

# A Remark on Using Gravitational Lensing Probability as a Probe of the Central Regions of CDM Halos

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**Abstract** We calculate the gravitational lensing probabilities by cold dark matter (CDM) halos with different density profiles, and compare them with current observations from the Cosmic Lens All-Sky Survey (CLASS) and the Jodrell-Bank VLA Astrometric Survey (JVAS). We find that the lensing probability is dramatically sensitive to the clumping of the dark matter, or quantitatively, the concentration parameter. We also find that our predicted lensing probabilities in most cases show inconsistency with the observations. It is argued that high lensing probability may not be an effective tool for probing the statistical properties of inner structures of dark matter halos.

**Key words:** cosmology: dark matter — gravitational lensing — cosmology: theory — galaxies: halos

## 1 INTRODUCTION

Numerical simulations in the standard Cold Dark Matter (CDM) model predict that the mass densities of dark matter halos, from galactic to cluster scales, follow a “universal density profile” known as the Navarro-Frenk-White profile (hereafter NFW profile, Navarro, Frenk & White 1996, 1997): the density profile is described by  $r^{-3}$  for the outer regions and by  $r^{-1}$  for the inner regions of dark halos. Although there is considerable disagreement on whether the inner slope of CDM halo density profiles follow exactly a  $r^{-1}$  cusp (e.g. Huss et al. 1999; Moore et al. 1999; Jing & Suto 2000; Navarro 2001), the central cusp is presented in all the CDM simulations, and the density profiles are found to go steeply toward the center. Dynamical Observations of dark halos, however, reveal a different picture. Observations from  $H_\alpha$  rotation curves of low surface brightness galaxies suggest the existence of soft cores instead of central cusps in galactic dark halos (e.g. Swaters et al. 2000; de Blok et al. 2001; Marchesini et al. 2002), and a similar result was also reached for a galaxy cluster from strong gravitational lensing observations (Tyson et al. 1998). This conflict, known as the “cuspy core problem”, leads to one of the main difficulties of the standard CDM model, which has motivated many recent attempts to modify this model. For example, the CDM particles may be self-interacting

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(Spergel & Steinhardt 2000), or the dark matter particles could be warm (Bode, Ostriker & Turok 2001). Clearly, the study of the core regions of dark halos is crucial for our understanding of the nature of dark matter.

Gravitational lensing probes directly the total mass (mainly dark matter) distribution in the universe. The lensing of light rays from distant quasars by foreground dark halos depends largely on the amount of mass in the central regions of the halos. Therefore, statistical studies of the lensing probabilities of the splitting angles of multiple images provide a *potentially powerful* tool for probing dark halo cores, and hence revealing the fundamental properties of dark matter. Li & Ostriker (2002) made a systematic study of the lensing probabilities produced by dark halos in different cosmogonies, and found that the lensing probability is extremely sensitive to the cosmological models. Their estimates of the concentration parameter, however, lacked accuracy. Some other papers (e.g. Oguri et al. 2001) have suggested that the predicted lensing number would also have a large scatter due to different values for the index of the inner slope of the dark halo density profile. However, a too negative inner slope would conflict conspicuously with the observed rotation curves of low surface brightness galaxies.

Following Li & Ostriker (2002), we calculate in this paper the lensing probabilities for different dark matter density profiles, especially for the standard NFW profiles with inner slope index  $-1$ , for different values of the concentration parameter  $C$ . The results are then compared with observations from the Cosmic Lens All-Sky Survey (CLASS) and Jodrell-Bank VLA Astrometric Survey (JVAS). We find that none of the density profiles match well the current observations, and that the predicted lensing probability is dramatically sensitive to the concentration parameter  $C$ . Throughout the paper we take a fiducial  $\Lambda$ CDM cosmogony with  $\Omega_M = 0.3$ ,  $\Omega_\Lambda = 0.7$ , and  $h = 0.68$ .

