Chin. J. Astron. Astrophys. Vol. 2 (2002), No. 5, 454–462
(http://www.chjaa.org or http://chjaa.bao.ac.cn)

Chinese Journal of
Astronomy and
Astrophysics

The Radial Structure of Pulsar Radio Emission Regions

Xin-Ji Wu[1,2,3 ⋆], Zhan-Kui Huang[1,2] and Xuan-Bin Xu[1,2]

[1] Department of Astronomy, Peking University, Beijing 100871
[2] The Chinese Academy of Sciences-Peking University Joint Beijing Astrophysics Center, Beijing 100871
[3] Urumqi Astronomical Observatory, National Astronomical Observatories, Chinese Academic of Science, Urumqi 830011

Received 2002 May 30; accepted 2002 July 24

Abstract An important parameter in the study of the radial structure of the pulsar radio emission region is the altitude of the emission, but this cannot be derived directly from the observations. The altitude can be expressed as a function of frequency, $r_\nu \propto \nu^{-\xi}$, and the method of K analysis can be used to calculate the power law index ξ from observations at different frequencies. We have calculated the value of ξ for 18 pulsars observed at two frequencies, 610 MHz and 1408 MHz and for three pulsars observed at three or more frequencies. The average value of ξ is 0.27, which indicates that the emission altitude increases with decreasing frequency and that the radial structure is compact.

Key words: stars: pulsars

1 INTRODUCTION

The radial structure of the emission region of pulsars has been of concern since these objects were discovered, but the altitude of the emission region cannot be determined directly from observation. It is widely believed that the radio emissions of different frequencies originate from different heights above the polar cap, the higher frequencies coming from lower altitudes.

Cordes (1978) assumed a radius to his frequency mapping model (RFM) to explain the systematic increase of the separation between the profile components and of the profile widths, with decreasing frequency. Several authors (Cordes 1978; Blaskiewicz et al. 1991 (BCW91); Philip 1992 (Phi92); Gil & Kijak 1993 (GK93); Kijak & Gil 1997 (KG97), 1998 (KG98); Wu 1999 (Wu99)) have put forward various methods of calculating the location of the radio emission at different frequencies.

The basic timing method of estimating the altitude of the emission region was first put forward by Cordes (1978) and applied to three pulsars. The separation between the emission regions for 430 MHz and 1400 MHz was found to be less than $\sim 10^2 - 10^3$ km, or a few percent of the light cylinder radius. Phi92 analyzed the timing data of four pulsars; after subtracting the

⋆ E-mail: wuxj@bac.pku.edu.cn

effect of retardation, aberration and magnetic field sweep back, he found the radial separation between the emission regions of 47 MHz and 4800 MHz to be less than 200 km.

BCW91 found that the relativistic electron emission and the rotation of the pulsar could delay the center of linear polarization of position angle by $\Delta t = 4r/c$ with respect to the center of the intensity profile. They obtained the values of the altitude $r_{\rm delay}$ at 430 MHz and 1408 MHz for 11 pulsars.

According to the polar cap model, the altitude of emission can be expressed as a function of the period (P) and the opening angle (ρ) of the pulsar beam (GK93, KG97)

$$\rho = 1.24 s r_6^{1/2} P^{-1/2} \,, \tag{1}$$

where $r_6 = r/R$ is the emission altitude in units of the pulsar radius, s is a parameter that labels the field lines that contribute to the emission, $0 < s \leq 1$, $s = 0$ is the axis of the magnetic field, $s = 1$ the last opened field line. In KG's papers $s = 1$ is assumed.

In order to obtain the opening angle of emission beams, GK93 and KG97 used the geometric relation of the polar cap

$$\rho = \arccos\left[\cos\beta - 2\sin\alpha\sin\theta\sin^2(\Delta\phi/4)\right] , \tag{2}$$

in which $\theta = \alpha + \beta$, α is the magnetic inclination, β is the angle between the line of sight and the magnetic axis, $\Delta\phi$ is the apparent width of the mean profile in degrees.

