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Determination of the Proper Motions and Membership of the Globular Cluster M3 and of its Orbit in the Galaxy

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Abstract We examine 14 plates of the globular cluster M3 (NGC 5272) taken with the 40 cm refractor at the Sheshan station of Shanghai Astronomical Observatory. The plates span over a period of about 77 years. The positions and absolute proper motions of eight stars in the Hipparcos Catalogue and of 49 stars in the Tycho-2 Catalogue are used as the reference frame. The astrometric reduction is made with the central overlapping principle. The absolute proper motions of 534 stars in a region of about $100' \times 100'$ around the cluster are measured. With the new proper motion data the membership probabilities of the stars are determined. The average absolute proper motion obtained for the cluster is -0.06 ± 0.30 mas yr^{-1} in R.A. and -2.6 ± 0.30 mas yr^{-1} in Decl. By combining this result with the known distance and radial velocity of the cluster, we also obtained the Galactic orbit of M3 for a chosen three-component Galactic potential.

Key words: globular clusters — star cluster: membership probabilities — star cluster: proper motions — star cluster: space motions — star cluster: Galactic orbits

1 INTRODUCTION

Globular clusters are among the oldest objects in the Milky Way. Their kinematics data are of great significance in the study of the structure and evolution of the Galaxy. In recent studies of the kinematics of globular clusters, the emphasis is to calculate their orbits for some given galactic potential model. For this, the position and velocity of the cluster are needed. And to obtain the velocity components, we must have observational data on its proper motion, radial velocity and distance. Of these, the present accuracy of radial velocity determination is typically ±1 km s^{-1} and the accuracy in the distance determination, is in general better than ±10 percent; but there are greater difficulties in the proper motion determination of globular clusters. First, the proper motion data of these clusters are poor and in the database of 147 known globular clusters (Harris 1999) only 37 have given absolute proper motions (Dinescu,

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Girard & van Altena 1999). Moreover, the systems on which the known absolute proper motions of the clusters are based are different and the average difference between the systems can be as high as 2 mas yr^{-1} (Geffert et al. 1993). This will introduce a velocity error about 100 km s^{-1} at a distance of 10 kpc.

For the determination of absolute proper motions of globular clusters, the Hipparcos Catalogue provides a quasi-inertial reference frame. Using stars in the Hipparcos Catalogues as reference stars, Odenkirchen, Brosche and Geffert (1997) determined the absolute proper motions of 15 globular clusters and studied their orbital motions.

The stars in the Hipparcos Catalogue, as reference stars in photographic astrometry, are sparse and relatively bright. Thus, their accident errors can easily slip into the system of plate reduction and so the derived absolute proper motions may significantly deviate from the system of the Hipparcos Catalogue. In view of this contingency, Wu and Wang (2000) included stars from the ACT Catalogue into the set of reference stars in the determination of absolute proper motion of the globular cluster M3 (NGC 5272). The ACT Catalogue is based on the system of the Hipparcos but the precision of proper motions is relatively low. In the Tycho-2 Catalogue, published in 2000, the observations of the satellite Hipparcos were combined with 144 catalogues of ground-based observations and the number of stars and precision of astrometry exceeded those of the ACT Catalogue. In the present work, we calculate the absolute proper motion of the globular cluster M3 using the Tycho-2 Catalogue and then study its Galactic orbit.

2 OBSERVATIONS AND REDUCTION OF PROPER MOTIONS

All the plates used in this work were taken with the 40 cm refractor with focal length 6895 mm at Sheshan station of Shanghai Astronomical Observatory. The relevant details of the plates are shown in Table 1. These plates were taken without filters and the sensitive wavelength of their emulsion range is close to the B-passband. The number of stars measured on each plate is 534 for both the early and late epoch sets of plates. These stars spread over a region of about $100' \times 100'$ around the center of the cluster.