## 2 HALO DENSITY PROFILE

The “universal” NFW halo density profile is described by

$$\frac{\rho(r)}{\rho_{\text{crit}}(z_0)} = \frac{\delta_c(z_0)r_s^3}{r(r+r_s)^2}, \quad (1)$$

where  $\delta_c$  is the characteristic density parameter which implies the clumping of a halo at redshift  $z_0$ ,  $\rho_{\text{crit}}$  is the critical density of the universe, and  $r_s$  is defined in terms of the virial radius  $R_{\text{vir}}$  of a halo and the concentration parameter  $C$ , such that  $r_s = R_{\text{vir}}/C$ . The N-body results of Navarro et al. (1997, hereafter NFW97) suggest that

$$\delta_c(z_0) = \bar{C} \cdot \Omega_m(z_0) \cdot \left( \frac{1+z_{\text{coll}}}{1+z_0} \right), \quad (2)$$

where  $\bar{C} \sim 3000$  is a fitting coefficient. According to the extended Press-Schechter formalism (Bond et al. 1991; Lacey & Cole 1993), the formation time of an NFW97 dark matter halo is defined such that half of the mass of a halo at redshift  $z_0$  is contained in its progenitors more massive than a fraction  $f = 0.01$  of the final mass. This is equivalent to setting the following probability to 1/2:

$$P(> fM_{z_0}, z_{\text{zoll}} | M_{z_0}, z_{\text{zoll}}) = \text{erfc} \left\{ \frac{\delta_{\text{crit}}(z_{\text{coll}}) - \delta_{\text{crit}}(z)}{\sqrt{2[\sigma^2(fM_{z_0}) - \sigma^2(M_{z_0})]}} \right\}. \quad (3)$$

Here  $\delta_{\text{crit}}(z_0)$  is the linear overdensity threshold of mass collapsing model with top-hat spherical filters, and  $\sigma(M)$  is the rms perturbation of the mass within this comoving filtering scale, both

of which are extrapolated to the present time. Assuming top-hat filter, we have

$$\sigma^2(M) = \frac{1}{2\pi^2} \int_0^\infty k^2 P(k) W^2(kR) dk, \quad (4)$$

where  $W(kR) = 3 [\sin(kR) - kR \cos(kR)] / (kR)^3$  is the top-hat window function, and  $P(k) = AkT^2(k)$  is the initial  $k$ -space power. For normalization, we take  $\sigma_8 = 0.85$ . As reported by Bardeen et al. (1986), the transfer function for the CDM scenario is

$$T(k) = \frac{\ln(1 + 2.34q)}{2.34q} \times [1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4]^{-0.25}, \quad (5)$$

where  $q = (k/h \text{Mpc}^{-1})/\Gamma$ ,  $\Gamma = \Omega_M h \exp[-\Omega_b(1 + \sqrt{2h}/\Omega_M)]$  is the shape parameter with the baryon density  $\Omega_b$ .

For an NFW halo, since the total mass distributed with the profile of Eq. (1) inside the virial radius is  $M_{z_0}$ , we have

$$\delta_c(z) = \frac{\Delta_c(z)}{3} \frac{C}{\ln(1+C) - C/(1+C)}, \quad (6)$$

where  $\Delta_c \sim 100$  denotes the true density contrast of a virialized object to the critical density for closure of the universe,  $\rho_{\text{crit}} = 3H^2/8\pi G$ . In this paper we take Bryan & Norman's results (1998), accurate to within 1% for  $0.1 < \Omega_M < 1$ ,

$$\Delta_c(z) = \begin{cases} 18\pi^2 + 82x - 39x^2, & \Omega_M + \Omega_\Lambda = 1; \\ 18\pi^2 + 60x - 32x^2, & \Omega_\Lambda = 0, \end{cases} \quad (7)$$

where  $x = \Omega_M(z) - 1$ . Note we simply take  $\Delta_c = 200$  in the NFW97 case, following the original paper.