The emission altitudes can be calculated by Equations (1) and (2), using the $\Delta\phi$ for different frequencies, and the θ, α published in journals. GK93 and KG97 obtained the emission altitudes for 430 MHz and 1420 MHz. They argued that the altitude of radio emission region depends on pulsar period approximately as $P^{0.5}$ (GK93) and as $P^{0.3}$ (KG97). However, GK's method is restricted by their adoption of the parameters β, α from others' papers. Obviously, the published data of θ and α given by different authors are based on different $\rho - P$ relations (LM88, Kuzmin 1984; Kuzmin & Wu 1992; Xu & Wu 1991; R93; G94). It is obvious that different opening angles ρ would be obtained by using these various $\rho - P$ relations. If one chooses the relation $\rho \propto P^{-0.5}$, then the emission altitude r must be a constant. The value of the parameter s in Equation (1) also bothers us in that only a part of the Goldreich-Julian polar cap is involved in the emission. It means that s is less than 1. In fact, the potential difference across the accelerating gap of pulsar can decrease from center so sharply that pair-avalanche cannot be produced in regions near the last open field line. GK (93) and Lin & Wu (1996) discussed the relationship of the parameter s and parameter B/P^2.

The form of the radius-frequency mapping depends on the physics of the emission mechanism. Usually a power-law relationship between the emission altitude and the frequency is assumed

$$r_\nu \propto \nu^{-\xi} \,, \tag{3}$$

where ξ is a constant. The index ξ is an important parameter for understanding the radial structure of the emission region in the pulsar magnetosphere. In this paper we use the multi-frequency K analysis to study the index ξ for 18 pulsars with good linear polarization curves at 610 MHz and 1408 MHz and three pulsars with good linear polarization curves at more than three frequencies. This study avoids the estimation of the values of β, α and s.

2 THE METHOD OF K PARAMETERS ANALYSIS

In order to study the evolution of the inclination angle and emission angle, Xu et al. (1991) and Wu et al. (1992) defined a parameter K under the polar cap model

$$K = \frac{\sin\Delta\Psi}{\sin\Delta\phi} = \frac{\sin\alpha}{\sin\rho}\,, \tag{4}$$

where $\Delta\Psi$ is the half polarization position angle swing, $\Delta\phi$ is the half apparent beam width. K is an observable parameter since both $\Delta\Psi$ and $\Delta\phi$ are observable parameters, α is the inclination angle and ρ is the half angle of emission cone.

The altitude ratio of emission at two frequencies can be obtained by using Equation (1) and Equation (4)

$$m = \frac{r\,(f_1)}{r\,(f_2)} = \frac{\rho^2\,(f_1)}{\rho^2\,(f_2)} \approx \frac{K^2\,(f_2)}{K^2\,(f_1)}\,, \tag{5}$$

from this formula it is straightforward to obtain the ratio of the emission altitudes at two frequencies. The power-law index ξ can then be derived

$$m = \frac{r\,(610)}{r\,(1408)} = \left(\frac{610}{1408}\right)^{-\xi}\,, \tag{6}$$

$$\xi = 2.75\log\left(\frac{K\,(1408)}{K\,(610)}\right)^2 = 2.75\log m\,. \tag{7}$$

The value of ξ can be obtained from the K parameters. So ξ is also determined by observational data. The great advantage of this method is that it does not require the calculation of the opening angle ρ from published values of θ and α, nor the consideration of the parameter s in Equation (1).

The apparent beam width $\Delta\phi$ can be obtained from mean pulse data with high signal to noise ratio, but to obtain the position angle swing $\Delta\Psi$ is much more difficult even for high precision radio observations. The linear polarization position angle is a function of the longitude in the profile. For some pulsars the swing of the position angle is an "S" curve, which can be well understood within the rotating vector model (Radhakrishnan & Cooke 1969). In such cases the parameter $\Delta\Psi$ can be measured easily. However, in real situations the polarization position angle curves are very complex, with depolarization in the some parts and jumps here and there. Some curves are asymmetric or lack a large part of the "S" curve. The depolarization and the jumps due to relative longitude shift of the pulsar beams were investigated by Xu et al. (1997, 2001)

For those pulsars with complex polarization position angle curves, $\Delta\Psi$ cannot be obtained. In this paper we need to know K parameters on two or more frequencies, which means that all of the pulsars in our sample must have good polarization data on at least two frequencies. This strict requirement prevents us from having a large data base.

