

## Close Approaches of Potentially Hazardous Asteroids during Two Centuries

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**Abstract** Asteroids are the most important small bodies in the solar system and the near-earth asteroids (NEAs) are of especial concern to the world. The reason is that they will make close approaches to the earth in the near future. We use a reasonable dynamical model and an efficient computing method to calculate the orbits of over 160 Potentially Hazardous Asteroids (PHAs) for two centuries.

**Key words:** celestial mechanics — asteroids — Earth

### 1 INTRODUCTION

Near-earth asteroids (NEAs) can move very near the earth or intrude into the earth orbit, hence they are of particular concern to the world. Even more so are the so-called Potentially Hazardous Asteroids (PHAs). PHAs (JPL, <http://neo.jpl.nasa.gov/neo/pha.html>) are currently defined by parameters that measure the asteroid's potential of making threateningly close approaches to the earth. Specifically, all asteroids with an earth Minimum Orbit Intersection Distance (MOID) of 0.05 AU or less and an absolute V-magnitude (H) of 22.0 or brighter are considered as PHAs. We should carefully watch and track these unexpected and dangerous "earth visitors", keeping a detailed record of their close-approaching status, such as the MOID and the encounter time. A lot of research work has been done in this field in recent years: Minor Planet Center (MPC, <http://cfa-www.harvard.edu/iau/mpc.html>) maintains the daily list of PHAs; Chesley et al. (Chesley & Milani 1999) have contributed an online website of NEODyS to provide asteroid information; Bowell et al. (Bowell 1999; [http://www.lowell.edu/users/elgb/current\\_moid.html](http://www.lowell.edu/users/elgb/current_moid.html)) have also presented asteroid ephemerides services via their website, etc. Another important motivation that drives us to the present work is that late last year a group of Hawaiian astronomers announced that an asteroid (2000 SG344) will collide with the earth in 2030, but one day later they corrected their statement and that the

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impact possibility would be down to a little more than one in thousand in 2071. Consequently, in this paper, after an introduction we shall report on our computation of 160 PHAs' orbits and show their encounter status during a period of two centuries.

As far as the tracking and observing of PHAs is concerned, many space-watching programs, such as LINEAR, NEAT, SPACEWATCH etc., have been set up; in addition, the project of 1.0m/1.2m Schmidt telescope in search of NEOs, sponsored by Purple Mountain Observatory of Chinese Academy of Sciences will be completed in 2002. International collaboration is so important to provide the new observations by monitoring PHAs in time, so that PHAs' orbits can be better determined and their future earth-impact threat will be well predicted.

This paper is organized as follows: Section 2 describes the dynamical model in the computation; Section 3 introduces the computation method we adopted; Section 4 presents our results and a brief discussion.

## 2 DYNAMICAL MODEL

In the dynamical model, the eleven large bodies are the sun, the nine major planets from Mercury to Pluto and the moon. In addition, the three biggest asteroids (the "Big 3", i.e., Ceres, Pallas and Vesta) in the main belt are also taken into account. As several NEAs such as (1566) Icarus can move so close to the sun, the relativity effect is also considered in the model. The orbits of the major planets, the moon and the Big 3 are calculated as well as the NEA's. In the heliocentric ecliptic coordinate system referred to J2000.0, the equations of motion can be uniformly written as

$$\begin{aligned} \ddot{\vec{r}}_i = & -(1 + \mu_i) \frac{\vec{r}_i}{r_i^3} - \sum_{j=1, j \neq i}^{14} \mu_j \left( \frac{\vec{\Delta}_{ji}}{\Delta_{ji}^3} + \frac{\vec{r}_j}{r_j^3} \right) \\ & + \left( \vec{A}_{\text{PN}} \right)_i + \left( \vec{A}_{\text{Fig}} \right)_e, \quad i = 1, \dots, 14 \end{aligned} \quad (1)$$

where  $\vec{r}_i, \vec{r}_j$  denote position vectors of the major planets ( $i = 1 \sim 9$ ), the moon ( $i = 10$ ), the Big3 ( $i = 11 \sim 13$ ) and the NEA ( $i = 14$ ), with  $\vec{\Delta}_{ji} = \vec{r}_i - \vec{r}_j$ ,  $\mu_i$  and  $\mu_j$  are these bodies' masses with respect to the sun. The mass of the NEA can be neglected, so in the right-sided expression of Equation (1) for the other bodies, the gravitational terms from the NEA will not appear.  $(\vec{A}_{\text{PN}})_i$  stands for Post-Newtonian terms, which stem from the gravitational effect of the sun.  $(\vec{A}_{\text{Fig}})_e$  represents the effect of the figuration of the earth, which becomes important when the NEA moves very close to the earth.