However, the above semi-analytic method to determine the parameters of NFW expression suffers from having two free parameters,  $\bar{C}$  and  $f$  that are somewhat arbitrary and their values lack reasonable physical meaning. Eke, Navarro & Steinmetz (2001, hereafter ENS) studied the dependence of dark halo concentration on the power spectrum and obtained a new fit to the NFW profile. The ENS fits define an "effective" amplitude of the power spectrum which leads to the halo collapsing time,

$$\begin{cases} \sigma_{\text{eff}}(M) = \left| M \frac{d\sigma(M)}{dM} \right|; \\ D(z_{\text{coll}}) \sigma_{\text{eff}}(M_s) = 1/C_\sigma, \end{cases} \quad (8)$$

where  $M_s$  is the mass contained within  $r_{\text{max}} = 2.17r_s$  where the circular velocity of NFW halo reaches its maximum, so

$$M_s = M_{z_0} \frac{0.47}{\ln(1+C) - C/(1+C)}, \quad (9)$$

and the linear positive perturbation growth factor,  $D = [g(z)/g(0)]/(1+z)$ , was presented in some previous papers (Heath 1977; Carroll et al. 1992) as

$$g(z) = \frac{5/2 \Omega_M(z)}{\Omega_M(z)^{4/7} - \Omega_\Lambda(z) + (1 + \Omega_M(z)/2)(1 + \Omega_\Lambda(z)/70)}. \quad (10)$$

In the ENS case, the scenario of halo formation via hierarchical clustering is described by setting the characteristic density  $\tilde{\rho}_s$  equal to the spherical collapse top-hat density at  $z_{\text{coll}}$ , that is,

$$\begin{cases} \tilde{\rho}_s = \Delta_c(z_{\text{coll}}) \rho_{\text{crit}}(z_{\text{coll}}); \\ M_{z_0} = \frac{4\pi}{3} \tilde{\rho}_s r_s^3. \end{cases} \quad (11)$$

This yields

$$C = \frac{\Delta_c(z_{\text{coll}})}{\Delta_c(z)} \frac{\Omega_M(z)}{\Omega_M(z_{\text{coll}})} \left( \frac{1+z_{\text{coll}}}{1+z} \right)^3. \quad (12)$$

Obviously, the ENS fitting has the advantage of containing only one free parameter,  $C_\sigma \sim 25$ . Moreover, with different  $C_\sigma$ , the ENS results can as well describe the Warm Dark Matter (WDM) halos.

### 3 LENSING EQUATIONS OF DARK HALOS

Supposing the universe is homogeneous on large scales, with relatively small mass clumps in it, we can accurately describe the universe by the Robertson-Walker geometry. We treat the radio quasars as point sources, and the dark halos as deflectors. When light travels through the center of a mass, the deflecting angle is about  $4Gm/pc^2$ , where  $p$  is almost exactly the distance from where the light travels through the lens to its center if the deflections involved in the lensing effect is small. So the lensing equation (e.g. Schneider & Ehlers 1992; Wu 1996) is

$$\bar{y} = \frac{D_s}{D_d} \bar{x} - D_{ds} \frac{4Gm(\bar{x})}{c^2 \bar{x}}, \quad (13)$$

where  $m(\bar{x})$  is the mass contained within the radius where the light rays travel through,  $\bar{y}$  and  $\bar{x}$  denotes the physical positions of the source and the images about the projection of the observer to the source and the lens planes,  $D$  is the angular diameter distance between two objects in the universe, and the subscripts  $d$  and  $s$  stand for the deflector and the source, respectively.

Combined with the density profiles of the deflectors (lenses), Eq. (13) can be solved to find the splitting angles subtended by multiple images. For NFW halos, the lensing equation is transformed to

$$\frac{D_d}{D_s} \frac{\bar{y}}{r_s} = \frac{\bar{x}}{r_s} - \frac{4\rho_s r_s}{\Sigma_{\text{crit}}} f\left(\frac{\bar{x}}{r_s}\right), \quad (14)$$

where  $\Sigma_{\text{crit}} = (c^2 D_s / 4\pi G D_d D_{ds})$  is the critical surface mass density, and

$$f\left(\frac{\bar{x}}{r_s}\right) = f(s) = \int_0^s y dy \int_0^\infty \frac{dz}{(y^2 + z^2)^{1/2} [1 + (y^2 + z^2)^{1/2}]^2}. \quad (15)$$

Since Eqs. (14) and (15) involve very small angles, one has to work them out numerically.