Table 1 Observational Record of Plates

Earlier epoch plates		Later epoch plates	
Plate No.	Observation date	Plate No.	Observation date
P345	1911–05–21	CL86011	1986–04–16
P403	1915–05–05	CL92009	1992–04–27
P404	1915–06–11	CL92010	1992–04–27
P405	1915–06–13	CL92011	1992–05–20
P486	1919–05–29	CL92012	1992–05–23
		CL92014	1992–05–24
		CL92015	1992–05–24
		CL92017	1992–05–27
		CL92021	1992–06–06

All of the plates were measured using the PDS-1010MS microdensitometer at the Purple Mountain Observatory. Window scanning was used in the measurement and the size of a scanning pixel was $10\mu m \times 10\mu m$. The R scanning mode and a scan speed of 5 mm s^{-1} were used. The scanning results were fitted to 2-D circular Gaussian function to obtain the measured coordinates of image centers. For the $200'' \times 200''$ central region containing the cluster, a fast and full-area scanning was first carried out to determine rough coordinates of the star images. Then, based on these rough coordinates, the precise coordinates were obtained with the window scanning (Wang, Zhao & Chen 1990; Wang & Chen 1992; Wang 1994, 1995). Of the 534 stars measured, eight Hipparcos stars and 49 Tycho-2 stars were identified. These 57 stars were used as reference stars. Their positions and proper motions were taken respectively from the Hipparcos Catalogue and the Tycho-2 Catalogue.

The reduction of proper motions was done by means of central overlap for successive improvement (Russell 1976; Wang, Chen & Zhao 1996). In the first loop of iteration, the 57 Hipparcos and Tycho-2 stars were used as reference stars for the plate reduction. After a test, six linear and six quadratic terms of coordinate were included in the plate reduction. In the second loop, the preliminary positions and proper motions of all the measured stars obtained in the first loop were referred to, and the plate reduction model was then tested again. The first-order term of magnitude was then added and the reduction model was not tested any more in the succeeding loops of iteration. In every loop, the catalogue positions and proper motions of the 57 Hipparcos and Tycho-2 stars, weighted by their errors, were included in the star reduction as observations. They provided some constraints for the reduction. Measurements with too large residuals were rejected in the star reduction but the results of position and proper motion for all 534 stars were derived in all the loops of iteration. The parameters of the internal standard errors of the final positions and proper motions are given in Table 2.

Table 2 Parameters of Internal Errors of Positions and Proper Motions

Parameter	Median Error	Maximum Error
α	$0.0024^{\rm s}$	$0.0670^{\rm s}$
δ	$0.027''$	$1.383''$
$\mu_\alpha \cos\delta$	0.75 mas yr^{-1}	25.60 mas yr^{-1}
μ_δ	0.61 mas yr^{-1}	20.74 mas yr^{-1}

3 ESTIMATION OF MEMBERSHIP PROBABILITIES

In our estimation of the cluster membership probabilities of the stars, both the distributions of positions and proper motions are taken into account. We assume a homogeneous distribution Ψ_f with average density n_f for the positions of field stars and a Gaussian distribution Ψ_c with central density n_0 and dispersion α for the cluster stars

$$\Psi_f = n_f \quad \text{and} \quad \Psi_c(r_i) = n_0 \exp\Big(-\frac{r_i^2}{2\alpha^2}\Big),$$

where r_i is the radial distance of the ith star from the cluster center. Let ψ_f and ψ_c be the normalized Ψ_f and Ψ_c: $\psi_f + \psi_c = 1$, and

$$\psi_f(r_i) = \frac{1}{1 + g/\exp(\frac{r_i^2}{2\alpha^2})} \quad \text{and} \quad \psi_c(r_i) = \frac{1}{1 + \exp(\frac{r_i^2}{2\alpha^2})/g},$$

where $g = n_0/n_f$. The distributions for the proper motions of field and cluster stars are independent of each other and we assume an elliptic Gaussian distribution Φ_f the field stars and a circular Gaussian distribution Φ_c for the cluster stars

$$\Phi_f(r_i,\mu_{xi},\mu_{yi},\varepsilon_{xi},\varepsilon_{yi}) = \psi_f(r_i)\cdot\frac{1}{2\pi(1-\rho^2)^{1/2}(\sigma_{x0}^2+\varepsilon_{xi}^2)^{1/2}(\sigma_{y0}^2+\varepsilon_{yi}^2)^{1/2}} \cdot\exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{(\mu_{xi}-\mu_{xf})^2}{\sigma_{x0}^2+\varepsilon_{xi}^2}-\frac{2\rho(\mu_{xi}-\mu_{xf})(\mu_{yi}-\mu_{yf})}{(\sigma_{x0}^2+\varepsilon_{xi}^2)^{1/2}(\sigma_{y0}^2+\varepsilon_{yi}^2)^{1/2}}+\frac{(\mu_{yi}-\mu_{yf})^2}{\sigma_{y0}^2+\varepsilon_{yi}^2}\right]\right\},$$

