

# A Method of Obtaining the Pitch Angle of Spiral Arms and the Inclination of Galactic Discs

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**Abstract** We investigate the mathematical form, the symmetry of spiral structure and the projected images of galactic discs. The measured pitch angles of spiral arms and inclination angles of galactic discs for 60 spiral galaxies are presented. The global spiral structure is emphasized in the study. It is found that, except for small-scale distortions, the spiral arms of those galaxies that were classified as AC 12 in the arm classification system of Elmegreen & Elmegreen, can be represented by the logarithmic spiral form.

**Key words:** galaxies: spiral galaxies — galaxies: spiral arm — disc: inclination

## 1 INTRODUCTION

A spiral galaxy inherently consists of a halo, a bulge and a thin disc with spiral structure which emerges from the central region or the end of a bright bar. Optical images of spirals, which are projections on the celestial sphere, are dominated by the light from the stars, as modified by the extinction and reddening of dust. Although Lord Rosse first discovered that M51 has two spiral arms in 1845, the form of the spiral structure had not been investigated until 1911 by von der Pahlen (1911). Then, several other authors (Danver 1942; Kennicutt 1981; Kennicutt & Hodge 1982) also studied the mathematical form of the spiral structure. They all corrected for inclination effects in which different authors used different methods. For example, Danver (1942) first obtained visual estimates of the inclination by rotating the image of the galaxy until circular with the help of a special display table. Then, using the obtained inclination angles, he deprojected the projected image on paper by transparent illumination to be used for the measurement. Kennicutt (1981) directly disposed photographic enlargement and gave an initial estimate of inclination angle and pitch angle to orient the spiral galaxy to a face-on geometry, and then used residual sinusoidal deviations in the spiral arms to make small corrections on the derived orientation by an iterative procedure. The Fourier analysis technique, which was proposed by Kalnajs (1975) and further developed by Considère & Athanassoula (1982, 1988), Iye et al. (1982) and Garcia Gómez & Athanassoula (1993), is often used to obtain the pitch

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angle of the spiral arms. If the spiral structure can be presented by logarithmic spirals

$$\ln r = c + \lambda\phi, \quad (1)$$

here  $c$  and  $\lambda$  are constants;  $r$  and  $\phi$  are the radius and azimuthal angle. It is useful to express the brightness  $I$  of the galactic disc as a Fourier sum of logarithmic spirals

$$I(u, \phi) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dp I(p, m) e^{(pu+m\phi)}. \quad (2)$$

$I(p, m)$  is the amplitude of an  $m$ -armed spiral of pitch angle,  $\mu = -\arctan(m/p)$ , and is treated as a single-peaked function of  $p$  for a given  $m$ . If  $I(p, m)$  can peak strongly at the values  $m_0$  of  $m$  and  $p_0$  of  $p$ , it can be said that the galaxy has pitch angle  $\mu_0 = -\arctan(m_0/p_0)$ .

Recently, Seigar & James (1998) imaged 45 face-on spiral galaxies in the  $K$ -band to determine the morphology of the old stellar population such as arm strengths, profiles and cross-sections, and spiral pitch angles. They concluded that the morphology of the old stellar population bears little resemblance to the optical morphology.

Since many observed data of a spiral galaxy must be corrected for the projection, the inclination angle of a galactic disc is an important parameter. For a spiral structure, if it is oriented at different angles of inclination, its images will be different too. Unless the value of the inclination angle is derived, the true shape of the spiral structure cannot be investigated.

The derivation of the inclination angle from axial ratio is commonly adopted, but it requires an assumption on the intrinsic thickness of the system. The standard formulation for the derivation of an inclination angle is

$$\gamma = \arccos \sqrt{(q^2 - q_0^2)/(1 - q_0^2)}, \quad (3)$$

where  $q = b/a$  is the observed ratio of the minor to major axes and  $q_0$  is the intrinsic axial ratio.  $q_0$  may depend on the Hubble type. For example, the thinnest systems are spirals of type Sc. For simplicity,  $q_0 = 0.2$  is often assumed. It is clear that an inclination angle derived by this equation is not exact enough.

Several other ways of measuring the inclination angle of a galactic disc, which are more or less independent, were also proposed and used (Warner et al. 1973; Bosma 1981; Pence et al. 1990; Boroson 1981; Grøsbøl 1985; Kent 1985; Iye et al. 1982; Considère & Athanassoula 1988; Garcia Gómez & Athanassoula 1991). In this paper, we directly study the mathematical form and symmetry of the spiral structure, and obtain the inclination angles of galactic discs by directly fitting the shape of the spiral structure on the images. Whether the fitting is good or bad is immediately obvious.

