthiness of the approximation of the area-flux correlation by a power law exceeds that for a linear law approximation by a factor of 3 or more. The attempts to describe this correlation by laws of other form resulted in even lower levels of trustworthiness.

3. It is also shown that there is no one single law, connecting the area and magnetic flux, for all ARs. Power laws with exponents exceeding unity have larger degrees of trustworthiness. The assumption of the existence of a discrete set of exponents in the area-flux correlation for the solar ARs has been made. However, such an assumption is not fully confirmed here and needs to be checked further using a larger sample.

4. High values of trustworthiness, found in some ARs, indicate that we can consider the corresponding exponents as invariants or quasi-invariants. In this function they can be effective for the elaboration of physically informal AR classification and also for the creation of prognosis algorithms of an AR evolution.

5. Some difficulties can arise in models which consider ARs to be hierarchical associations of the *stationary* extended MFTs.

8 DISCUSSION

It is well known that the magnetic field of a solar AR is highly structured, that it consists of aggregates of magneto-flux tubes with different cross-section sizes and magnetic fluxes. Furthermore, as one can see from a previous section, it is impossible to describe the magnetic fields of ARs with a simple model of hierarchical association of the stationary MFT. Hence we should consider alternative models. A possible scenario has been proposed in (Chumak 1990, 1995, 1996a, 1996b). In these works the author considered possible observational consequences of expanded three-dimensional structures arising in the medium of magneto-turbulent plasma as a result of a self-organization of circular currents (short magneto-flux tubes in the medium of magneto-turbulent plasma). Another possible mechanism of magneto-flux tube initiation with a changing field could be the mechanism of kinematics intensifier on convective motions (Parker 1979, 2001). However, some theoretical limitations and the absence of a clear correlation between magnetic velocity fields cause serious difficulties in this scenario.

A magnetic flux of any kinds (of separate magnetic features, pores, sunspots, active regions, large-scale places, etc.) has a value and an area. So one can ask the question: what is the ratio between these two parameters in different cases? The ratio has important physical sense. It gives some information about the magneto-plasmic structures on various spatial scales that are dominant in the given case. So it is important to analyze this ratio on different spatial scales including one of the entire AR.

In recent work (Li et al. 2001) this question was analyzed in respect to sunspots and pores. The authors have shown that there are various power low relationships for different magnetic elements. In the present work we have analyzed this ratio for the AR as a whole. We have revealed that for some of the ARs the power laws holds true with a high degree of trustworthiness for the area-flux ratio (see Table 1). It means that a structural self-similarity in a wide range of spatial scales takes place in these regions. In other cases the power law does not work well, because the spatial self-similarity is absent. Moreover, in some ARs this law can hold for only one polarity. As a rule it is the preceding polarity (mainly the S polarity in the present paper). It allows us to distinguish ARs by the degree or order of their magneto-plasmic structures. What are the physical mechanisms which cause such a structure of solar ARs? We hope that further theory will be found to answer such questions.

The authors would like to point out that the conclusions of the present paper have a preliminary character because of our small statistics. So the results can only be considered as a reference line for further investigations.

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