It is worth noting that in the above model the perturbation of the moon is well separated from that of the earth. The reason is very obvious: when an asteroid comes near the earth, it will also come near the moon, necessitating a more detailed evaluation of the gravitational perturbation of the moon.

We take the initial elements of all the asteroids from the asteroid orbital elements database given by Bowell (Bowell, <ftp://ftp.lowell.edu/pub/elgb/astorb.dat.gz>) on Dec. 22, 2000. The starting positions and velocities of the planets and moon, as well as their masses, come from the JPL planetary and Lunar Ephemerides DE405/LE405, in which the slight difference between ICRF and J2000.0 is neglected.

### 3 COMPUTATIONAL METHOD

For the qualitative study of asteroid orbits, the symplectic algorithm (Feng 1984, 1986; Forest et al. 1990; Wisdom et al. 1991), which has many advantages such as holding the symplectic structure of the Hamiltonian system, is preferably used in the dynamical evolution of the asteroids. Liu et al. (Liu 1998, 1999a, 1999b) have come to the conclusion that the algorithm is fairly efficient and the computations are credible as long as the asteroid does not move too close to some of the major planets. However, when the asteroid is close to one or several major planets, the stepsize has to be frequently changed in the course of the integration because of the accumulating gravitation, then the merit of the symplectic algorithm is completely lost and it becomes not as good as the automated stepsize-adjusted non-symplectic algorithm, such as the RKF7(8) integrator (Fehlberg 1968), RADAU integrator (Everhart 1985) and Bulirsch-Stoer (Stoer & Bulirsch 1980) integrator.

As for the numerical method, though it is convenient to change the stepsize by using variable-stepsize integrators, yet the orbital period of the moon is so short with respect to those of the planets and other asteroids, that the total integration stepsize cannot but be kept smaller, which will lead to loss of computational efficiency. However, we have got a solution of the problem, that is, in the integration the geocentric orbital elements of the moon are adopted as the integration variables, which are further transformed into heliocentric positions and velocities with the help of the heliocentric vector of the earth (Ji & Liu 2000), while for the other bodies we still use the position and velocity vectors. Using this method the stepsize in the integration can be adjusted to much larger values, so greatly improving the computational efficiency.

### 4 RESULTS AND DISCUSSIONS

All of the numerical integration was carried out on Alpha workstations. The results are given for 100 years for the limited size of this paper, but the full results are available. In Table 1 the name (and the number) of the asteroids, the encountering time and the MOID (less than 0.05 AU) are listed, sorted by the distances. And we have also compared our results with those of MPC, and found most of them are in good agreement; but the dynamical model in this paper is based on more detailed considerations. We can draw the conclusion that the dynamical model is reasonable for the computation of close approaches of the PHAs and that the algorithm used is effective.

**Table 1** 160 PHAs Close Encounters with the Earth during the Next Century  
(JD=2451900.5, 2000.12.22)

Object	Name	Date of Encounter (TT) Calendar	Distance (AU)	Object	Name	Date of Encounter (TT) Calendar	Distance (AU)
	1999 AN10	2027 Aug. 7.26	0.002651	4660	Nereus	2060 Feb. 14.38	0.008033
	1999 RQ36	2060 Sept.23.21	0.004047		1998 SC15	2095 Apr. 9.31	0.008497
	2000 QK130	2036 Mar. 14.94	0.004634		1997 WQ23	2068 Nov. 4.42	0.009871
	1997 XF11	2028 Oct. 26.25	0.006168		1999 JU3	2076 Dec. 6.05	0.01048
2340	Hathor	2086 Oct. 21.82	0.006184	4179	Toutatis	2004 Sept.29.18	0.01064
	2340 Hathor	2069 Oct. 21.31	0.006606		2000 UG11	2051 Nov. 10.23	0.01068
	1998 HH49	2023 Oct. 16.81	0.007692		2000 QK130	2083 Mar. 14.57	0.01153
	1999 DB7	2048 Feb. 28.09	0.007925		1999 AQ10	2009 Feb. 18.76	0.01179