The outlines of this paper are as follows. In Sect. 2, we outline the principles of deriving the pitch angle of a spiral feature and the inclination angle of the galactic disc; Sect. 3 presents the method; Sect. 4 shows how our sample is selected; and the results are listed in Sect. 5; Conclusions and a discussion are given in Sect. 6.

## 2 PRINCIPLE OF DERIVING PITCH AND INCLINATION ANGLES

When the line of intersection (i.e., the major axis of the image) between the galactic plane and tangent plane is taken as the polar axis, it is easily proved that

$$r = \rho \sqrt{1 + \tan^2 \gamma \sin^2 \theta}. \quad (4)$$

and

$$\tan \phi = \frac{\tan \theta}{\cos \gamma}, \tag{5}$$

where  $r$  and  $\phi$  are the polar co-ordinates in the galactic plane,  $\rho$  and  $\theta$  are the corresponding co-ordinates in the tangent-plane, and  $\gamma$  is the inclination angle of the galactic disc. To represent the arms by equiangular spirals,

$$r = r_0 e^{\lambda(\phi - \phi_0)} \tag{6}$$

and

$$\mu = \arctan \lambda, \tag{7}$$

where  $r$  and  $\phi$  are the polar co-ordinates on the spiral arm in the galactic plane, and  $\mu$ , the pitch angle, is the angle between the tangent to the arm and the circle with at the given  $r$ . The mathematical form of Eq. (6) in the tangent plane of the celestial sphere is

$$\rho(\theta, \gamma, \lambda) = \rho_0 \frac{f(\theta_0, \gamma)}{f(\theta, \gamma)} e^{\lambda B(\theta, \gamma)}, \tag{8}$$

here

$$f(\theta, \gamma) = \sqrt{\sin^2 \theta + \cos^2 \theta \cos^2 \gamma} \tag{9}$$

and

$$B(\theta, \gamma) = g(\theta, \gamma) - g(\theta_0, \gamma), \tag{10}$$

where

$$g(\theta, \gamma) = \begin{cases} \arctan(\tan \theta / \cos \gamma), & k\pi - \frac{\pi}{2} < \theta < k\pi + \frac{\pi}{2}, \\ \pi + \arctan(\tan \theta / \cos \gamma), & k\pi + \frac{\pi}{2} < \theta < k\pi + \frac{3\pi}{2}, \end{cases} \tag{11}$$

$k$  is an integer. Let  $(\rho_i, \theta_i)$  be the measured co-ordinates of points of the arms; according to the least squares method, we set

$$\sum_{i=1}^n [\rho_i - \rho(\theta_i, \gamma, \lambda)]^2 = \text{minimum}. \tag{12}$$

By direct differentiation with respect to  $\lambda$ , we obtain the equation for determining the parameter  $\lambda$  of

$$\sum_{i=1}^n B(\theta_i, \gamma) \rho(\theta_i, \gamma, \lambda) [\rho_i - \rho(\theta_i, \gamma, \lambda)] = 0. \tag{13}$$

This is a transcendental equation. When the value of  $\gamma$  is given,  $\mu$  can be derived.

### 3 METHOD OF DERIVING PITCH AND INCLINATION ANGLES

We derive the pitch angles of spiral arms and the inclination angles of galactic discs by directly fitting the images of the spiral structure. The DISPLAY task of IRAF software can enlarge the image and change its grey scale to minimize any personal prejudice on the regularity and prominence of the arms. We can “DISPLAY” the image clearly by changing the values of z1 and z2 (minimum and maximum greylevels to be displayed) in the DISPLAY task.

The main points of our method of deriving the pitch angle of spiral arm and inclination angle of galactic disc are:

1. Adjust the minimum and maximum image intensity in the DISPLAY task in order to display the image clearly.

2. Enlarge the image and change its grey scale to minimize any personal prejudice on the regularity and prominence of the arms.

3. Find the position of the starting point of the spiral arm from which the arm emerges, and measure its coordinate  $(\rho_0, \theta_0)$  relative to the image center ( $\theta_0$  is relative to the main axis of the arm).

4. Sample as many of points as possible situated on the central line of the mass of spiral arm, and measure their coordinates  $(\rho_i, \theta_i)$  relative to the image center ( $\theta_i$  is relative to the polar axis of the arm). In the last two processes, the RIMCURSOR task of IRAF software is used.

