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Entropy of Dilatonic Black Hole due to Arbitrary Spin Fields

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Abstract Using the membrane model based on the brick-wall model, we calculate the free energy and entropy of dilatonic black hole due to arbitrary spin fields. The result shows that the entropy of scalar field and the entropy of Fermionic field have similar formulas. There is only a numerical coefficient between them.

Key words: black hole physics — entropy — spin

1 INTRODUCTION

In theoretical physics, the thermodynamics of black holes remains an enigma, it turns out to be a junction of general relativity, quantum mechanics, and statistical physics.

Since Bekenstein and Hawking proposed, in 1970s, that the black hole entropy is proportional to the area of the event horizon (Bekenstein 1972, 1973, 1974; Hawking 1975; Kallosh et al. 1993), many efforts are devoted to the study of the statistical origin of the black hole entropy,

One such effort is the widely used brick wall model proposed by Hooft (1985). Using this model, Hooft investigated the statistical properties of a free scalar field in the Schwarzschild black hole background, got an expression for the entropy in terms of the area of the event horizon which verifies the proportional relationship between them. Furthermore, when the cut-off parameter satisfies a certain condition, the entropy can be written as $S = A_h/4$, while for the case of a vanishing cut-off parameter, the entropy would be divergent, which was attributed to the infinite density of states at the vicinity of the horizon. Another different but actually equivalent (Callan et al. 1994; Kabat et al. 1994) approach is adopted by Bombelli et al. (1986) and Srednicki (1993). Solodukhin used Gibbons-Hawking (1977) Euclidean path integral approach to study the quantum corrections to the entropy of a Schwarzschild black hole starting with the one-loop effective action of scalar matter (Solodukhin 1995a, 1995b). In quantum mechanics, the following convention applies. If the particles are scalar Bosons obeying Bose-Einstein statistics, the entropy obtained is conventionally called the Bosonic entropy; if the

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quantum-mechanical geometric entropy is calculated by counting the Fermionic particle states, the corresponding entropy is called the Fermionic entropy.

Since the mid-1990s, various problems have aroused much interest among many researchers (Ghosh 1994, 1995; Teitelboin 1995; Kabat 1995; Larsen 1995, 1996; Congnola 1998; Lee 1996a, b, c, d; Cvetic 1996; Alwis 1995; Hawking 1995; Carlip 1995; Zhao 1999; Wang 2000; Shen 1998a, b, c, d, 1999a, b, 2000a, b, c, d, 2001a, b; Pinto-Neto 1995; Guber 1996; Fila 1994; Jocason 1995; Russo 1995; Zhou 1995; Brown 1995; Demers 1995; Solodukhin 1995a, b, c, 1996; Carlip 1995; Kim 1997; Mann 1996). But up to now, the method mainly used is the brick wall model; furthermore, in order to get the proportional-to-area result, we must use the small mass approximation. Considering the divergence of wave function near the event horizon and the introduction of cutoff near the event horizon, we ask: why can we not assume that the free energy of the black hole only comes from a layer near the event horizon? Such a physical picture is very clear.

The membrane model (Gao et al. 2000; Li 2000) assumes that the area in which the wave function is not zero lies in $r_H + \varepsilon \leq r \leq r_H + \varepsilon + \delta$. That is, the interval of integration of r is only a membrane of thickness δ at distance ε to the event horizon.

In this paper, we use the membrane model to obtain the free energy and entropy of the gravitational field (spin $s = 2$), electromagnetic field (spin $s = 1$) and neutrino field (spin $s = \frac{1}{2}$). The formulas of the entropy are spelt out. We find that the entropy of the scalar field and the entropy of the Fermionic field have similar formulas. There is only a numerical coefficient between them. This result is similar to the previous result (Alwis et al. 1995; Zhou et al. 1995; Larsen et al. 1995; Congnola et al. 1998; Shen et al. 1999a, b, 2000a, b, c, d).