Table 1 Continued

Object	Name	Date of Encounter (TT) Calendar	Distance (AU)	Object	Name	Date of Encounter (TT) Calendar	Distance (AU)
4581	1998 OR2	2079 Apr. 16.69	0.01185	2340	Hathor	2045 Oct. 21.68	0.02430
	Asclepius	2051 Mar. 24.55	0.01205	1999 UR	2033 Oct. 27.22	0.02430	
	1998 WT24	2001 Dec. 16.18	0.01249	1998 CS1	2080 Jan. 20.78	0.02441	
	1988 TA	2053 Oct. 1.51	0.01253	1999 KW4	2089 May 26.30	0.02447	
	1998 WT24	2099 Dec. 18.16	0.01266	6037	1988 EG	2041 Feb. 28.16	0.02449
	1999 VP11	2017 Oct. 22.78	0.01267	1998 HE3	2059 May 11.05	0.02472	
	1998 SF36	2004 June 26.67	0.01291	1998 DV9	2095 Feb. 7.32	0.02480	
7482	2000 AA6	2060 Jan. 13.09	0.01296	7822	1991 CS	2065 Aug. 31.16	0.02498
	1994 PC1	2022 Jan. 18.98	0.01326	4769	Castalia	2046 Aug. 26.65	0.02512
	1994 WR12	2046 Nov. 25.91	0.01360	2000 CO101	2057 Sept.17.19	0.02523	
	1999 MN	2036 June 1.87	0.01406	1996 RG3	2096 Feb. 27.75	0.02524	
	1998 WT24	2054 Dec. 19.26	0.01435	8566	1996 EN	2070 Sept. 8.71	0.02528
4660	1997 WQ23	2052 Nov. 3.28	0.01442	1999 MN	2010 June 3.63	0.02532	
	Nereus	2071 Feb. 4.29	0.01547	1999 FA	2095 Mar. 7.65	0.02560	
	1999 KW4	2036 May 25.57	0.01554	1994 PM	2003 Aug. 16.24	0.02562	
	1989 UQ	2093 Aug. 13.08	0.01592	1998 VD35	2042 May 12.05	0.02580	
	2000 DO1	2094 Mar. 10.52	0.01627	2000 RS11	2014 Mar. 12.30	0.02584	
	1997 QK1	2039 July 14.35	0.01682	1999 GS6	2071 Sept.10.95	0.02585	
	1994 CC	2009 June 10.44	0.01702	2000 GJ147	2099 May 17.68	0.02586	
	1997 NC1	2026 June 27.29	0.01713	1994 CC	2074 June 15.62	0.02607	
	2000 TU28	2051 Apr. 12.66	0.01726	2000 RD53	2043 Sept.19.73	0.02614	
	1999 SO5	2086 Apr. 7.05	0.01782	1999 JT6	2076 Dec. 14.81	0.02615	
	1999 KW4	2071 May 26.26	0.01787	4660	Nereus	2021 Dec. 11.73	0.02632
	1999 MN	2004 July 11.62	0.01805	1989 UP	2050 Dec. 8.06	0.02675	
	1995 CR	2014 Feb. 20.79	0.01833	7335	1989 JA	2022 May 27.75	0.02693