Since 1990s several group (Wu et al. 1990, 1993; Qiao et al. 1995; Manchester et al. 1998; G94; GL98) have carried out systematic polarization observations of the mean pulse profile because of their fundamental importance. The largest such sample was given in the thesis of G94. We have selected from this source 18 pulsars with good $\Delta\phi$ and $\Delta\Psi$ data at 610 MHz and 1408 MHz.

The pulse profile has no precise boundary, so we use the apparent beam width at the 10% intensity level. Rankin (1983) found the phenomena of profile absorption in some pulsars. We have excluded all the pulsars that might suffer from profile absorption.

3 THE POWER LAW INDEX ξ

Table 1 shows the observational parameters at 610 MHz and 1408 MHz and the calculated results of the altitude ratio m and the parameter ξ for the 18 selected pulsars. It is obvious that the K analysis at more than two frequencies is valuable for understanding the structure of the pulsar emission region. However, there are a few pulsars with good polarization data at three or more frequencies. Table 2 shows the results of K analysis for three such pulsars. Figure 1 shows the relation between the relative altitude r/r_{1408} and frequency. The curves are the fitted results with power law index (ξ) 0.18 for PSR 0525+21, 0.21 for PSR 0628–28 and 0.18 for PSR 2045–16. There are small differences in the value of ξ using two and three or more frequencies.

Table 1 Observed Data and Derived Parameters

PSR	$\Delta\phi_{610}$	$\Delta\phi_{1408}$	$\Delta\psi_{610}$	$\Delta\psi_{1408}$	K_{610}	K_{1408}	m	ξ
0301+19	18.2	16.3	117.5	122.5	5.41	6.18	1.31	0.32
0525+21	20.4	18.8	153.2	153.2	5.49	5.96	1.18	0.19
0628-28	38.0	35.6	120.3	120.3	2.66	2.84	1.13	0.15
1039-19	19.6	17.7	131.5	142.7	5.36	6.16	1.32	0.33
1133+16	12.1	11.4	88.6	88.6	6.63	7.03	1.13	0.14
1702-19	18.0	17.3	101.3	101.3	4.94	5.14	1.08	0.09
1737+13	23.8	22.0	154.4	177.2	4.73	5.24	1.23	0.24
1811+40	15.7	15.1	139.2	145.6	6.86	7.27	1.12	0.14
1839-04	81.9	70.7	182.3	183.5	1.53	1.73	1.28	0.30
1916+14	9.4	8.0	139.2	139.2	11.4	13.4	1.38	0.38
2011+38	51.1	43.7	31.6	32.9	0.63	0.76	1.45	0.45
2021+51	19.7	16.9	62.0	62.0	3.01	3.50	1.36	0.36
2045-16	17.4	16.1	143.0	158.2	6.27	7.01	1.25	0.27
2154+40	27.5	25.6	126.6	125.3	3.76	4.01	1.14	0.15
2306+55	26.6	24.1	153.2	154.4	4.23	4.67	1.22	0.24
2319+60	26.1	22.9	101.3	101.3	3.42	3.90	1.29	0.31
2323+63	37.0	34.2	183.5	183.5	3.15	3.40	1.16	0.18
2324+60	31.3	26.4	145.6	177.2	3.54	4.38	1.53	0.51

Table 2 The K Parameter and Relative Altitude r/r_{1408}

PSR		410 MHz	610 MHz	925 MHz	1408 MHz
0525+21	K	—	5.49	5.81	5.92
	r/r_{1408}	—	1.16	1.04	1
0628–28	K	2.45	2.64	2.71	2.82
	r/r_{1408}	1.32	1.14	1.08	1
2045–16	K	5.98	6.28	—	6.99
	r/r_{1408}	1.37	1.24	—	1