Meanwhile, for comparison, we can also solve the lensing equations with deflectors of the Singular Isothermal Sphere (SIS) profile,

$$\rho_{\text{SIS}}(r) = \frac{\sigma_v^2}{2\pi G} \cdot \frac{1}{r^2}, \quad (16)$$

where  $\sigma_v = (GM/3R_{\text{vir}})^{1/2}$  is the velocity dispersion of the halo. One can analytically work out the splitting angle  $\Delta\vartheta$  and the lensing cross section  $\sigma_{\text{CS}}$  produced by SIS-profiled deflectors:

$$\begin{cases} \Delta\vartheta = 8\pi \frac{D_{ds}}{D_s} \left(\frac{\sigma_v}{c}\right)^2, \\ \sigma_{\text{CS}} = 16\pi^3 \left(\frac{D_d D_{ds}}{D_s}\right)^2 \left(\frac{\sigma_v}{c}\right)^4. \end{cases} \quad (17)$$

Obviously, the SIS profile has a more ‘‘cuspy’’ core than the improved NFW profile. Hence a SIS halo is expected to have a higher efficiency of lensing.

#### 4 THE PROBABILITIES OF LENSING

Once the sources are located within the lensing cross section, each background object with redshift less than  $z_s$  (in this paper we take  $z_s = 1$ ) will give rise to a probability of forming multiple images. Integrating from  $z = 0$  to  $z_s = 1$ , one has

$$P = \int_0^{z_s} (1+z)^3 \frac{cdt}{dz} \int \sigma_{cs}(M, z) \frac{dN}{dM dV} dM dz, \quad (18)$$

where  $\frac{cdt}{dz}$  can be easily derived from the Friedman equation and  $\frac{dN}{dM dV}$  is the comoving spatial halo number density within  $M \sim M + dM$ . Press & Schechter proposed their halo counting formula in 1974, known as the PS theory, which agrees well with the simulation results (Sheth et al. 1999, Jenkins et al. 2001),

$$\frac{dN}{dM dV} = -\sqrt{\frac{2}{\pi}} \frac{\bar{\rho}_0}{M} \frac{\delta_{\text{crit}}(z)}{\sigma^2} \frac{d\sigma}{dM} \exp(-\delta_{\text{crit}}^2(z)/2\sigma^2). \quad (19)$$

Here  $\delta_{\text{crit}}(z)$  is the same as in Equation (3), given by  $\delta_{\text{crit}} = \delta_{\text{crit}0}/D(z)$ , with  $\delta_{\text{crit}0} \approx 1.68$  demonstrating the linear overdensity threshold for halo collapsing with top-hat filters.

#### 5 RESULTS AND DISCUSSION

We apply the splitting angles and the cross sections produced by all the deflectors presented in Section 3 to Eqs. (18) and (19) to obtain the integrated and differential lensing probabilities. Figure 1 shows that the predicted probability from SIS halos marginally agrees with the CLASS and JVAS lensing surveys: one candidate out of  $10^3 \sim 10^4$  sources over arc second separation. The NFW profile, however, produces much smaller probabilities, both in the NFW97 case and the ENS case (even when we take into account the magnification bias for magnitude-limited sample of lensed QSO survey [e.g. Turner et al. 1984; Takahashi et al. 2001]). For NFW dark halos, the lensing probabilities differ by several orders of magnitude, though the NFW97 and ENS estimates predict similar concentration parameters.