	1998 WB2	2037 Dec. 4.69	0.01856	1998 WT24	2040 Dec. 20.73	0.02738	
	1999 AN10	2081 Aug. 5.95	0.01903	1998 WT24	2015 Dec. 11.65	0.02798	
3200	Phaethon	2093 Dec. 14.46	0.01981	3671	Dionysus	2085 June 18.29	0.02802
4179	1999 FA	2046 Mar. 4.82	0.01984	2000 HA24	2079 Mar. 17.83	0.02810	
	Toutatis	2069 Nov. 5.72	0.01985	1998 QP	2053 Aug. 25.93	0.02856	
3361	2000 EW70	2059 Mar. 25.33	0.01987	2000 RW37	2047 Feb. 23.98	0.02888	
	1997 QK1	2025 Aug. 20.78	0.02003	4660	Nereus	2002 Jan. 22.35	0.02903
	2000 SP43	2076 Sept.19.14	0.02050	13651	1997 BR	2051 July 23.73	0.02908
	2000 PN9	2006 Mar. 6.43	0.02083	1998 BY7	2018 June 28.40	0.02912	
	2000 AA6	2086 July 29.25	0.02083	12538	1998 OH	2042 May 3.24	0.02921
	1999 MM	2088 July 14.13	0.02089	1998 CS1	2009 Jan. 17.59	0.02927	
	3362	Orpheus	2091 Apr. 18.80	0.02114	2000 RD53	2074 Sept.25.16	0.02930
	3362	Khufu	2045 Aug. 22.23	0.02120	2000 EW70	2069 Mar. 25.15	0.02933
	1997 XF11	2095 Oct. 26.20	0.02210	1997 WQ23	2013 Nov. 7.37	0.02959	
	1999 MN	2076 July 11.44	0.02222	8566	1996 EN	2033 Sept. 8.43	0.03042
4953	1997 NC1	2088 June 26.23	0.02239	9856	1991 EE	2065 Sept. 6.29	0.03056
	1994 CN2	2042 Aug. 6.66	0.02244	1999 MN	2041 June 3.91	0.03068	
	1998 BB10	2033 July 21.30	0.02300	4953	1990 MU	2027 June 7.01	0.03083
	2000 AF6	2081 Dec. 13.70	0.02302	2000 PD3	2097 Jan. 10.25	0.03106	
	1990 MU	2058 June 5.39	0.02314	1998 FW4	2046 Feb. 13.77	0.03114	
	1999 MN	2030 July 11.11	0.02319	2000 EJ26	2070 Apr. 2.38	0.03120	
	1998 HE3	2073 May 10.79	0.02345	1999 MN	2082 June 4.21	0.03137	
	1999 AN10	2076 Feb. 2.68	0.02372	1998 VO	2046 Apr. 30.80	0.03148	
	1999 RM45	2021 Mar. 3.09	0.02402	2000 CO101	2096 Sept.17.19	0.03148	
	1989 UQ	2087 Dec. 2.50	0.02403	1999 JV6	2098 Jan. 6.18	0.03195	
	1997 WQ23	2084 Nov. 7.38	0.02412	2000 AZ93	2050 Dec. 31.53	0.03197	
	1998 BB10	2076 July 22.02	0.02422	1999 YR14	2069 Oct. 22.62	0.03211	