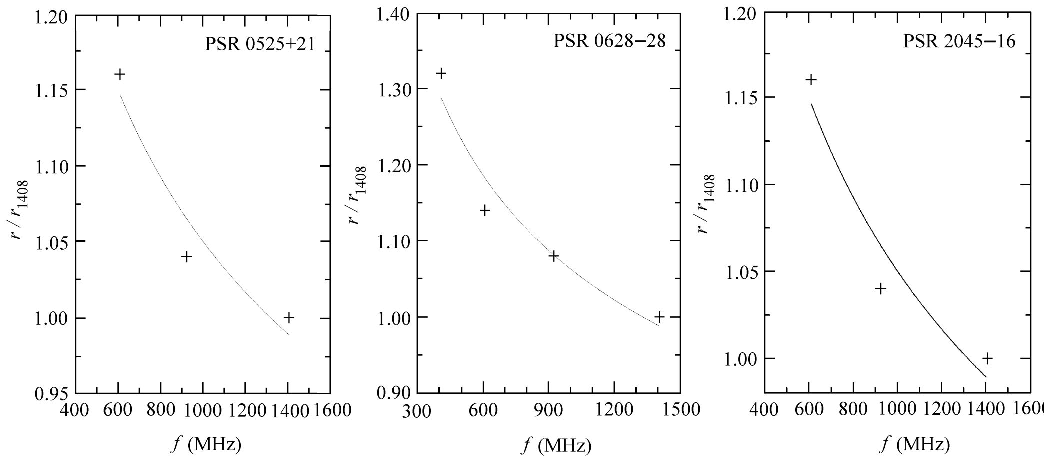

Fig. 1 Relation between the relative altitude r/r_{1408} and frequency for three pulsars.

All of the 18 pulsars have $m > 1$ and $\xi > 0$, which shows that the emission region of 610 MHz is higher than that of 1408 MHz, in agreement with the RFM model. The value of m is 1.25 ± 0.12 (ranging from 1.08 to 1.53), the power law index (ξ) is 0.27 ± 0.12 (ranging from 0.09 to 0.51). These estimates show that the pulsar emission region is very compact.

Theoretically the value of the spectral index (ξ) is a constant ranging from $\xi = 0$ (Barnad & Arons 1986) to $\xi = 2/3$ (Ruderman & Sutherland 1975). The results of 18 pulsars in this paper are clearly desirable for discriminating the competing theories, and our results are consistent with the assumption of dipolar field in the radio emission zone.

The average value ξ we derived in this paper is comparable to those obtained by using other methods (Table 3). All the values of ξ given by this and other papers and listed in Table 3 are less than 0.66, the upper limit given by Phi92.

For some individual pulsars, the relation between emission altitude and frequency has been known: For PSR 1451–68 at 0.17–1.62 GHz: $r(\mathrm{km}) = 225 f_{\mathrm{GHz}}^{-0.37}$ (Wu et al. 1998); for PSR 1857–26 at 0.17 – 2.65 GHz: $r(\mathrm{km}) = 249 f_{\mathrm{GHz}}^{-0.30}$ (R93); for PSR 2111+40 at 0.408–4.85 GHz: $r(\mathrm{km}) = 235.5 f_{\mathrm{GHz}}^{-0.14}$ (Xu et al. 2002).

Table 3 Comparison of the Values of ξ Given by This and Other Papers

ξ	Frequency Range (GHz)	Number of Pulsars	Ref.
0.21±0.1	0.43–1.42	11	BCW91
≤0.66	0.05–4.80	4	Phi92
0.12±0.08	0.43–1.42	6	GK93
0.11±0.02	1.41–10.6	5	K94
0.21±0.07	0.43–1.42	10	KG97
0.26±0.09	0.43–1.42	16	KG98
0.21±0.1	0.43–1.42	8	Wu99
0.27±0.12	0.61–1.41	18	This paper

Figure 2 shows the relation between ξ and the period P. The correlation coefficient between ξ and P is -0.244, which means the correlation is very weak. It seems that there is a boundary for ξ in Fig. 2, and all the pulsars are located to the left of this boundary. The upper limit of $\xi_{\rm up}$ in the more certain period interval is related to the period, increasing with decreasing P.