2 FIELD EQUATION

The metric of the Garfinkle-Horowitz-Strominger dilatonic black hole is (Garfinkle et al. 1991)

$$\mathrm{d}s^2 = \left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 - \left(1 - \frac{2M}{r}\right)^{-1}\mathrm{d}r^2 - r\,(r-a)\,(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2), \tag{1}$$

$$e^{-2\phi} = e^{-2\phi_0}\left(r - \frac{a}{r}\right), \tag{2}$$

$$F_{\theta\varphi} = Q\sin\theta, \tag{3}$$

where M and Q are the mass and magnetic charge of the black hole, respectively. The parameter a is defined to be

$$a = \frac{Q^2}{2M}e^{-2\phi_0}, \tag{4}$$

where ϕ_0 is an arbitrary constant.

Choose the null tetrad as follows:

$$\begin{aligned} l^\mu &= \left(\left(1 - \tfrac{2M}{r}\right)^{-1}, 1, 0, 0\right), \\ n^\mu &= \tfrac{1}{2}\left(1, -\left(1 - \tfrac{2M}{r}\right), 0, 0\right), \\ m^\mu &= \tfrac{1}{\sqrt{2r(r-a)}}\left(0, 0, 1, \tfrac{i}{\sin\theta}\right), \\ \bar{m}^\mu &= \tfrac{1}{\sqrt{2r(r-a)}}\left(0, 0, 1, -\tfrac{i}{\sin\theta}\right). \end{aligned} \tag{5}$$

The corresponding covariant null tetrad is

$$\begin{aligned} l_\mu &= \left(1, -\left(1-\tfrac{2M}{r}\right)^{-1}, 0, 0\right), \\ n_\mu &= \tfrac{1}{2}\left(\left(1-\tfrac{2M}{r}\right), 1, 0, 0\right), \\ m_\mu &= -\tfrac{\sqrt{r(r-a)}}{\sqrt{2}}(0, 0, 1, i\sin\theta), \\ \bar{m}_\mu &= -\tfrac{\sqrt{r(r-a)}}{\sqrt{2}}(0, 0, 1, -i\sin\theta). \end{aligned} \tag{6}$$

The above null tetrad satisfies the null vector conditions

$$l_\mu l^\mu = n_\mu n^\mu = m_\mu m^\mu = \bar{m}_\mu \bar{m}^\mu = 0, \tag{7}$$

pseudo-orthogonality conditions

$$\begin{aligned} & l_\mu n^\mu = -m_\mu \bar{m}^\mu = 1, \\ & l_\mu m^\mu = l_\mu \bar{m}^\mu = n_\mu m^\mu = n_\mu \bar{m}^\mu = 0, \end{aligned} \tag{8}$$

and metric conditions

$$g_{\mu\nu} = l_\mu n_\nu + n_\mu l_\nu - m_\mu \bar{m}_\nu - \bar{m}_\mu m_\nu\,. \tag{9}$$

Set

$$\Delta = r^2 - 2Mr, \tag{10}$$

then the nonvanishing spin coefficients are (Newman et al. 1962)

$$\rho = -\frac{2r-a}{2r\,(r-a)}, \tag{11}$$

$$\gamma = \frac{M}{2r^2}, \tag{12}$$

$$\mu = -\frac{(2r-a)\,\Delta}{4r^3\,(r-a)}, \tag{13}$$

$$\alpha = -\beta = -\frac{\cot\theta}{2\sqrt{2r(r-a)}}\,. \tag{14}$$

Only one of the Weyl tensors is not zero, i.e.,

$$\Psi_2 = -\frac{M\,(2r-a)}{2r^3\,(r-a)} + \frac{a\,(r-2M)}{8r^3\,(r-a)^2}. \tag{15}$$