$$\Phi_c(r_i,\mu_{xi},\mu_{yi},\varepsilon_{xi},\varepsilon_{yi}) = \psi_c(r_i)\cdot\frac{1}{2\pi(\sigma_0^2+\varepsilon_{xi}^2)^{1/2}(\sigma_0^2+\varepsilon_{yi}^2)^{1/2}} \cdot\exp\left\{-\frac{1}{2}\left[\frac{(\mu_{xi}-\mu_{xc})^2}{\sigma_0^2+\varepsilon_{xi}^2}+\frac{(\mu_{yi}-\mu_{yc})^2}{\sigma_{y0}^2+\varepsilon_{yi}^2}\right]\right\},$$

where (μ_{xi},μ_{yi}) are the two components of proper motion for the i-th star and $(\varepsilon_{xi},\varepsilon_{yi})$ are the (known) estimated observing errors of (μ_{xi},μ_{yi}), (μ_{xc},μ_{yc}) and (μ_{xf},μ_{yf}) are the centers of the cluster and field stars on the proper motion vector point diagram, σ_0 and $(\sigma_{x0},\sigma_{y0})$ are the corresponding intrinsic dispersions, and ρ is the correlation coefficient. All the last eight free parameters plus α and g were determined by the maximum likelihood procedure and the maximum of the likelihood function was searched by a bisection method (Wang 1997). The uncertainties of the parameter estimates were estimated from the second derivatives of the likelihood function (Zhao & He 1987).

By using the estimates of parameters found above, the membership probability of the i-th star can be calculated by

$$p_i = \frac{\Phi_c(r_i,\mu_{xi},\mu_{yi},\varepsilon_{xi},\varepsilon_{yi})}{\Phi_f(r_i,\mu_{xi},\mu_{yi},\varepsilon_{xi},\varepsilon_{yi})+\Phi_c(r_i,\mu_{xi},\mu_{yi},\varepsilon_{xi},\varepsilon_{yi})}.$$

Figure 1 shows the vector point diagram of proper motions of 534 stars, where different symbols denote different grades of membership probability. Table 3 gives the maximum likelihood estimates of the parameters of these distributions and their uncertainties. The astrometry and membership probabilities of 534 stars are listed in an electronic catalogue available on request from the authors.

Table 3 Estimates of Proper Motion Distribution Parameters

Parameter	Estimation and Uncertainty
μ_{xc}	-0.06 ± 0.17 mas yr^{-1}
μ_{yc}	-2.60 ± 0.17 mas yr^{-1}
μ_{xf}	-7.87 ± 0.21 mas yr^{-1}
μ_{yf}	-5.43 ± 0.81 mas yr^{-1}
σ_o	2.14 ± 0.43 mas yr^{-1}
σ_{xo}	17.01 ± 0.04 mas yr^{-1}
σ_{yo}	14.37 ± 0.44 mas yr^{-1}
ρ	-0.20 ± 0.03
α	5.56 ± 0.15 arcmin
g	7.37 ± 0.70

4 DISCUSSION ON THE PROPER MOTIONS

Figure 2 shows a comparison between the proper motions given in the catalogues and those obtained in this work for the 57 reference stars. From this figure we cannot find any significant systematic differences. So the proper motions obtained in this work may just be the absolute proper motions based on the Hipparcos system.