5. Give an initial value of the inclination angle  $\gamma_1$

$$\gamma_1 = \arccos \sqrt{(q^2 - 0.2^2)/(1.0 - 0.2^2)}, \quad (14)$$

fit the spiral arm from that starting point with a logarithmic spiral curve to derive its pitch angle  $\mu_1$  by Eq. (13). Here  $q = d_{25}/D_{25}$ ,  $D_{25}$  and  $d_{25}$ , taken from the *Third Reference Catalogue of Bright Galaxies* (hereafter RC3) (de Vaucouleurs et al. 1991), are the apparent major and minor isophotal diameters measured at or reduced down to the surface brightness level  $\mu_B = 25.0$  B magnitudes per square arcsecond. In this fitting process, the method of the least squares is used.

6. Deproject the logarithmic spiral curve ( $\mu_1$ ) on the image by using the value of inclination angle  $\gamma_1$ . In this procedure, the TVMARK task of IRAF software is used.

7. Compare the spiral arm in question with the logarithmic spiral curve ( $\mu_1$ ). If the fitting is not good, change the value of the inclination angle near  $\gamma_1$ , and repeat the processes of 5 and 6.

8. Determine the inclination angle  $\gamma$  and the corresponding pitch angle of the spiral arm  $\mu$ , i.e.,  $\gamma$  and  $\mu$  that correspond to “the best-fitted logarithmic spiral curve” are the inclination angle of galactic disc and the pitch angle of spiral arm in question.

In the fitting procedure, we emphasize the global spiral structure. Except for the small-scale distortions, the spiral arms can be represented by the logarithmic spiral forms.

#### 4 SAMPLE

There are several possible approaches to the classification of galaxies such as morphology, photometry, colorimetry and spectroscopy. The first and simplest one is the morphological approach proposed by Hubble (1926, 1936). It is based on qualitative, empirical criteria regarding the shape, concentration, and structure of galaxies as seen on optical (in blue light) photographs. Its concepts are still in use, a sequence starting from elliptical to spiral galaxies including lenticulars. This scheme has been extended by some other authors (Holmberg 1958; de Vaucouleurs 1956, 1959; Morgan 1958, 1959; van den Bergh 1960a, b, 1976; Sandage 1961; Sandage & Tammann 1981, 1987; Sandage & Bedke 1993) over the years, who tried to employ multiple classification criteria. The two main systems commonly used are derived from Hubble’s original classification criteria. One is the Hubble system as explained in detail by Sandage (1961), Sandage & Tammann (1981, 1987) and Sandage & Bedke (1993). Another, developed by de Vaucouleurs (1956, 1959), adds more detailed descriptions to the notation and makes a division and extension of the Sc and SBc families by introducing the Scd, Sd, Sdm, Sm and Im subdivisions.

Apart from Hubble’s classification, Elmegreen & Elmegreen (1982) introduced a new classification system for spiral galaxies that is designed to emphasize arm symmetry and continuity.

This system contains 12 distinct arm classes (AC), corresponding to a systematic change from the ragged and patchy arms in ‘flocculent’ galaxies to the two symmetric and continuous arms in ‘grand design’ ones. Intermediate arm classes show characteristics of both the ‘flocculent’ and ‘grand design’ types. Based on this system, Elmegreen & Elmegreen (1982) and Elmegreen, Elmegreen & Dressler (1982) classified spiral galaxies in the general field, in binary systems, in groups and in clusters. They concluded that bars tend to correlate with spiral density waves, that companions may influence (or generate) symmetric density waves, and that grand design galaxies are preferentially in dense groups. Then, Elmegreen & Elmegreen (1987) changed slightly the arm classification system, and presented the best possible sample of arm classification. In the arm classification system, spiral galaxies classified as AC 12 have two long symmetric arms dominating the optical discs. They constitute a good sample that can be used to study the mathematical form of spiral arms. The sample of this paper consists of the AC 12 spiral galaxies in Table 2 of Elmegreen & Elmegreen (1987). The right ascensions and declinations (for the equinox 2000.0) of these galaxies are from RC3 (de Vaucouleurs et al. 1991), and their images are taken from the Digitized Sky Survey (DSS)\* at Sheshan Station of Shanghai Astronomical Observatory. There are 84 galaxies with the AC 12 classification in Table 2 of Elmegreen & Elmegreen (1987), one of which (0235-02) is not included in RC3 (de Vaucouleurs et al. 1991). However, because of the low resolution of the images or the irregularity of the arms, the arms of 23 of the galaxies cannot be fitted. So, the sample of this paper includes 60 spiral galaxies.