It is obvious in Fig. 1, that the number of predicted lenses varies greatly for different estimates of the NFW profile. Note that in our paper, all the work is done within one cosmological model. The dashed line is plotted with the parameter estimate of  $C = 7/(1+z)$  for the flat universe (Bartelmann 1998; Li & Ostriker 2002): this estimate is rather coarse because N-body simulation has shown that the concentration level increases with decreasing halo mass in the CDM model, hence is mass-dependent. The dotted line is for the ENS estimate with  $C_\sigma = 25$ , where for a  $10^{14} M_\odot$  halo,  $C \simeq 6$  at  $z = 0$ . We can see that this value is less than Li & Ostriker's estimate, and such a difference produces the divergence of several orders of magnitude of separation over arc seconds. A low-redshift dark halo of the ENS case has a similar concentration parameter as in the NFW97 case. However, the NFW97 halo produces a greater lensing probability partly due to their simulation only producing a small number of halos identified at  $z = 0$ , so we could not check the redshift dependence of halo concentration or any over-prediction of  $C$  for high-redshift halos. In addition, the result of NFW97 is believed to be reliable only in their  $\Lambda$ CDM model with  $\sigma_8 = 1.3$ , so it is possible that, in our calculations (where  $\sigma_8 = 0.85$ ), NFW97 could not predict very precise concentration parameters. Compared to other estimates, the ENS case also includes an examination of the dependence of the halo concentration on  $\sigma_8$ .