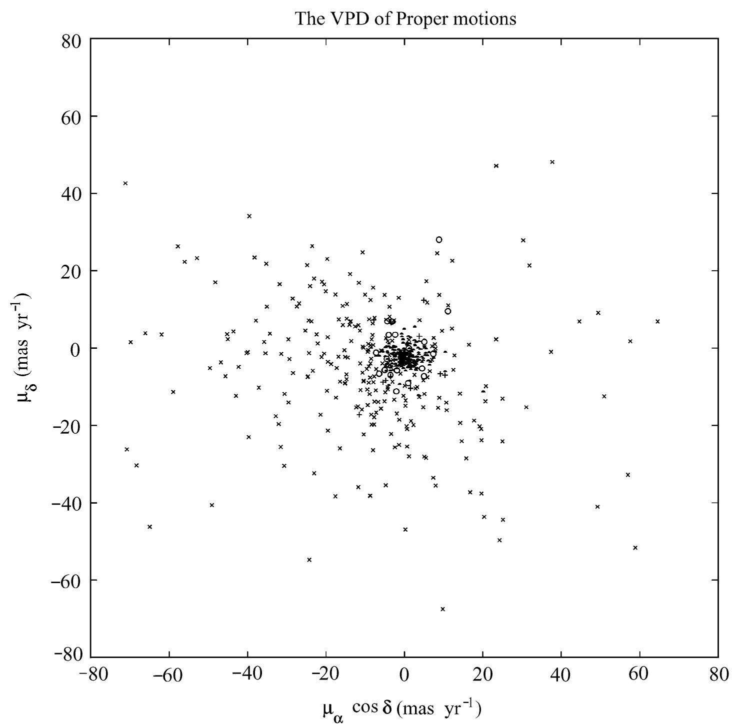

Fig. 1 Vector point diagram of proper motions of 534 stars in the region of globular cluster M3 (×: $p < 0.1$; +: $0.1 \leq p < 0.5$; ○: $0.5 \leq p < 0.9$; •: $0.9 \leq p \leq 1.0$).

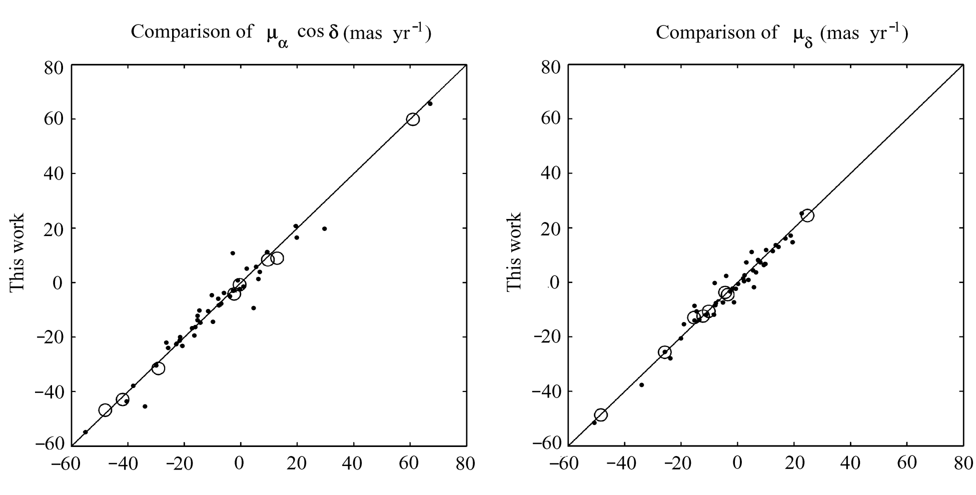

Fig. 2 Comparison between the proper motions from this work and from the Hipparcos and Tycho-2 Catalogues for 57 stars in common (○: Hipparcos stars; •: Tycho-2 stars).

To further examine possible systematic trends in the proper motions obtained in this work, Fig. 3 plots the proper motions in R.A. and Decl. of stars with membership probabilities $p \geq 0.9$ against the coordinates x and y, the magnitude B and the color $B - V$. This figure shows no significant systematic trends with either position, magnitude, or color. Accordingly it may be considered that the plate reduction model used in this work is adequate and has taken into account most of the factors that affect the reduction of proper motions.