### 5 RESULTS

Figure 1 shows the images of the spiral galaxies superposed with the fitted logarithmic spiral curves. The measured pitch angles of the spiral arms and inclination angles of the galactic discs for the sample galaxies are listed in Table 1. In this table,  $T$ , which are taken from RC3 (de Vaucouleurs et al. 1991), is the mean numerical Hubble stage index; RA and DEC, also from RC3, are the right ascension and declination for the equinox 2000.0;  $\gamma$ , which is obtained by comparing the spiral arm with the logarithmic spiral curve (see Section 3), is the inclination angle of the galactic disc. The pitch angles of the spiral arms are listed in column 7, A and B correspond to the A-arm and B-arm on the image when there are two arms to be fitted in one galaxy; Column 8 lists the polar angle of the main axis of the arms, which is anticlockwise from the east in the frame of the image.

**Table 1** Measured Inclination Angles of Discs and Pitch Angles of Spiral Arms

PGC	Other Name	$T$	RA (2000.0)	DEC (2000.0)	$\gamma$ ( $^{\circ}$ )	$\mu$ ( $^{\circ}$ )	P.A. ( $^{\circ}$ )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
PGC 303	NGC 7819	3.0	0 4 24.8	31 28 25	44.7	35.0	148.0
PGC 2052	NGC 150	3.0	0 34 15.8	-27 48 18	66.7	7.0	23.5
PGC 2901	NGC 266	2.0	0 49 48.2	32 16 43	13.0	12.0(A) 7.5(B)	21.4
PGC 2949	NGC 271	1.5	0 50 42.0	-1 54 32	41.3	11.4	40.3
PGC 5939	NGC 622	3.3	1 35 59.9	0 39 50	49.1	6.1	131.3
PGC 6624	NGC 673	5.0	1 48 22.8	11 31 23	47.0	17.5(A) 14.3(B)	75.8
PGC 6833	IC 167	5.0	1 51 8.3	21 54 50	38.0	27.9(A) 25.2(B)	44.0

\* Based on photographic data of the National Geographic Society – Palomar Observatory Sky Survey (NGS-POSS) obtained using the Oschin Telescope Palomar Mountain, or based on photographic data obtained using The UK Schmidt Telescope.

