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The Prediction of Maximum Amplitudes of Solar Cycles and the Maximum Amplitude of Solar Cycle 24

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Abstract We present a brief review of predictions of solar cycle maximum amplitude with a lead time of 2 years or more. It is pointed out that a precise prediction of the maximum amplitude with such a lead-time is still an open question despite progress made since the 1960s. A method of prediction using statistical characteristics of solar cycles is developed: the solar cycles are divided into two groups, a high rising velocity (HRV) group and a low rising velocity (LRV) group, depending on the rising velocity in the ascending phase for a given duration of the ascending phase. The amplitude of Solar Cycle 24 can be predicted after the start of the cycle using the formula derived in this paper. Now, about 5 years before the start of the cycle, we can make a preliminary prediction of 83.2–119.4 for its maximum amplitude.

Key words: Sun: sunspots – Sun: activity

1 INTRODUCTION

The maximum smoothed monthly mean sunspot number of a solar cycle is very often called the maximum amplitude of the cycle. Increasingly more attention is being paid to the prediction of the maximum amplitude, because its great significance in applications (Elison 1959; Hirman & Goldberg1978; Allen & Wilkison 1993). Since the 1960s, there have been many attempts at precise prediction of the maximum amplitude. In particular, many papers using geomagnetic precursors or statistical characteristics of solar cycles have been published in this field (e.g. Ohl 1966; Brown 1974, 1979; Brown & Butcher 1981; Teuber et al. 1984; Wilson, Richmana & Teuber 1986; Wang et al. 1986; Thompson 1993; Wang &Han 1997; Lantos & Richard 1998; Hathaway, Wilson & Richmana 1999; Xu 2001; Wang et al. 2002). However, the long-term prediction with a lead time of 2 years or more has remained a problem. Generally, the longer the lead time is, the less precise the prediction will be. In the present paper, we limit our study only to those predictions that are made for a cycle before the sunspot number of that cycle has shown an obvious rise.

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Ohl (1966) suggested using a geomagnetic index as the precursor for predicting the maximum amplitude of the next cycle. He was successful in predicting Solar Cycle 20, with an error of about 2%. In a similar way, Ohl & Ohl (1979) made, for Cycle 21, a prediction of 183, which differed from the observed value by 11%. The error here is larger than that for Cycle 20. Sargent (1977) made a good prediction of the maximum amplitude of Cycle 21 using his own magnetic precursor method, with an error of only about 5.5%. For Cycle 22 his prediction made in 1987 has a larger error of 25% (Sargent 1987). It appears that various methods using geomagnetic precursors have been severally successful in predicting the amplitudes for Solar Cycle 20, 21 and 22. On the other hand, predictions have been made for Cycles 21 and 22 using statistical characteristics of solar cycles. For these, most of the predictions have errors greater than 20%. A method combining an extremal distribution with a time sequence was used to predict the amplitude of Cycle 21 (Wang et al. 1975) and regression equations of parameters of solar cycles were used to predict the maximum amplitude of Cycle 22 (Wang et al. 1986). The errors were as high as 44% and 37%, respectively. Brown (1986a) and Kunches (1993) published separate reviews of the predictions for Cycle 21 and 22. Their reviews showed (1) that methods using statistical characteristics of solar cycles generally give predictions lower than the observed value and with larger errors, and (2) that methods using geomagnetic precursors generally give predictions approaching or higher than the observed value. It was thought that the only prediction methods which had the approval of most of the scientific community were precursor techniques (Thomposon 1993). Quite a number of papers have been dedicated to the development of some geomagnetic precursor in long-term solar cycle prediction and these have been comprehensively reviewed by Lantos & Richard (1998).

However, a comparison of the predicted and observed amplitudes for Cycle 23 presents something new. Typical geomagnetic precursor methods predicted Cycle 23 to be a rather high cycle with a maximum amplitude higher than that of Cycle 22 and much higher than the observed maximum amplitude of Cycle 23, 120.8 (e.g. Peng et al. 1997; Thompson 1997; Lantos & Richard 1998; SEC 2001*). In contrast, some of the predictions using a non-geophysical method are quite successful. These methods had a lead time of 2 years or more, although they were published just before or after the appearance of the maximum of Cycle 23 (e.g. Kane 1999; Hathaway, Wilson & Reichmann 1999; Landscheidt 1999; Dmitrieva, Kuzayan & Obridko 2000; Rameshi 2000). In 1992 we used a method based on statistical characteristics of solar cycles and obtained a prediction of 119 for Cycle 23, with an error of 1.5% (Wang 1992; Wang & Zhang 1993) In 1997, we obtained in a similar cycle method, a predicted amplitude of 126.7 for Cycle 23, which differs by only 5% from the observed amplitude and a predicted peak time of 2000 February, only 2 months off the observed peak time (Wang & Han 1997).