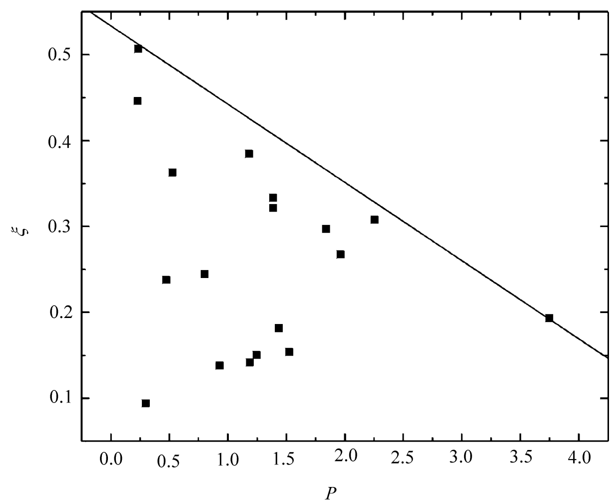

Fig. 2 Relation between ξ and P.

4 THE ABSOLUTE ALTITUDE OF THE EMISSION REGION

The K analysis method cannot give the absolute altitude of the emission region directly. For this we use the method of R93 and GK93. The formula for the absolute emission altitude given by R93 is

$$r(\mathrm{km}) = 6.66\rho^2 P\,. \tag{8}$$

We prefer to choose the method of LM88 to avoid the difficulty arising from different $\rho - P$ relations. There are 15 pulsars with the values of α and β in the tables 1 and 2 of LM88, based on the relation $\rho = 6.5P^{-0.33}$ and the $\Delta\phi_{10}$ at about 400 MHz. We found that the values of $\Delta\phi_{10}$ at 400 MHz are more or less different from the results given by GL98. For some pulsars the GL98 pulse profiles have higher S/N ratios than the LM88 ones. PSR1916+14 was taken out, because the values of $\Delta\phi_{10}$ at 400 MHz from GL98 and LM88 are not consistent and the S/N ratio is low in both cases. For three pulsars with no α and β values in LM88, the GL98 data were used to estimate the $\Delta\phi_{10}$ and the maximum rate of position angle swing $(\mathrm{d}\Psi/\mathrm{d}\phi)_{\max}$ for PSR 2011+38 and PSR 2021+51 at 400 MHz and for PSR 1839–04 at 600 MHz. The obtained values of $(\mathrm{d}\Psi/\mathrm{d}\phi)_{\max}$ are 23, 1.2 and 2.0 for PSR 1839+04, PSR 2011+38 and PSR 2021+51, respectively.

We calculated the values of α and β again using the new data by the method of LM88 and the altitude of emission region r(km) using Equations (2) and (8). The results are given in Table 4.

In Table 4, r_{610} and r_{1408} are the two emission altitudes given by Equation (8) and Equation (2). Those altitudes are underestimates, because $s = 1$ was adopted.

The average altitude of emission above the surface of the neutron star is 290.1 ± 100.6 (km) at 610 MHz, and 236.4 ± 86.4 (km) at 1408 MHz. The total extent of the radio emission region between 0.61 GHz and 1.408 GHz is only 54.2 ± 29.2 (km). Figures 3 and 4 show the relations between the emission altitude and the period.

Table 4 Emission Altitudes at 610 MHz and 1408 MHz

PSR	P (s)	α (degree)	β (degree)	r_{610} (km)	r_{1408} (km)
0301+19	1.388	36.4	–1.8	287.6	236.7
0525+21	3.745	23.0	0.7	418.9	357.7
0628-28	1.244	16.3	–3.7	295.1	273.0
1039–19	1.386	34.6	–1.8	302.2	252.0
1133+16	1.188	48.4	3.7	279.3	260.1
1702–19	0.299	90.0	–4.1	194.5	182.2
1737+13	0.803	40.7	–2.5	338.3	294.0
1811+40	0.931	69.9	2.7	386.4	360.9
1839-04	1.84	7.4	0.3	341.1	257.2
2011+38	0.23	9.0	7.4	127.5	115.9
2021+51	0.529	14.9	7.3	221.0	212.2
2045–16	1.962	33.7	–1.1	311.1	268.7
2154+40	1.525	20.3	2.7	333.4	298.9
2306+55	0.475	37.0	1.4	214.8	177.5
2319+60	2.256	20.8	–2.3	366.8	300.8
2323+63	1.436	14.1	2.1	262.9	231.0
2324+60	0.234	30.9	3.8*	133.7	101.7