Table 1 Continued

PGC	Other Name	T	RA (2000.0)	DEC (2000.0)	$\gamma$ ( $^{\circ}$ )	$\mu$ ( $^{\circ}$ )	P.A. ( $^{\circ}$ )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
PGC 8961	0218+39A	3.0	2 21 28.8	39 22 31	48.0	21.9(A) 34.7(B)	140.7
PGC 9236	NGC 918	5.3	2 25 50.6	18 29 49	53.6	19.9(A) 16.9(B)	61.6
PGC 9426	NGC 945	4.5	2 28 37.3	-10 32 22	33.5	13.9(A) 11.2(B)	120.6
PGC 10488	NGC 1097	3.0	2 46 18.9	-30 16 21	45.8	10.6	47.2
PGC 12184	0313+31	5.0	3 16 59.3	31 34 2	32.5	22.3	37.2
PGC 12412	NGC 1300	4.0	3 19 40.8	-19 24 41	49.0	12.1(A) 11.0(B)	14.6
PGC 13166	NGC 1357	2.0	3 33 17.2	-13 39 54	25.5	4.4(A) 3.9(B)	163.1
PGC 13179	NGC 1365	3.0	3 33 36.6	-36 8 17	63.5	13.8(A) 17.8(B)	115.6
PGC 13584	NGC 1417	3.0	3 41 57.3	-4 42 21	66.5	9.3(A) 12.9(B)	94.3
PGC 14897	NGC 1566	4.0	4 20 0.4	-54 56 18	36.3	19.1(A) 14.0(B)	114.2
PGC 15018	NGC 1530	3.0	4 23 28.5	75 17 50	23.3	29.3(A) 33.3(B)	111.3
PGC 18709	0609+71A	3.0	6 15 7.9	71 8 12	55.3	8.1(A) 8.7(B)	130.0
PGC 22957	NGC 2535	5.0	8 11 13.2	25 12 22	31.0	22.0	120.8
PGC 23028	NGC 2543	3.0	8 12 58.4	36 15 15	54.6	13.1(A) 11.2(B)	126.0
PGC 24723	NGC 2633	3.0	8 48 6.6	74 5 58	45.4	10.8	103.8
PGC 24996	IC 2421	5.0	8 54 21.6	32 40 50	20.5	14.6(A) 17.7(B)	73.0
PGC 26666	NGC 2857	5.0	9 24 38.0	49 21 20	17.0	12.5(A) 13.4(B)	0.0
PGC 27777	NGC 2964	4.0	9 42 53.9	31 50 51	65.5	10.4	0.0
PGC 28630	NGC 3031	2.0	9 55 33.5	69 4 0	57.3	13.6	68.0
PGC 30323	NGC 3183	3.5	10 21 50.4	74 10 41	48.6	7.7(A) 23.5(B)	57.8
PGC 31926	NGC 3347	3.0	10 42 46.4	-36 21 14	66.6	25.0(A) 20.2(B)	85.0
PGC 32302	NGC 3381	99	10 48 25.1	34 42 44	27.7	3.3	51.9
PGC 33410	NGC 3513	5.0	11 3 45.7	-23 14 41	41.3	18.2(A) 8.1(B)	53.5
PGC 33860	IC 2627	4.0	11 9 53.3	-23 43 35	36.1	28.3(A) 20.2(B)	150.3
PGC 34232	NGC 3583	3.0	11 14 11.5	48 19 12	54.2	20.7(A) 12.6(B)	44.8
PGC 36875	NGC 3893	5.0	11 48 39.1	48 42 40	46.5	14.0(A) 10.5(B)	64.3
PGC 36902	NGC 3897	4.0	11 48 59.5	35 0 58	27.0	16.1(A) 15.2(B)	49.9
PGC 37386	NGC 3963	4.0	11 54 59.3	58 29 37	25.0	14.0(A) 11.0(B)	46.8
PGC 38024	1200+41	4.0	12 2 36.0	41 3 18	54.1	17.5(A) 10.0(B)	95.2
PGC 38240	NGC 4079	3.5	12 4 50.4	-2 22 58	32.7	11.6(A) 10.0(B)	20.6
PGC 38392	NGC 4102	3.0	12 6 23.4	52 42 41	54.6	16.4	115.1
PGC 39479	NGC 4246	5.0	12 17 58.2	7 11 7	60.4	12.7(A) 16.4(B)	172.4
PGC 40153	NGC 4321	4.0	12 22 55.2	15 49 23	30.7	21.0(A) 14.3(B)	30.6
PGC 42174	NGC 4580	1.0	12 37 48.4	5 22 9	43.0	9.6(A) 1.5(B)	67.5
PGC 45170	NGC 4939	4.0	13 4 14.6	-10 20 21	57.2	8.1(A) 10.8(B)	98.4
PGC 47404	NGC 5194	4.0	13 29 53.3	47 11 48	30.9	16.7(A) 15.8(B)	127.0
PGC 48130	NGC 5248	4.0	13 37 31.9	8 53 8	43.7	22.7(A) 23.8(B)	43.7
PGC 48371	NGC 5260	4.5	13 40 20.1	-23 51 28	28.1	11.1(A) 10.0(B)	59.6
PGC 49881	NGC 5430	3.0	14 0 45.7	59 19 42	43.3	18.7(A) 10.4(B)	91.7
PGC 51169	1416-26	5.0	14 19 22.7	-26 38 34	0.0	13.5(A) 7.7(B)	0.0
PGC 54018	NGC 5874	4.5	15 7 52.4	54 45 12	53.5	15.9(A) 15.6(B)	131.1
PGC 54097	NGC 5861	5.0	15 9 16.0	-11 19 17	56.5	13.6(A) 12.7(B)	59.9
PGC 54445	NGC 5905	3.0	15 15 23.2	55 31 5	43.0	29.0(A) 24.1(B)	19.6
PGC 56479	1555+30	4.0	15 57 27.8	30 3 23	28.1	8.3(A) 13.3(B)	134.8
PGC 58470	NGC 6181	5.0	16 32 20.9	19 49 30	60.9	22.9	85.3
PGC 59280	IC 1237	4.5	16 56 15.5	55 1 32	72.4	29.8(A) 30.9(B)	120.6
PGC 64650	NGC 6907	4.0	20 25 6.6	-24 48 30	49.5	23.5	149.9
PGC 64652	2022+05	4.0	20 25 6.0	5 15 0	29.6	29.1(A) 19.0(B)	49.1
PGC 65086	NGC 6951	4.0	20 37 15.2	66 6 22	46.5	25.7	18.5
PGC 65269	NGC 6956	3.0	20 43 53.7	12 30 39	58.6	25.7(A) 20.0(B)	48.6
PGC 65375	NGC 6962	2.0	20 47 19.0	0 19 17	55.3	10.7(A) 5.6(B)	163.5
PGC 69439	2237+37	4.0	22 39 49.8	38 12 58	32.5	21.1	148.4
PGC 72387	NGC 7753	4.0	23 47 4.7	29 29 2	53.4	13.7	150.3