Table 1 Continued

Object	Name	Date of Encounter (TT) Calendar	Distance (AU)	Object	Name	Date of Encounter (TT) Calendar	Distance (AU)
	2000 GE2	2024 Oct. 3.70	0.03225	3361	Orpheus	2021 Nov. 21.72	0.03856
	1998 SC15	2042 Apr. 7.14	0.03226		2000 AZ93	2070 Dec. 31.36	0.03872
	1999 KW4	2001 May 26.12	0.03237		1997 QK1	2067 Aug. 26.09	0.03890
	1999 JV6	2016 Jan. 6.45	0.03248	7822	1991 CS	2090 Sept. 1.16	0.03907
	1989 UP	2078 Dec. 13.50	0.03257		1996 JG	2049 Nov. 25.26	0.03919
	1999 SG10	2078 Oct. 10.40	0.03285	8014	1990 MF	2080 Sept.12.29	0.03933
	1999 RQ36	2005 Sept.20.41	0.03305		1998 WT	2040 Mar. 4.44	0.03938
1862	Apollo	2082 May 12.27	0.03336		1998 FH12	2047 Feb. 15.41	0.03972
	1998 HE3	2012 May 10.40	0.03356		1998 XN2	2032 Aug. 28.90	0.03979
5604	1992 FE	2017 Feb. 24.28	0.03361		1991 GO	2073 Oct. 25.08	0.03987
	1997 XF11	2090 May 19.40	0.03365		1994 XD	2012 Dec. 1.29	0.03995
	2000 ED14	2090 Mar. 17.15	0.03416		2000 JG5	2090 Apr. 27.16	0.04000
6037	1988 EG	2079 Aug. 22.76	0.03427		2000 EE104	2031 Nov. 12.32	0.04007
	1999 KW4	2019 May 25.99	0.03463		1994 WR12	2090 Nov. 22.86	0.04016
	2000 JG5	2045 Apr. 25.67	0.03488		2000 EV70	2055 Aug. 9.44	0.04023
	1998 WT24	2085 Dec. 6.85	0.03501		1992 UY4	2005 Aug. 8.63	0.04041
	2000 HA24	2038 Sept.27.82	0.03506		2000 EJ26	2083 Apr. 2.11	0.04068
11500	1989 UR	2053 Nov. 26.90	0.03514	6037	1988 EG	2023 Aug. 23.16	0.04070
	1999 LX1	2068 June 5.52	0.03514		2000 HA24	2034 Mar. 18.79	0.04090
	1988 TA	2080 May 14.62	0.03518	4450	Pan	2008 Feb. 19.67	0.04091
	2000 QW7	2019 Sept.14.91	0.03520		2000 QW7	2087 Sept.19.73	0.04095
	1999 RM45	2095 Sept. 5.64	0.03525		1998 HE3	2096 May 10.71	0.04095
1862	Apollo	2046 Nov. 13.12	0.03530		1999 MN	2071 July 10.15	0.04097
	1999 BJ8	2054 Feb. 24.50	0.03533		1997 BQ	2020 May 21.77	0.04117
2101	Adonis	2036 Feb. 7.32	0.03570		1998 HL1	2019 Oct. 25.45	0.04163
	2000 AA6	2051 July 28.05	0.03573		2000 AZ93	2030 Dec. 31.98	0.04202
	1991 GO	2089 Oct. 27.69	0.03580		2000 EE104	2029 Nov. 10.81	0.04204
	1994 XL1	2088 Dec. 5.74	0.03605		1998 OR2	2020 Apr. 29.68	0.04205
	1998 FW4	2009 Oct. 1.07	0.03614		1994 WR12	2069 Nov. 29.65	0.04209
	2000 AZ93	2071 Jan. 3.31	0.03623		2000 EV70	2069 Mar. 6.18	0.04220
	1998 VO	2056 May 3.47	0.03665		1999 YR14	2016 Sept. 4.36	0.04234
	1994 XD	2052 June 10.52	0.03670		1998 XN2	2094 Nov. 30.04	0.04235
	2000 GJ147	2094 May 16.14	0.03689		1999 FA	2027 Mar. 4.33	0.04248
	2000 ET70	2059 Feb. 19.59	0.03696		1998 FG2	2032 Mar. 1.99	0.04249
4450	Pan	2060 Feb. 17.60	0.03700		1998 WT24	2068 Dec. 22.07	0.04254
	1998 SH36	2091 Oct. 12.20	0.03746		2000 ET70	2047 Feb. 19.65	0.04267
	1994 XL1	2077 Dec. 4.60	0.03756		1994 XL1	2005 Dec. 6.88	0.04279
	2000 EE104	2030 Nov. 11.10	0.03764		2000 DP107	2067 Sept.20.56	0.04280
	1994 CN2	2038 Oct. 20.10	0.03764		2000 EH26	2053 May 12.39	0.04295
	1998 FG2	2003 Oct. 21.58	0.03781		1999 JV6	2015 Dec. 31.91	0.04306
3361	Orpheus	2025 Nov. 19.03	0.03792		2000 GE2	2022 Sept.29.27	0.04306
3757	1982 XB	2074 Dec. 18.17	0.03793		2000 UL11	2038 Mar. 26.51	0.04317
	1994 XL1	2099 Dec. 7.13	0.03800		1994 XL1	2066 Dec. 4.47	0.04360
	1991 GO	2081 Oct. 25.78	0.03815	1566	Icarus	2090 June 14.45	0.04362
	1998 VF32	2090 Nov. 22.83	0.03818		1988 TA	2013 May 9.29	0.04380
	1998 SF36	2001 Mar. 29.49	0.03831		1998 VF32	2079 Nov. 22.91	0.04394
	2000 AZ93	2031 Jan. 4.52	0.03834	2102	Tantalus	2038 Dec. 27.20	0.04446
	1999 MN	2097 July 9.65	0.03835		1991 VH	2065 Aug. 6.52	0.04450
	2000 QS7	2093 Sept. 8.72	0.03835		1999 JV6	2097 Dec. 31.64	0.04452
	2000 LB16	2051 Dec. 14.65	0.03839		1998 FW4	2029 Sept.28.61	0.04471
	1998 QC1	2052 Jan. 20.58	0.03841		1998 SH36	2010 June 7.80	0.04487
	1999 RQ36	2054 Sept.29.76	0.03843		1991 VH	2088 Aug. 15.47	0.04533