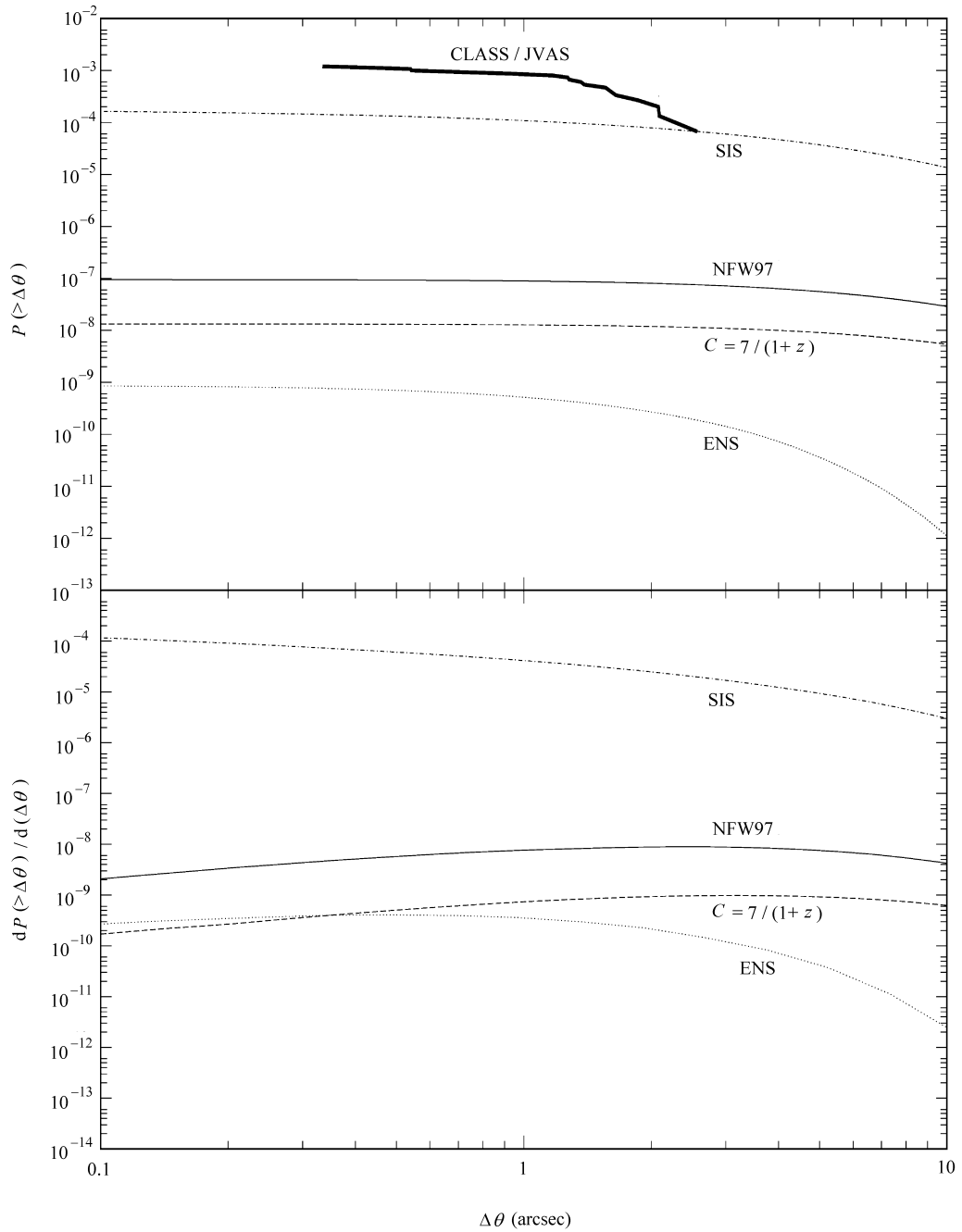


Fig. 1 Upper panel: Total lensing probabilities of angular separations greater than  $\Delta\theta$ . From top to bottom, the heavy solid line represents the CLASS-JVAS survey and the predicted probabilities from different estimates of  $C$ : with  $\bar{C} = 3400$  for NFW97,  $C_\sigma = 25$  for the ENS case. The lower panel shows the corresponding differential probabilities of each prediction.

With the different choices of  $C_\sigma$ , one can precisely predict the concentration of dark halos in various cosmogonies.

In order to explicitly show how sensitive the predicted lensing probability is to the concentration parameter in the NFW model, we also plot in Fig. 2 and Fig. 3, from top down, for three different values of  $C_\sigma = 30, 25, 20$ , for the ENS case. The large divergence seen in Fig. 3 stems simply from the difference of  $C$  in Fig. 2.

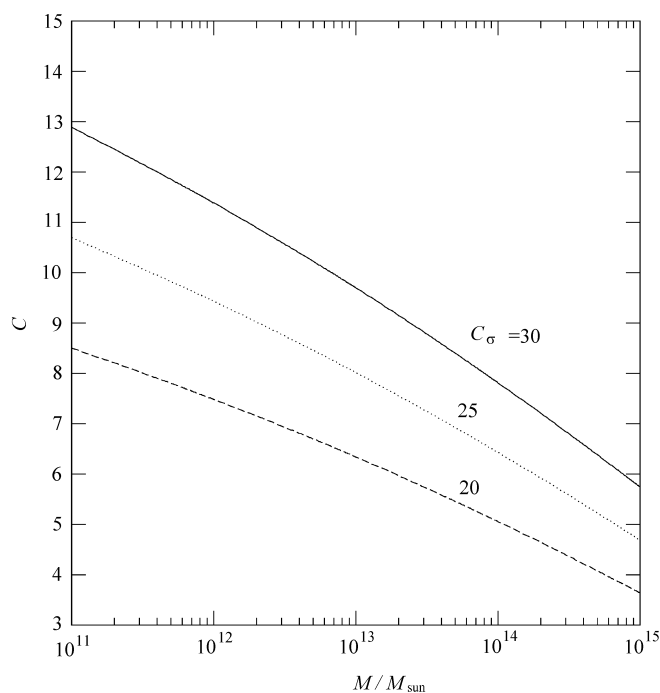


Fig. 2 The concentration parameter  $C$  versus halo mass for  $C_\sigma = 30, 25, 20$  in the ENS case.  $z = 0$ .

N-body simulations (e.g. Bullock 2001) suggest that the density profile of dark halos is subject to the variance of a log-normal distribution, so the uncertainties presented above would still remain even if we use their results in our lensing calculations. This seems to be a fatal flaw of current semi-analytic predictions of statistical lensing caused by dark halos. Moreover, recent research has shown that the condensed baryonic components (stellar bulges and disks) within galactic dark halos are very likely to change, and even to dominate the inner part of the mass distribution within a radius the order of 1 kpc (e.g., Mo et al. 1998). Hence, condensed baryons are expected to contribute much to the lensing efficiency of galactic halos. Improvement in N-body simulation and in our understanding of the formation of baryonic structures within dark halos can be expected to help resolving the problem in the calculation of lensing probabilities.