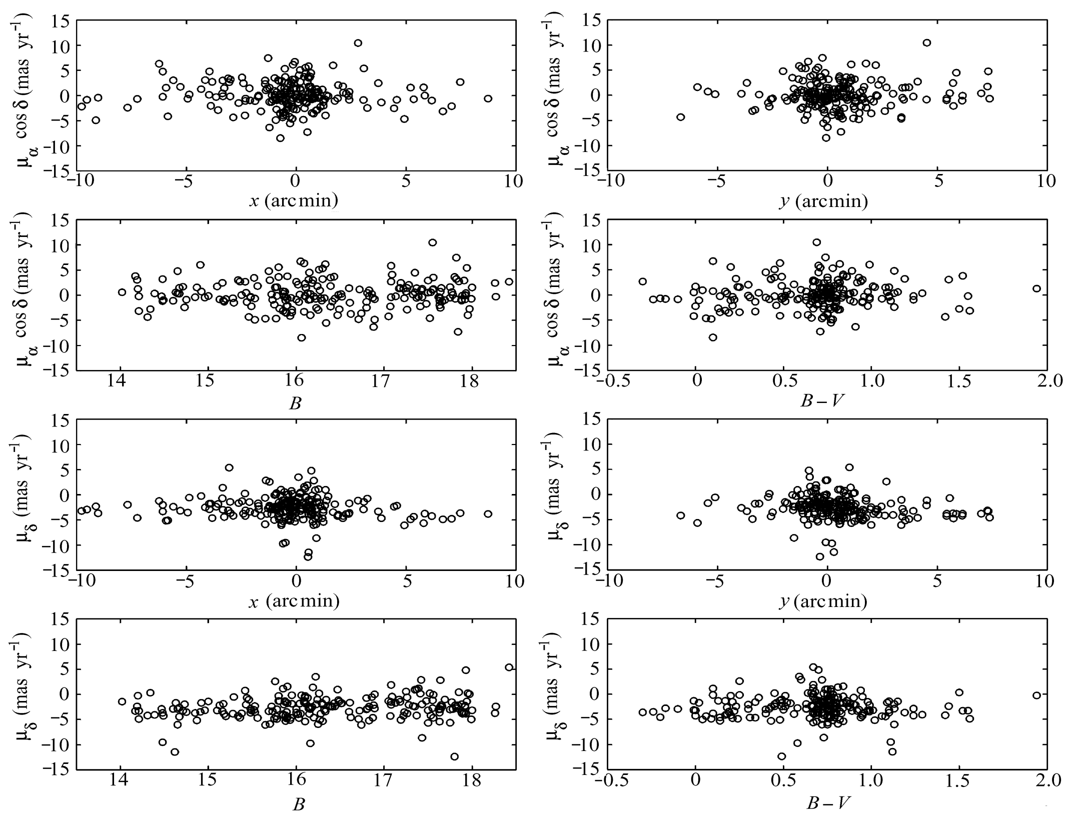

Fig. 3 Examination of possible systematic trends in the proper motions derived in this work for $p \geq 0.9$ stars only.

The photometry used in this examination was obtained as follows. (1) For the eight Hipparcos stars, V and $B - V$ from the Hipparcos Catalogue were adopted directly. (In fact, the membership probabilities of these stars are all equal to zero and they were not used in the examination.) (2) For the 49 Tycho-2 stars, the Tycho-2 Catalogue magnitudes V_T and B_T were transformed to V and $B - V$ using the formulae given in the Guide to the Tycho-2 Catalogue. (3) For stars other than the Hipparcos and Tycho-2 ones, the data were taken from Ferraro et al. (1997); Buonanno et al. (1994); Cudworth (1979) or Scholz and Kharchenko (1994).

There were a few stars which were not identified and they were not used in the analysis in spite of their high membership probabilities. Stars whose B magnitudes come from Tucholke, Scholz and Brosche (1994), or are adjusted with known data of the identified stars, are given in an electronic catalogue available on request from the authors.

A straight average of the absolute proper motions of the stars with membership probabilities $p \geq 0.9$ is calculated, and the result is (0.30 ± 0.21) mas yr^{-1} in R.A. and (-2.59 ± 0.16) mas yr^{-1} in Decl. On the other hand, if the calculation is made with weights equal to the membership probabilities, then we have the values (0.24 ± 0.14) mas yr^{-1} in R.A. and (-2.39 ± 0.14) mas yr^{-1} in Decl. Table 3 lists the maximum likelihood value of the absolute proper motion for the cluster, (-0.06 ± 0.17) mas yr^{-1} in R.A. and (-2.60 ± 0.17) mas yr^{-1} in Decl. These estimates agree with each other very well within their errors. In the present work the last mentioned maximum likelihood estimate is adopted as the estimate for the absolute proper motion of the globular cluster M3. If we take into account the rotation error of the Hipparcos reference frame (on which this work is based), with respect to the ICRS (Kovalevsky et al. 1997), the error estimates should be modified and the result then reads (-0.06 ± 0.30) mas yr^{-1} in R.A. and (-2.60 ± 0.30) mas yr^{-1} in Decl.