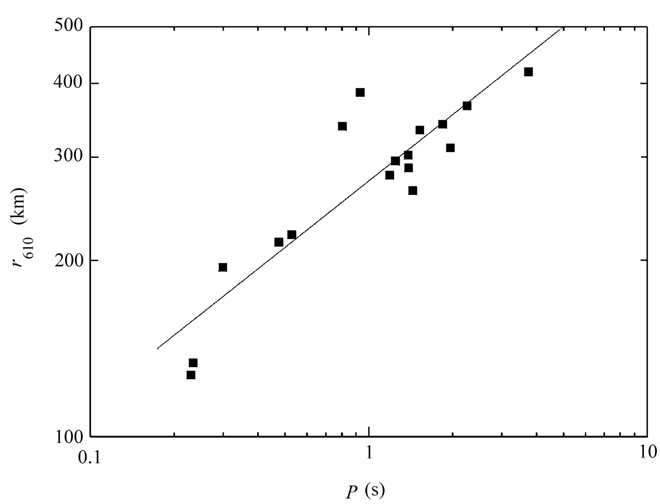

Fig. 3 Altitude of emission at 610 MHz versus pulsar period.

The regression equations of the altitude on the period at 610 MHz and 1408 MHz are

$$r(610) = 273.0\, P^{0.38\pm0.06}(\mathrm{km})\,, \tag{9}$$

$$r(1408) = 236.6\, P^{0.36\pm0.08}(\mathrm{km})\,. \tag{10}$$

The correlation coefficient R of $\log P$ and $\log r$ is 0.89 for 630 MHz and 0.84 for 1408 MHz. The emission altitude r_6 is proportional to the $P^{0.38}$ (610 MHz) and $P^{0.36}$ (1408 MHz). GK93, KG97 and G98 found $r \propto P^{0.35}$ for data of Arecibo at 1.42 GHz, $r \propto P^{0.39}$ for data of Effelsberg at 1.41 GHz and $r \propto P^{0.32}$ for data of Arecibo at 0.43 GHz (KG97). All of these results show that the emission altitude at different frequencies is a function of the period.

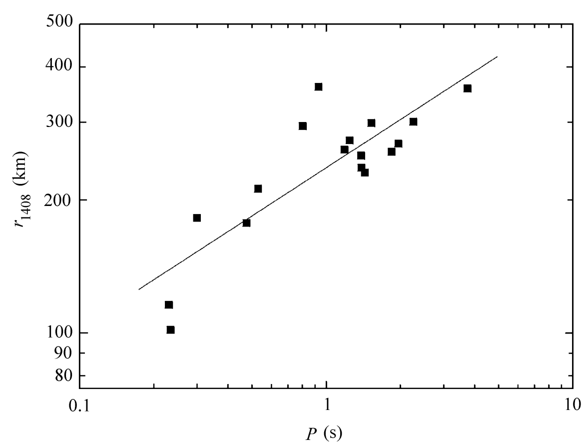

Fig. 4 Altitude of emission at 1408 MHz versus pulsar period.

5 SUMMARY AND DISCUSSION

The method of the K parameter analysis was used to calculate the power law index ξ of the altitude-frequency relation ($r_\nu \propto \nu^{-\xi}$) directly from observational data at different frequencies. The values of ξ are obtained for 18 pulsars observed at two frequencies, 610 MHz and 1408 MHz, and for three pulsars observed at three of more frequencies. The average of the power law index ξ is 0.27, which is consistent with those estimated using different methods (Table 3). The emission altitude increases with decreasing frequency and the radial structure is compact.

The altitudes of the emission regions of 610 MHz and 1408 MHz are calculated using the method of R93 and GK93. The values of α and β were calculated using the new data of apparent beam width ($\Delta\phi_{10}$) according the method of LM88. The average value of the emission altitude is 290.1 $\pm$100.6 (km) above the surface of neutron star at 610 MHz, and the 236.4 ± 86.4 (km) at 1408 MHz. The total extent of the radio emission region between 0.61 GHz and 1.408 GHz is only 54.2 ± 29.2 (km).