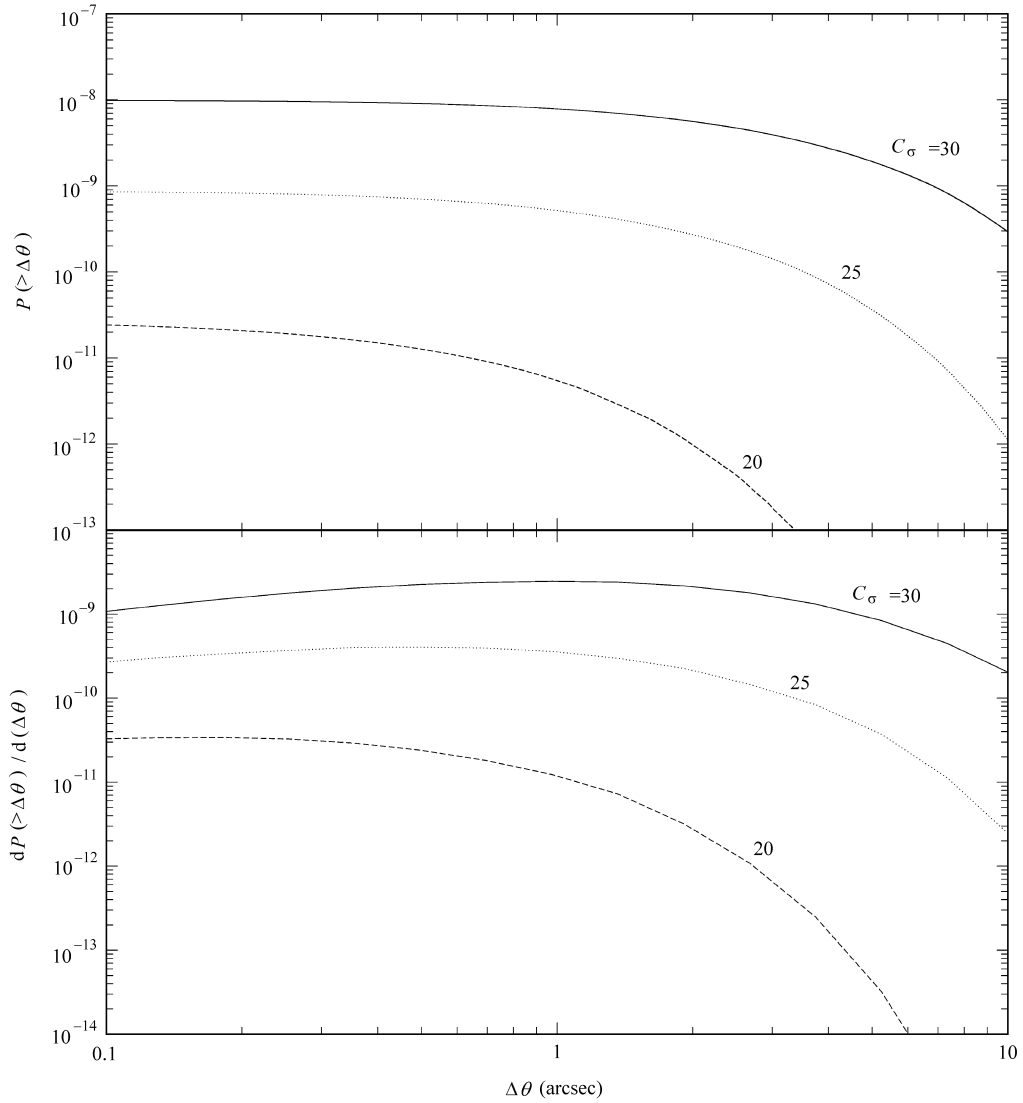


Fig. 3 Total and differential probabilities predicted by the cases of Fig. 2.

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