Table 4 shows a comparison between this work and some previous works (Cudworth & Hanson 1993 = CH93; Scholz, Odenkirchen & Irwin 1993 = SOI93; Odenkirchen et al. 1997 = OBGT97; Geffert 1998 = G98), where the differences are in the sense "this work - the other". In general, this work agrees well with the other works and agrees particularly well with the average of OBGT97 and G98 which were also based on the Hipparcos. Also, the comparison shows that the system of this work is closer to the ICRS reproduced by the Hipparcos than that in Wu and Wang (2000) with the ACT Catalogue.

Table 4 Comparison with the Average Absolute Proper Motions for the Cluster from Different Works

author	$\mu_\alpha \cos\delta$ (mas yr^{-1})	μ_δ (mas yr^{-1})	$\Delta\mu_\alpha \cos\delta$ (mas yr^{-1})	$\Delta\mu_\delta$ (mas yr^{-1})
This work	-0.06 ± 0.30	-2.60 ± 0.30	–	–
CH93	-1.2 ± 2.5	-2.4 ± 3.0	-1.14	5.00
SOI93	-3.1 ± 0.2	-2.3 ± 0.4	3.04	0.30
OBGT97	0.9 ± 1.0	-2.2 ± 1.0	0.96	0.40
CH93	-1.2 ± 0.8	-3.2 ± 0.8	-1.14	-0.60

5 SPACE MOTION AND GALACTIC ORBIT OF M3

From the absolute proper motion $(\mu_\alpha \cos_\delta, \mu_\delta)$ found above, combined with the distance from the Sun ($R_{\rm SUN}$) and the radial velocity (V_r) of the cluster given by Harris (1999), the present space motion is derived. In the calculation, the Galactocentric distance of the Sun given by Harris (1999), the peculiar motion of the Sun given by Ratnatunga, Bahcall and Casertano (1989), and the Galactic rotation velocity of the Local Standard of Rest ($V_{\rm LSR}$) given by Kerr and Lynden-Bell (1986) are adopted. The relevant data are given in Table 5, where x, y, z are Galactic rectangular coordinates and U, V, W are the velocities in respect to the Galactic Standard of Rest in the Galactic rectangular coordinate system. In this work, the x-axis of the Galactic coordinate system points from the center of the Galaxy towards the antisolar direction. In Table 5, the velocity components Π, Θ and W in the Galactic cylindrical coordinate system are also listed, where Π points radially outwards from the Galactic center to the cluster, Θ increases in the direction of the Galactic rotation, and the component W is the same as in the rectangular coordinate system.

Table 5 Space Motion of Globular Cluster M3

Observational data		Parameters of the Sun		Derived values	
α_{2000}	$13^{\rm h}42^{\rm m}11.2^{\rm s}$	x	–8.0 kpc	x	–6.5 kpc
δ_{2000}	$+28°22'32''$	y	0 kpc	y	1.3 kpc
$R_{\rm SUN}$	10.4 kpc	z	0 kpc	z	10.2 kpc
V_r	-148.5 ± 0.4 km s^{-1}	U	11 km s^{-1}	U	67 ± 16 km s^{-1}
$\mu_\alpha \cos\delta$	-0.06 ± 0.30 mas yr^{-1}	V	14 km s^{-1}	V	112 ± 17 km s^{-1}
μ_δ	-2.60 ± 0.30 mas yr^{-1}	W	7 km s^{-1}	W	-136 ± 2km s^{-1}
		$V_{\rm LSR}$	220 km s^{-1}	Π	$-42\pm$ 16 km s^{-1}
				Θ	124 ± 17 km s^{-1}