A consensus appears in our simple review mentioned above that the long-term prediction of the maximum amplitude of a solar cycle, when the lead time is two years or more, is still an open question facing us.

In Section 2 of this paper, a method using statistical characteristics of solar cycles is developed and applied to predict the maximum amplitude of Solar Cycle 24. Finally, a discussion is given in the last Section 3.

2 METHOD AND RESULTS

As illustrated in Figure 1. we denote by $A(n)$ and $D(n)$ the durations of the ascending phase and descending phase, respectively, of solar cycle n, by $K(n, n+1)$ the time interval between

* Space Environmental Center, gopher//sec.noaa.gov/weekly/recent indices.txt

the successive maxima of solar cycles n and $n+1$ and by $M(n)$ the maximum amplitude of solar cycle n. Units for time are years and for $M(n)$, sunspot numbers.

A plot of $A(n)$ versus $M(n)$ for $n = 1, 2, \cdots 23$ is shown in Figure 2. The numbers beside the points are the cycle numbers. One may easily find from the distribution of these points in Figure 2 (a) that Solar Cycles 1 to 23 may be divided into two groups: there are two values of M corresponding to a given A in the figure, the upper one belongs to the group with a high rising velocity (HRV hereafter) in its ascending phase, while the lower one belongs to the other group with a low rising velocity (LRV hereafter); (b) that the HRV group consists of some, but not all, of the odd number cycles (Cycles 1, 5, 7, 9, 19 and 21);

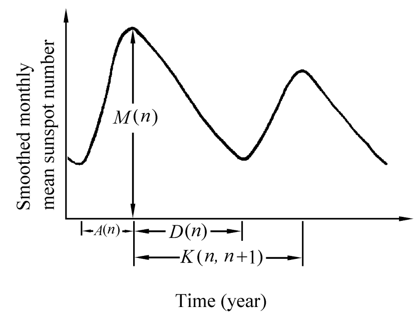

Fig. 1 Definitions of $A(n)$, $D(n)$, $K(n, n+1)$ and $M(n)$.

and (c) that the LRV group consists of all the even number cycles and the remaining odd number cycles (Cycles 2, 3, 4, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 22 and 23). Also shown in Figure 2 are the two regression lines for the two groups (Eqs. (1) and (2)). We have

$$M(n) = 311.64 - 36.96A(n)\,, \tag{1}$$

coefficient $\gamma = 0.969$ and critical coefficient $\gamma_c = 0.811$ ($\alpha = 5\%$) for the HRV cycles, and

$$M(n) = 263.30 - 37.93A(n)\,, \tag{2}$$

coefficient $\gamma = 0.963$ and critical coefficient $\gamma_c = 0.482$ ($\alpha = 5\%$) for the LRV cycles.

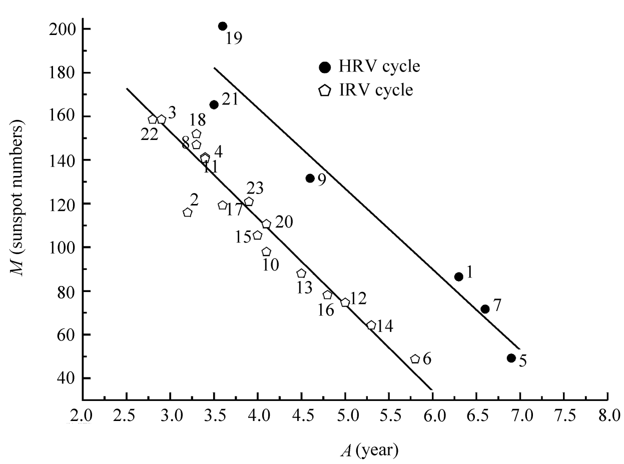

Fig. 2 A plot of $M(n)$ (in sunspot numbers) versus $A(n)$ (in years) for $n = 1, 2, \cdots 23$.

A physical or statistical link between successive cycles possibly provides a way by which we may able to predict some of parameters of the following cycle using the parameter of the preceding cycle (i.e. Ohl 1969; Thompson 1993). We find that there is a good correlation of $D(n)$ with $K(n, n+l)$, which may be one of such links (Wang et al. 1975; Wang et al. 1986; Wang 1992; Wang & Zhang 1993). To try to make a successful prediction of the maximum amplitude of Solar Cycle 24, we should find a regression equation with a high coefficient, valid for D (23) and K (23, 24). Cycles 2, 4, 9, 11, 17 and 20 are selected to be the "similar" cycles of Solar Cycle 23. The following conditions are used for the selection of "similar" cycles of Cycle 23: (a) its maximum amplitude in the range from 106 to 145 (Cycle 23 has a maximum amplitude of 120.8); and (b) its ascending phase in the range from 3.2 to 4.6 years (Cycle 23 has an ascending phase of 3.9 years). Then from the observed values of the ascending phases and maximum amplitudes of these "similar" cycles, we obtain the following relation of $D(n)$ and $K(n, n+1)$:

$$K(n, n+1) = 1.95D(n) - 3.14\,, \tag{3}$$

with a coefficient $\gamma = 0.988$ and critical coefficient $\gamma_c = 0.811$ ($\alpha = 5\%$). Equation (3) should be a reasonable approximation of the relation between Cycle 23 and Cycle 24 when we take $n = 23$ in Eq. 3.

Inserting Equation (3) into Equations (1) and (2) result in

$$M(24) = 427.77 - 35.26D(23)\,, \tag{4}$$

if Solar Cycle 24 is an HRV cycle, and

$$M(24) = 382.48 - 36.19D(23)\,, \tag{5}$$

if Solar Cycle 24 is an LRV cycle.

Very possibly, Cycle 24 is an LRV cycle because it is an even number cycle. It is therefore suggested to use Equation (5) for predicting the maximum amplitude of Cycle 24, when we have available $D(23)$ from the observed time of the solar minimum between Cycles 23 and 24.

At present the end minimum of Cycle 23 has not yet appeared. We may take the average length of the descending phases of Cycles 2, 4, 9, 11, 17 and 20 which are similar cycles to cycle 23, 7.77 years, as an estimate of $D(23)$, and by allowing 0.5 years either way, we have an estimated range from 7.77 to 8.77 years for $D(23)$. Thus, two very preliminarily predicted ranges for the maximum amplitude of Cycle 24 can be made by substituting 7.27 and 8.27 years for $D(23)$ in Eq. (4) and Eq. (5), respectively. They are 136.2–171.4 if Cycle 24 is an HRV cycle, and 83.2–119.4 if it is an LRV cycle.

Very probably, Solar Cycle 24 belongs to the LRV cycle group (see the second part of this paper), and its maximum amplitude is preferably predicted to be in the range from 83.2 to 119.4. At the same time, we have from Eq. (3), a prediction of the peak time of Cycle 24: between 2011 March and 2013 March, on taking the peak time of Cycle 23 to be at 2000 March.

3 DISCUSSION

(1) Recent history of long-term predictions of the maximum amplitude of solar cycles shows that while some methods might give good predictions for one or two cycles, no methods have been found to work well for all the recent cycles, Cycles, 20, 21, 22 and 23. This suggests that there is still room for improvement in the present long-term predictions.

(2) Differences and relations between amplitudes and between the magnetic fields of the odd and even solar cycles have been studied for many years (e.g. Gnevyshev & Ohl 1948; Durney 2000) and were used to predict the amplitudes (Tritakis 1986; Wilson 1988, 1992; Obridko 1995). For Cycle 23, Obridko (1994) obtained a very high prediction of 203.2 based on a relation of solar cycle pairs, and a low prediction of 74.7 using his new method. He pointed out that Cycle 23 seems likely to violate the regularities established for the past 125 years. In our view, however, Cycle 23 belongs to the LRV cycle group shown in Figure 2 and it is not abnormal. We should note that not all the odd cycles are HRV cycles. Different formulae were used to describe the M-A relation for odd- and even-numbered cycles, and applied to predict the amplitudes. However, according to Bray & Loughhead (1979), only a limited reliance can be placed on such forecasts or on the formulae.

(3) There has been a trend to seek some relationship between the parameters measured in the descending phase of a cycle and those measured in the ascending phase of the following cycle, for solar cycle predictions (e.g. Ohl 1966; Brown 1986b; Tompson 1993; Rameshi 2000). In our case, the physical link between the preceding cycle and the following cycle is realized by the interaction of the descending phase of cycle n with the ascending phase of cycle $n+1$, implied by Equation (3).

(4) It is essential to know the time of appearance of the end minimum of Cycle 23 when using our prediction method. To have a very preliminary prediction of Cycle 24 before the appearance of the beginning minimum of the cycle, the concept of similar cycles (Wang & Han 1997) can be used. An approximately predicted range of the maximum amplitude of 83.2–119.4 and one of the peak time of 2011 March–2013 March for Solar Cycle 24 can then be obtained. The predicted amplitude is comparable with the predicted result of 105 by Kane (1999) and is consistent with the trend pointed out by Landscheidt (1999) that weak sunspot cycles are to be expected after Cycle 23, but might be higher than that predicted by Badalyan et al. (2001) who forecast a low Wolf number cycle 24 with a maximum Wolf number not exceeding 50.

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