Equations (11)–(15) show that the GHS (Garfinkle-Horowitz-Strominger) metric is of Petrov-type D. Using the result of Teukolsky (Teukolsky 1973; Teukolsky et al. 1974), the field equation of spin $s = \frac{1}{2}, 1$, and 2 for the source free case can be combined into

$$\begin{aligned} & [D - (2s+1)\,\rho]\,[\Delta - 2s\gamma + \mu]\,\Phi_{+s} \\ & -\left\{[\delta + (2s-2)\,\alpha]\left[\bar{\delta} - 2s\alpha\right] + (2s-1)\,(s-1)\,\Psi_2\right\}\Phi_{+s} = 0, \\ & [\Delta + (2s-2)\,\gamma + (2s+1)\,\mu]\,[D - \rho]\,\Phi_{-s} \\ & -\left\{\left[\bar{\delta} + (2s-2)\,\alpha\right][\delta - 2s\alpha] + (2s-1)\,(s-1)\,\Psi_2\right\}\Phi_{-s} = 0, \end{aligned} \tag{16}$$

where

$$D \equiv l^\mu \partial_\mu = \frac{r^2}{\Delta}\partial_t + \partial_r, \tag{17}$$

$$\Delta \equiv n^\mu \partial_\mu = \frac{1}{2}\partial_t - \frac{\Delta}{2r^2}\partial_r, \tag{18}$$

$$\delta \equiv m^\mu \partial_\mu = \frac{1}{\sqrt{2r\left(r-a\right)}}\left(\frac{\partial}{\partial\theta} + \frac{i}{\sin\theta}\frac{\partial}{\partial\varphi}\right), \tag{19}$$

$$\bar{\delta} \equiv \bar{m}^\mu \partial_\mu = \frac{1}{\sqrt{2r\left(r-a\right)}}\left(\frac{\partial}{\partial\theta} - \frac{i}{\sin\theta}\frac{\partial}{\partial\varphi}\right). \tag{20}$$

The first one of Equations (16) is for spin states $p = s$ and the other one, for $p = -s$. Make the transformations (Teukolsky 1973),

$$\Phi_{+s}, \Phi_{-s} = r^{p-s}{}_pR_{lE}\left(r\right){}_pY_l^m\left(\theta,\varphi\right)e^{-iEu}. \tag{21}$$

Put Equations (11)–(15), (17)–(21) into Equations (16), and we obtain the radial equation

$$\begin{aligned}\partial_r^2{}_pR_{lE}\left(r\right) \; &+ \; \frac{2r^2}{\Delta}\left[\frac{\left(p+1\right)M}{r^2} + \frac{p+1}{r^2}\frac{2r-a}{2r\left(r-a\right)}\Delta\right]\partial_{r}{}_pR_{lE}\left(r\right) \\ &+ \; \left[\frac{r^4}{\Delta^2}E^2 + iEA\left(r\right) + B\left(r\right) - \frac{r}{\left(r-a\right)\Delta}\lambda^2\right]{}_pR_{lE} = 0,\end{aligned} \tag{22}$$

where

$$A\left(r\right) = -\frac{2r^2}{\Delta}\left[\frac{pM}{\Delta} - \frac{p\left(2r-a\right)}{2r\left(r-a\right)}\right], \tag{23}$$

$$\begin{aligned}B\left(r\right) = \; &- \; \frac{2r^2}{\Delta}\left[\frac{2Mp}{r^3} - p\left(2p+1\right)\frac{M\left(2r-a\right)}{2r^3\left(r-a\right)} + \frac{r-4M}{4r^3}\frac{2r-a}{r-a}\right] \\ &- \; \frac{2r^2}{\Delta}\left[\frac{\Delta}{4r^3}\frac{a}{\left(r-a\right)^2} - \frac{2p+1}{8}\frac{\Delta}{r^4}\frac{\left(2r-a\right)^2}{\left(r-a\right)^2}\right],\end{aligned} \tag{24}$$

and the angular equation

$$\begin{aligned}&\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right]{}_pY_l^m\left(\theta,\varphi\right) \\ &+ \left[\frac{2ip\cos\theta}{\sin^2\theta}\frac{\partial}{\partial\varphi} - p^2\cot^2\theta - p + \lambda^2\right]{}_PY_l^m\left(\theta,\varphi\right) = 0.\end{aligned} \tag{25}$$