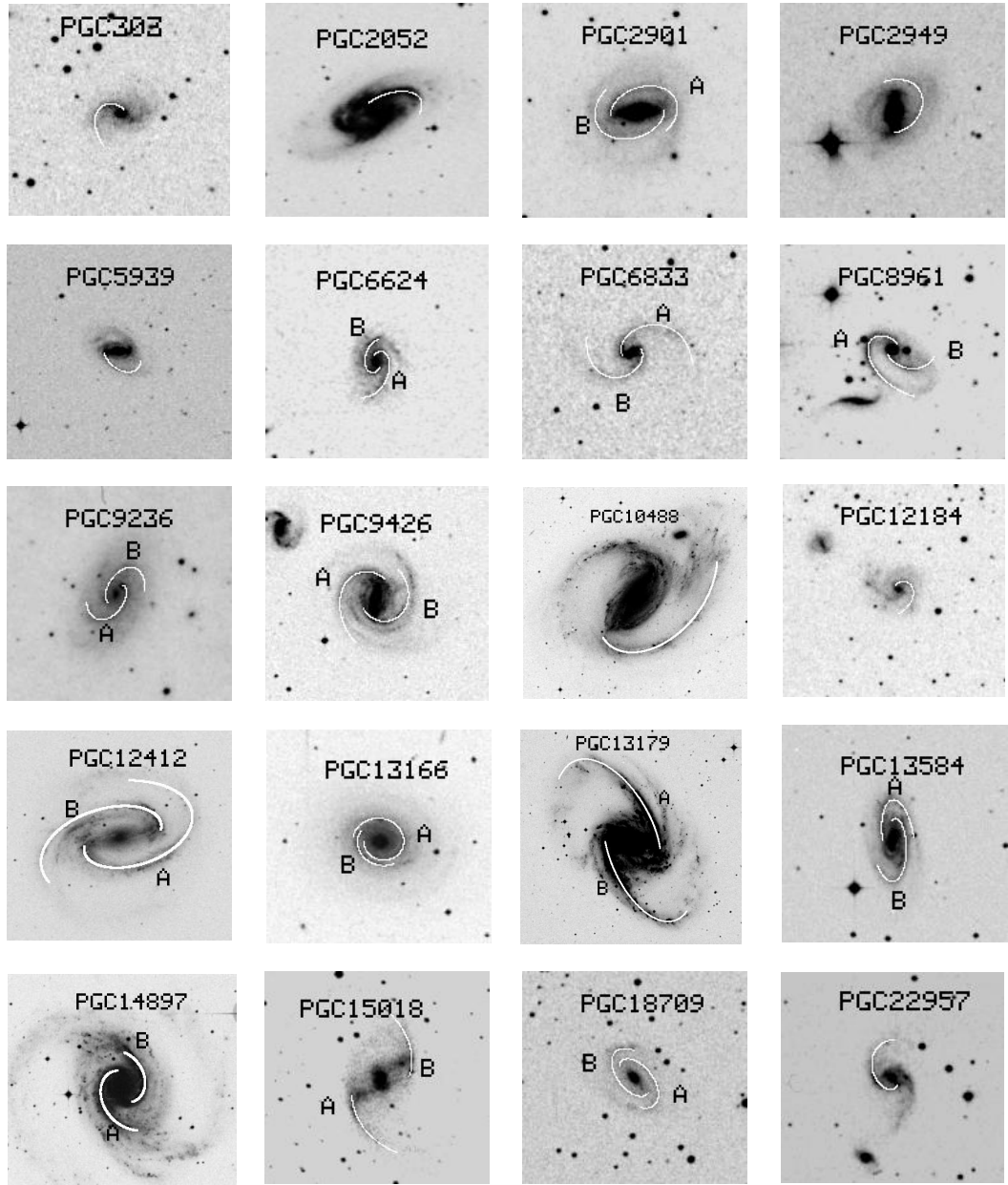


Fig. 1 Images of spiral galaxies with superposed fitted logarithmic spiral curves