There are close relations between the period and the altitudes at 610 MHz and 1408 MHz, and they are in agreement with the results of GK93 and KG97. However, we need to consider that the method of calculating α and β may affect the resulting altitude-period relation, due to the $\rho - P$ relations introduced into their simultaneous equations, such as $\rho = 6.5P^{-0.33}$ at 400 MHz of LM88. If this relation of $\rho \propto P^{-0.33}$ is applicable to different frequencies, then we can obtain the relation $r \propto P^{0.33}$ from Equation (1), which is similar to the results of $r \propto P^{0.38}$ at 610 MHz and $r \propto P^{0.36}$ at 1408 MHz given by this paper.

Acknowledgements This work was supported by the National Natural Science Foundation of China under Grant 10073001.

References

Barnard J. J., Arons J., 1986, ApJ, 302, 138
Blaskiewicz M., Cordes J. M., Wasserman I., 1991, ApJ, 370, 643 (BCW91)
Cordes J. M., 1978, ApJ, 222, 1006
Manchester R. N., Han J. L., Qiao G. J., 1998, MNRAS, 259, p.280
Gil J., Kijak J., 1993, A&A, 273, 563 (GK93)
Gould D. M., Ph. D thesis, University of Manchester of UK, 1994 (G94)
Gould D. M., Lyne A. G., 1998, MNRAS, 301, 235 (GL98)
Kijak J., Gil J., 1997, MNRAS, 288, 631 (KG97)
Kijak J., Gil J., 1998, MNRAS, 299, 855 (KG98)
Kramer M., Weilebinski R., Jess ner A., Seiradakis J. H., 1994, A&AS, 107, 515 (K94)
Kuzmin D. A., Dagkesamanskaya I. M., Pugachev V. D., 1984, Soviet Stron. Letters 10, 854
Kuzmin D. A., Wu Xinji, 1992, ApSS, 190, 209
Lin J. L., Wu X. J., 1996, In: Qiao G.J., He X.T., Han J. L., eds., Theory and Observation of Compact Astronomical Objects, The 8th Guo Shoujing workshop of astrophysics, p.243
Lyne A. G., Manchester R. N., 1988, MNRAS, 234, 477 (LM88)
Manchester R. N., Han J. L., Qiao G. J., 1998, MNRAS, 259, p.280
Philip J. A., 1992, ApJ, 385, 282 (Phi92)
Qiao G. J., Manchester R. N., Lyne A. G., Gold D. M., 1995, MNRAS, 274, p.572
Radhakrishnan V., Cooke D. J., 1969, ApJ, 3, L225
Rankin J. M., 1983, ApJ, 274, 333
Rankin J. M., 1993, ApJ, 405, 285 (R93)
Rankin J. M., 1993, ApJS, 85, 145
Ruderman M. A., Sutherland P. G., 1975, ApJ, 196, 51
Wu Xinji, Manchester R. N., Lyne A. G., Qiao Guojun, 1993, MNRAS, 261, 630
Wu Xinji, Manchester R. N., Lyne A. G., 1990, Acta Astrophysica Sinica, 11, 227 (Chin. Astron. Astrophys., 15, 423)
Wu Xinji, Xu Wen, Rankin J. M., 1992, In: Proceeding of IAU Colloquium 128, Poland, p.172
Wu Xinji, Gao Xueyan, Rankin J. M., Xu Wen, Malofeev V. M., 1998, AJ, 116, 1984 (Wu98)
Wu Xinji, Xu Wen, 1992, In: Proceeding IAU Colloquium 128, Poland, p.18
Wu Xinji, Yu K. Y., Zhang C. M., Wen Xueshi, 1999, Chinese Physics Letters, 16(1), p.74 (Wu99)
Xu W, Wu X. J., 1991, ApJ, 380, 550
Xu R. X., Qiao G. J., Han J. L., 1997, A&A, 323, 395
Xu R. X., Xu J. W., Qiao G. J., 2001, Chinese Journal of Astronomy and Astrophysics, 1, 152
Xu X. B., Wu X. J., 2002, Science Sinica, in press