Equation (25) shows that ${}_pY_l^m$ is the spin-weighed spherical harmonic (Goldberg et al. 1967; Jenson et al. 1995), and the separation constant λ satisfies

$$\lambda = \sqrt{\left(l-p\right)\left(l+p+1\right)}, \tag{26}$$

where

$$l \geq \left|p\right|, \qquad -l \leq m \leq l. \tag{27}$$

3 FREE ENERGY AND ENTROPY

In this section we calculate the black hole entropy with the membrane model. In simplest terms, the membrane model says: in the vicinity of event horizon there is a layer of radiation whose thickness is δ and whose distance to the event horizon is ε. The entropy of the black hole is equal to that of the membrane. So the boundary condition of the wave function is

$$\Phi(r) = 0, \quad \text{when}, \quad r \leq r_h + \varepsilon, \tag{28}$$

$$\Phi(r) = 0, \quad \text{when}, \quad r \geq r_h + \varepsilon + \delta, \tag{29}$$

where $\varepsilon \ll r_h$, $\delta \ll r_h$, $r_h = 2M$ is the equation of event horizon.

Let

$$_pR_{lE}(r) = e^{iZ}, \tag{30}$$

and using the WKB approximation, we obtain

$$K^2 = (\partial_r Z)^2 = \left[\frac{r^4}{\Delta^2}E^2 + B(r) - \frac{(l-p)(l+p+1)}{(r-2M)(r-a)}\right], \tag{31}$$

where K is the radial wave number.

The constraint of semi-classical quantum condition imposed on K reads

$$n\pi = \int_{r_h+\varepsilon}^{r_h+\varepsilon+\delta} K\mathrm{d}r, \tag{32}$$

where n is a non-negative integer. As in the case of the brick wall model, the energy E is positive and the wave number K is real.

According to the ensemble theory, the free energy is

$$\beta F = \mp \sum \ln\left(1 \pm e^{-\beta\omega}\right), \tag{33}$$

where β is the inverse of the Hawking temperature, i.e.,

$$\frac{1}{\beta} = T_h = \frac{\kappa}{2\pi} = \frac{1}{8\pi M}.$$

Regarding the states of energy as continuous and changing summation into integration, we obtain

$$\Sigma \to \int_0^\infty \mathrm{d}Eg(E), \tag{34}$$

where $g(E)$ is the density of states, i.e.,

$$g(E) = \frac{\mathrm{d}\Gamma(E)}{\mathrm{d}E},$$

$\Gamma(E)$ is the number of the microscopic states, that is,

$$\Gamma(E) = \sum_p \sum_l (2l+1)\,n. \tag{35}$$

Transform the summation of l into integration and require $K \geq 0$, then we obtain

$$\begin{aligned}
\Gamma(E) &= \sum_P \int (2l+1)\,\mathrm{d}l \frac{1}{\pi} \int K \mathrm{d}r \\
&= \frac{1}{\pi} \sum_p \int_{r_h+\varepsilon}^{r_h+\varepsilon+\delta} \mathrm{d}r \int_{|p|}^{l_{\max}} \mathrm{d}l\,(2l+1)\,K \\
&= \frac{2}{3\pi} \sum_P \int_{r_h+\varepsilon}^{r_h+\varepsilon+\delta} \mathrm{d}r\,(r-2M)\,(r-a) \\
&\quad \left[\frac{r^4}{\Delta^2} E^2 + B(r) - \frac{(|p|-p)}{(r-2M)\,(r-a)}\right]^{\frac{3}{2}} .
\end{aligned} \tag{36}$$

The free energy can be written as

$$\begin{aligned}
F &= -\frac{2}{3\pi}\frac{1}{\beta} \sum_P \int_0^\infty \mathrm{d}E \frac{1}{e^{\beta E} \mp 1} \int_{r_h+\varepsilon}^{r_h+\varepsilon+\delta} \mathrm{d}r\,(r-2M)\,(r-a) \\
&\quad \left[\frac{r^4}{\Delta^2} E^2 + B(r) - \frac{(|p|-p)}{(r-2M)\,(r-a)}\right]^{\frac{3}{2}} \\
&= -\frac{2}{3\pi}\frac{1}{\beta} \sum_P \int_0^\infty \frac{E^3 \mathrm{d}E}{e^{\beta E} \mp 1} \int_{r_h+\varepsilon}^{r_h+\varepsilon+\delta} \mathrm{d}r \left(1 - \frac{2M}{r}\right)^{-2} \left(r^2 - ra\right).
\end{aligned} \tag{37}$$