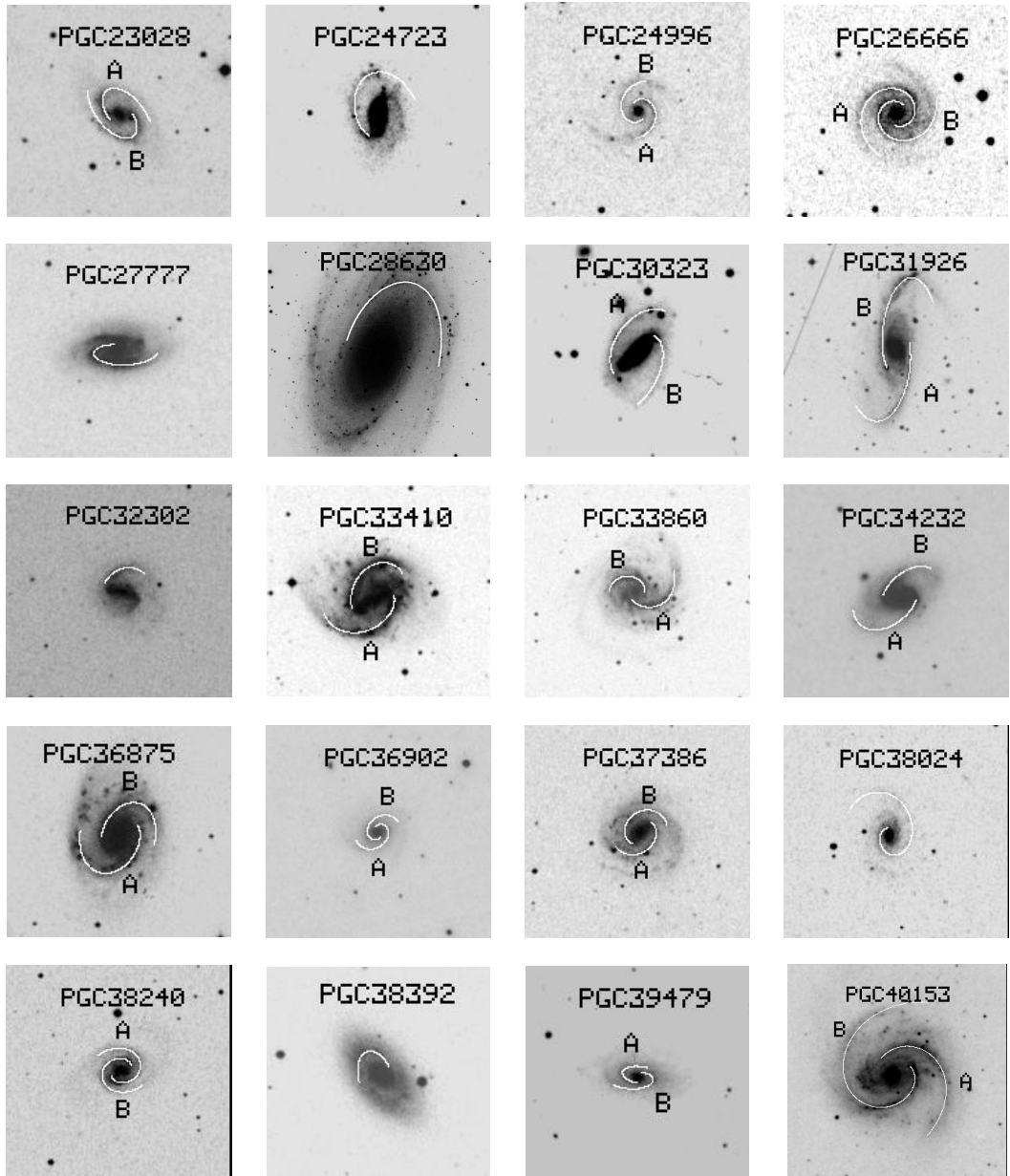


Fig.1 Continued



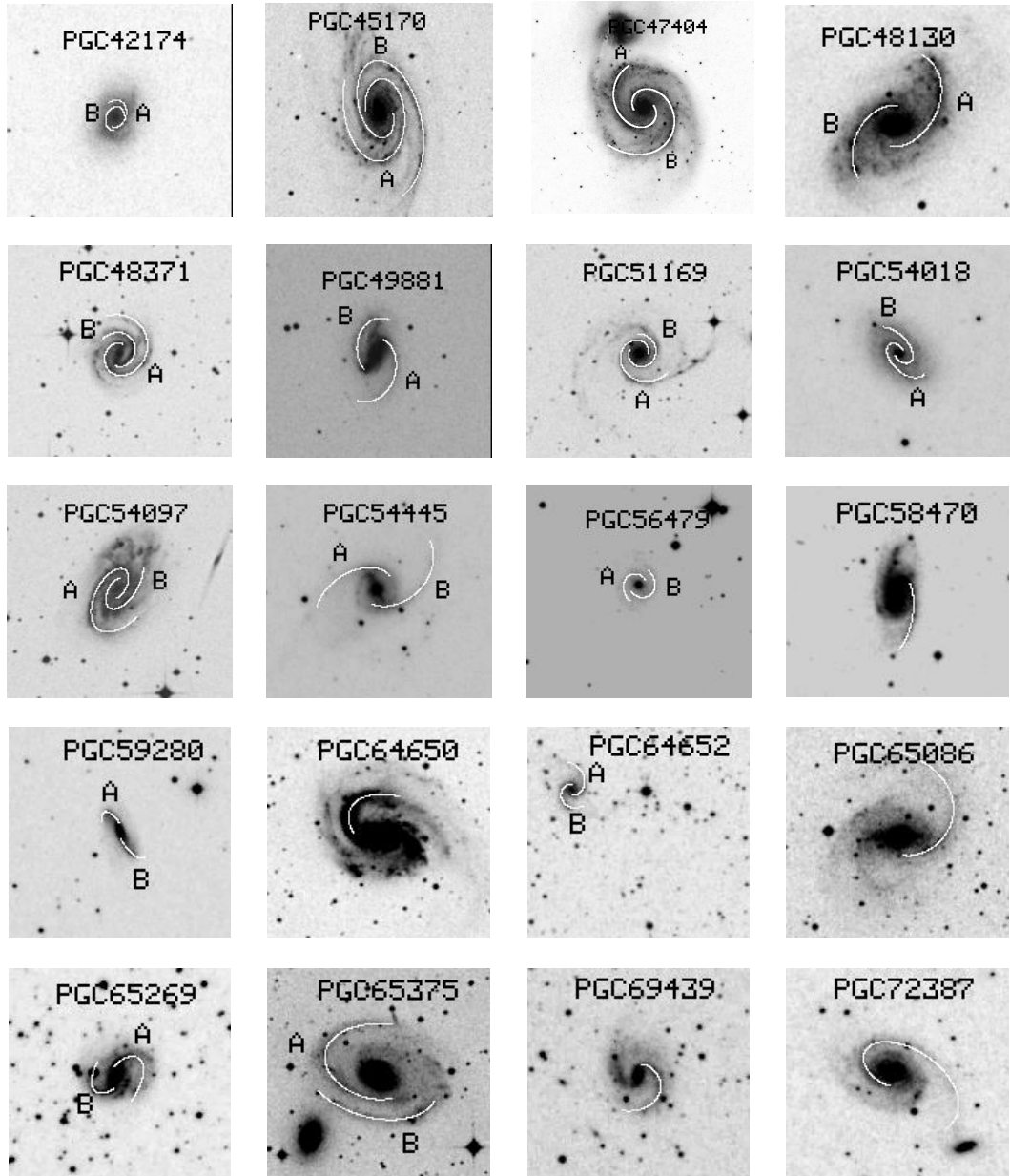


Fig.1 Continued

## 6 CONCLUSIONS AND DISCUSSION

In this paper, we study the mathematical form, the symmetry of spiral structure and the projection of the galactic discs, and present the measured pitch angles of the spiral arms and inclination angles of the galactic discs for 60 spiral galaxies. We find that, except for small-scale distortions, the spiral arms of the galaxies that are assigned to AC 12 in the arm classification system of Elmegreen & Elmegreen (1987), can be represented by the logarithmic spiral form. There are 15 galaxies, in which only one arm is clear enough to be fitted and the other is weak or short. These galaxies are PGC 303, 2949, 5939, 12184, 22957, 24723, 28630, 32302, 38024, 38392, 58470, 64650, 65086, 69439 and 72387. For PGC 2052, one of the arms is split and cannot be fitted. For PGC 10488, one of the arms is peculiar, and its end part is not joined to the beginning part and defied fitting. The rest 43 galaxies have two arms that can be fitted. But, for PGC 47404, the A arm is influenced by another galaxy, and its end part cannot be fitted by the same logarithmic spiral form as the rest and looks like a straight line; for PGC 51169, the end part of the A arm also looks like a straight line and is separate from the rest, the end part of the B arm cannot be fitted by the same logarithmic spiral form as the rest. From Table 1, we can see that, for most spiral galaxies, the pitch angles of the two arms are not equal, even very different, such as PGC 26666, PGC 30323, and others. There are only a few galaxies that have symmetric arms, for example, PGC 47404, PGC 48130, PGC 59280 and PGC 54018. Finally, we should emphasize that, in our method, there exist some uncertainties in obtaining the pitch angles of the spiral arms and inclination angles of the galactic discs because of the uncertainties of sampling the points on the the central line of the mass of the spiral arm. The reliability of this method for obtaining the measured pitch angles of the spiral arms and inclination angles of the galactic discs has been discussed by Ma et al. (1999), who presented the statistical correlation of 72 northern spiral galaxies. Further statistical study on the comparison of the obtained parameters in this paper with other authors will be addressed in a separate paper.

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