Table 1 Continued

Object	Name	Date of Encounter (TT) Calendar	Distance (AU)	Object	Name	Date of Encounter (TT) Calendar	Distance (AU)
	2000 ET70	2012 Feb. 19.63	0.04546	4581	Asclepius	2099 Aug. 23.53	0.04715
	1998 DV9	2058 Feb. 12.35	0.04550		1993 BX3	2021 Jan. 17.56	0.04734
	1998 SF36	2036 June 8.04	0.04555		1993 VB	2023 Feb. 22.33	0.04736
	1999 KW4	2054 May 25.67	0.04573		1999 VP11	2077 Oct. 20.85	0.04739
	2000 CO101	2009 Sept.17.75	0.04575		1999 YG3	2067 Jan. 14.77	0.04739
	1991 VH	2008 Aug. 15.27	0.04579		1999 JU3	2033 Dec. 21.03	0.04742
	1998 FW4	2095 Feb. 14.67	0.04588		1998 HH49	2097 Oct. 12.42	0.04762
	1998 FH12	2088 Feb. 17.02	0.04595	1620	Geographos	2051 Aug. 22.97	0.04789
	2000 DP107	2075 Sept.13.37	0.04597		1998 HD14	2050 Apr. 5.93	0.04791
	1999 UR	2046 Dec. 18.60	0.04600		1996 GT	2003 Nov. 12.52	0.04805
	2000 AZ93	2091 Jan. 9.93	0.04622		1998 WT	2079 Mar. 7.89	0.04823
	2000 AZ93	2099 Dec. 17.62	0.04632	2340	Hathor	2014 Oct. 21.70	0.04827
	1998 FH12	2052 June 30.35	0.04637		1999 GS6	2058 Sept.11.45	0.04838
4179	Toutatis	2012 Dec. 12.57	0.04637		1998 SH36	2015 Oct. 14.60	0.04841
	2000 QK130	2066 Mar. 20.78	0.04646		1999 MN	2077 June 1.67	0.04855
	2000 AZ93	2011 Jan. 10.58	0.04671		2000 CE59	2057 Aug. 16.91	0.04946
	1999 JD6	2054 July 25.60	0.04703	13651	1997 BR	2068 July 24.81	0.04948
	2000 BF19	2055 Aug. 26.36	0.04712		1998 WT24	2029 Nov. 25.85	0.04958
					1996 GT	2062 Oct. 19.47	0.04986

As more and more PHAs are being or will be discovered and their orbits will be further refined and better determined with more observations, it is clear that precise orbits of the PHAs are important for their future orbital motion. The predictions of the PHAs' close approaches are sensitive to the initial conditions, so we will construct an online website on this topic to update the results everyday. In our future papers, we will introduce our work on the dynamics of the orbital evolution of the NEAs.

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## References

- Bowell E. A, DPS meeting #31, #28.03, 1999  
Chresley S. R., Minali A., DPS meeting #31, #28.06, 1999  
Feng K., Proc. 1984, In: Feng K. ed., Beijing Symposium on Differential Geometry and Differential Equations, Beijing: Science Press, 1985, p.42  
Feng K., 1986, J. Comp. Math., 4, 279  
Fehlberg E., Classical Fifth-, Sixth, Seventh-and Eight-order Runge-Kutta formulas with stepsize control., NASA TR R-287, 1968  
Forest E., Ruth R. D., 1990, Physica D, 43, 105  
Everhart E., In: A. Carusi, G. B. Valsecchi, eds., Dynamics of Comets: Their Origin and Evolution, Reidel: Dordrecht, 1985, p.185  
Liu L., Ji J. H., 1998, Chin. Astron. Astrophys., 22, 218  
Liu L., Ji J. H., Liao X. H., 1999a, Chin. Astron. Astrophys, 23, 108  
Liu L., Ji J. H., 1999b, Chin. Astron. Astrophys., 23, 273  
Ji J. H., Liu L., 2000, Science in China (A), 43, 1114  
Wisdom J., Holman M., 1991, AJ, 102, 1528