In the case of $a \neq 2M$, the free energy writes

$$F_{1b} \approx -\frac{\omega\pi^3}{90\varepsilon}\left(1 - \frac{2M}{a}\right)\left(\frac{2M}{\beta}\right)^4, \tag{38}$$

$$F_{1f} \approx -\frac{7}{8}\frac{\omega\pi^3}{90\varepsilon}\left(1 - \frac{2M}{r}\right)\left(\frac{2M}{\beta}\right)^4. \tag{39}$$

In the case of $a = 2M$, the free energy writes

$$F_{2b} \approx -\frac{\omega\pi^3}{180M} \ln\left(\frac{\varepsilon+\delta}{\varepsilon}\right)\left(\frac{2M}{\beta}\right)^4, \tag{40}$$

$$F_{2f} \approx -\frac{7}{2}\frac{\omega\pi^3}{180M} \ln\left(\frac{\varepsilon+\delta}{\varepsilon}\right)\left(\frac{2M}{\beta}\right)^4, \tag{41}$$

where ω is the degeneracy due to spin. For the gravitational and electromagnetic fields we have $\omega = 2$; for the neutrino and scalar fields we have $\omega = 1$, where $a = 2M$ is for the extreme GHS dilatonic black hole case.

Considering the relationship between entropy and free energy below

$$S = \beta^2 \frac{\partial F}{\partial \beta}, \tag{42}$$

we obtain

$$S_{1b} = \frac{8\omega\pi^3}{45\varepsilon}(2M-a)\left(\frac{2M}{\beta}\right)^3, \tag{43}$$

$$S_{1f} = \frac{7}{8}\frac{8\omega\pi^3}{45\varepsilon}\left(2M - a\right)\left(\frac{2M}{\beta}\right)^3, \tag{44}$$

and

$$S_{2b} = \frac{8\omega\pi^3}{45M}\ln\left(\frac{\varepsilon+\delta}{\varepsilon}\right)\left(\frac{2M}{\beta}\right)^3, \tag{45}$$

$$S_{2f} = \frac{7}{8}\frac{8\omega\pi^3}{45M}\ln\left(\frac{\varepsilon+\delta}{\varepsilon}\right)\left(\frac{2M}{\beta}\right)^3. \tag{46}$$

In the case of $a \neq 2M$ when we choose

$$\varepsilon = \frac{1}{720\pi M}, \tag{47}$$

which is exactly the ultraviolet cutoff in Reference (Hooft 1985), the entropy can be written as

$$S_{1b} = \omega 2\pi M\left(2M - a\right), \tag{48}$$

$$S_{1f} = \frac{7}{8}\omega 2\pi M\left(2M - a\right). \tag{49}$$

The area of GHS dilatonic black hole writes

$$A_h = 4\pi r_h\left(r_h - a\right), \tag{50}$$

so the formulas of entropy become

$$S_{1b} = \omega\frac{A_h}{4}, \tag{51}$$

$$S_{1s} = \frac{7}{8}\omega\frac{A_h}{4}. \tag{52}$$

The factor ω vanishes if we put

$$\varepsilon = \frac{\omega}{720\pi M}.$$

Equations (45)–(46) tell us that in the case of $a = 2M$, i.e., extreme black hole, the entropy of GHS dilatonic black hole is not zero although its area is zero. Equations (43)–(44), (51)–(52) and Reference (Ghosh et al. 1994) show that GHS dilatonic black hole entropy due to scalar field ($s = 0, 1, 2$) and Fermionic field ($s = \frac{1}{2}$) have similar formulas. There is only a numerical coefficient between them.

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