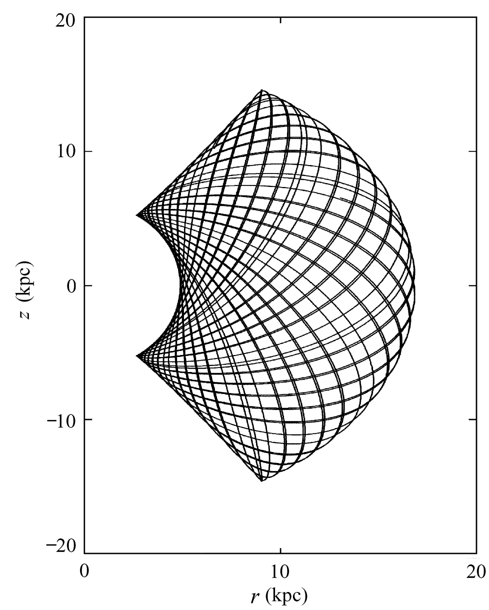

Fig. 4 Orbit of the globular cluster M3 projected onto the meridional plane

From the present space position and velocity of the cluster in Table 5, the Galactic orbit and some parameters of the orbit in the last 10 Gyr were calculated for the cluster with the Galactic potential given by Allen and Santillan (1991). The values of some of the parameters of the orbit are given in Table 6. In the table, R is the Galactocentric distance, z is the perpendicular distance to the Galactic plane, v, the velocity of space motion, J, the angular momentum, e and i, the eccentricity and inclination of the orbit, and E and T, the total energy and the period of the orbital motion. The subscript $t = 0$ represents the present value, $N_{\rm rev}$ represents the number of revolutions in the last 10 Gyr, J_z is the z-component of the angular momentum. E/m is the specific total energy, or the total energy of orbital motion per unit mass. Figure 4 shows that the meridional orbit of M3 is box-like, filling the box almost symmetrically already in the 10 Gyr time-interval.

In order to check the influence of uncertainties in the initial data on the orbit, we calculate the orbits of M3 with some "new" data. Because most of the errors come from the proper motions of the cluster, we only change the initial proper motions in the new calculation. Specifically, in case a, we add the errors of proper motions to their mean values; in case b, we subtract the errors from the mean values. The results are listed in the Table 7. It can be seen from Table 7 that the orbit of M3 is not much changed by such small changes in the initial proper motion data.

Table 6 Orbital Parameters of the Globular Cluster M3

Invariables		Geometry		Velocity		Angular momentum and incl.	
J_z	–814 kpc km s^{-1}	$R_{t=0}$	12.2 kpc	$v_{t=0}$	189 km s^{-1}	$J_{\rm mean}$	1820 kpc km s^{-1}
E/m	–118918 km^2 s^{-1}	$R_{\rm max}$	17.1 kpc	$v_{\rm max}$	365 km s^{-1}	$J_{\rm max}$	2187 kpc km s^{-1}
		$R_{\rm min}$	5.0 kpc	$v_{\rm min}$	90 km s^{-1}	$J_{\rm min}$	1546 kpc km s^{-1}
		e	0.55	$v_{\rm mean}$	245 km s^{-1}	$i_{\rm mean}$	116.8°
		$z_{\rm max}$	14.5 kpc	$T_{\rm min}$	0.37 Gyr	$i_{\rm max}$	121.8°
		$z_{\rm min}$	–14.5 kpc	$N_{\rm rev}$	26.8	$i_{\rm min}$	111.9°

Table 7 Variation of Orbital Parameters with Initial Values

Initial values	$R_{\rm max}$ (kpc)	$R_{\rm min}$ (kpc)	e	$Z_{\rm max}$ (kpc)	$Z_{\rm min}$ (kpc)
a	18.2	6.0	0.51	15.3	–15.3
b	16.4	4.0	0.61	14.0	–13.5

6 CONCLUSIONS

We have obtained the absolute proper motions of 534 stars in a region of about $100' \times 100'$ around the cluster M3, based on eight stars in the Hipparcos Catalogue and 49 stars in the Tycho-2 Catalogue. The membership probabilities of the stars were determined, and an average absolute proper motion of -0.06 ± 0.30 mas yr^{-1} in R.A. and -2.6 ± 0.30 mas yr^{-1} in Decl., was determined for the cluster. By combining this result with the known distance and radial velocity of the cluster, its Galactic orbit in the given three-component Galactic potential was derived. The Galactic orbit shows that the cluster is a typical outer halo globular cluster. Its apogalactic distance is about 17 kpc, the maximum perpendicular distance to the Galactic plane exceeds 14 kpc, the inclination of orbit on the Galactic plane is very large, and the z-component of angular momentum is small, relative to its projected angular momentum in the Galactic plane. These conclusions agree basically with those given by other authors (Scholz, Odenkirchen & Irwin 1993; Odenkirchen et al. 1997; Geffert 1998) and are consistent with inferences from the metallicity of the cluster, [Fe/H] = –1.57 (Harris 1999).

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