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Harmonic Structures of Gyrosynchrotron Radiation and Cyclotron Maser Instability

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Abstract The harmonic structures of the gyrosynchrotron radiation are studied and compared with the results of the electron cyclotron maser instability. The structures only appear at lower harmonics ($s < 10$), and their peaks deviate by a small amount from integer harmonic numbers. The amplitudes of harmonic structures of the extraordinary modes are usually larger than that of the ordinary modes. The numbers and amplitudes of harmonic structures increase with increasing electron energy spectral index δ and propagation angle θ. All the properties of harmonic structures are consistent with the predictions of the electron cyclotron maser instability in an extremely magnetized plasma on the basis of Maxwell and Vlasov equations. The physical relations between gyrosynchrotron radiation and cyclotron maser instability and a possible application of the properties of the harmonic structures are discussed.

Key words: Sun: activity — Sun: radio radiation — Sun: magnetic fields

1 INTRODUCTION

Solar microwave bursts are usually produced by different radiation mechanisms: gyroresonance absorption, Coulomb bremsstrahlung of the thermal background plasma, as well as gyrosynchrotron radiation of the nonthermal electrons (Hildebrandt et al. 1998). Meanwhile, cyclotron or synchrotron maser instabilities may be responsible for microwave spikes, blips, and type III bursts (Wu 1985; Huang 1987; Huang et al. 1996; Huang 1998), which are usually superposed on the ambient microwave bursts (Aschwanden 1998). The magnetic field and density of nonthermal electrons in the source region of microwave bursts calculated from the gyrosynchrotron model have been found to be comparable with the ambient plasma and nonthermal parameters calculated from the electron cyclotron maser theory for the event on 1997 November 2, (Huang et al. 2000).

It is interesting that there are some harmonic structures at the lower harmonics or the optically thick part of microwave burst spectrum from the calculations of gyrosynchrotron theory,

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which are explained by the strong gyroabsorption effects (Hildebrandt et al. 1998). Note, however, that the harmonic structures are also predicted by the model of electron cyclotron maser instability, especially, when the non-resonant effect is taken into consideration, the separate harmonic peaks are broadened and connected each other (Huang et al. 1996).

The question considered in this paper is how to understand the relation between gyrosynchrotron radiation and cyclotron maser instabilities. The first step is to calculate the emissivity, $\eta_{\nu\pm}/BN$, and absorptivity, $\kappa_{\nu\pm}B/N$, and to study the properties of the harmonic structures of the gyrosynchrotron radiation excited by nonthermal electrons (Section 2). Then, the properties are compared with the results of electron cyclotron maser instability (Section 3). Finally, the physical relations between gyrosynchrotron radiation and cyclotron maser instability and a possible application of harmonic structure properties are discussed (Section 4).

2 HARMONIC STRUCTURES OF GYROSYNCHROTRON RADIATION

2.1 Emissivity and Absorptivity of Gyrosynchrotron Radiation

We now consider such plasma media in which the density is so low that the index of refraction is nearly unity, i.e., $n_\pm \approx 1$. This situation applies in the microwave burst regions. If the distribution of pitch angle ϕ of the energetic electrons is isotropic and the energy distribution is $N(\varepsilon) = G\varepsilon^{-\delta}$, then

$$4\pi p^2 N\,(p,\phi)\,\mathrm{d}p = N\,(\varepsilon)\,\mathrm{d}\varepsilon. \tag{1}$$

For the propagation angle $\theta \neq \pi/2$ and the electron energy spectral index $\delta > 1$, the emissivity $\eta_{\nu\pm}/BN$ and absorptivity $\kappa_{\nu\pm}B/N$ of gyrosynchrotron radiation excited by the energetic electrons are (cf. Takakura and Scalise 1970)

$$\frac{\eta_{\nu\mp}}{BN} = \frac{e^3}{4mc^2}\,(\delta-1)\,\varepsilon_0^{\delta-1}\frac{s}{|\cos\theta|}\sum_{n>s\sin\theta}^{\infty}\int_{p_0}^{p_m}(a\pm b)^2\frac{\left(\sqrt{1+p^2}-1\right)^{-\delta}}{(1+p^2)}\mathrm{d}p, \tag{2}$$

$$\begin{aligned}\frac{\kappa_{\nu\mp}B}{N} &= \pi^2 e\,(\delta-1)\,\varepsilon_0^{\delta-1}\,(s\cos\theta)^{-1}\sum_{n>s\sin\theta}^{\infty}\int_{p_0}^{p_m}(a\pm b)^2\mathrm{d}p \\ &\times\left\{-\frac{\mathrm{d}}{\mathrm{d}p}\left[p^{-1}\left(1+p^2\right)^{-1/2}\left(\sqrt{1+p^2}-1\right)^{-\delta}\right]\right\},\end{aligned} \tag{3}$$

where

$$a = \frac{1}{s\cos\theta}\left[2ns\sqrt{1+p^2}-s^2(1+p^2\sin^2\theta)-n^2\right]^{1/2}J_n'(x),$$

$$b = \frac{n-s\sqrt{1+p^2}\sin^2\theta}{s\cos\theta\sin\theta}J_n(x),$$

$$x = \tan\theta\left[2ns\sqrt{1+p^2}-s^2(1+p^2\sin^2\theta)-n^2\right]^{1/2},$$

$$p = mv/m_0c, \varepsilon = K/m_0c^2, (1+\varepsilon)^2 = 1+p^2\,.$$

The terms $J_n(x_n)$ and $J_n'(x_n)$ are, respectively, Bessel function of order n and its derivative. s is the harmonic number.

For $\theta = \pi/2$ and $\delta > 1$ we have (Zhou, Huang and Wang 1999)

$$\frac{\eta_{\nu\pm}}{BN} = \frac{e^3}{2mc^2}(\delta-1)\,\varepsilon_0^{\delta-1} s \sum_{n=s+1}^{\infty}\left(1+\frac{s}{n}\right)\left(\frac{n}{s}-1\right)^{1-\delta}\int_{-1}^{1} Y_{\pm}\mathrm{d}\,(\cos\phi), \tag{4}$$

$$\frac{\kappa_{\nu\pm}B}{N} = 2\pi^2 e\,(\delta-1)\,\varepsilon_0^{\delta-1}s^{-1}\sum_{n=s+1}^{\infty}\left(\frac{n}{s}-1\right)^{-\delta}\left[2-\left(\frac{s}{n}\right)^2+\delta\left(1+\frac{s}{n}\right)\right]\int_{-1}^{1} Y_{\pm}\mathrm{d}(\cos\phi)\,, \tag{5}$$

where

$$Y_+ = \cos^2\phi J_n^2(x_n),\;\; Y_- = \sin^2\phi J_n'^2(x_n),$$

$$\gamma_n = n\nu_B/\nu, \qquad \beta_n = (1-\gamma_n^{-2})^{1/2}, \qquad x_n = n\beta_n\sin\phi.$$

When $\delta = 1$, the term $(\delta-1)\varepsilon_0^{\delta-1}$ is substituted by $1/\ln\varepsilon_m$ in Equations (2)–(5).

2.2 Properties of Harmonic Structures

The emissivity and absorptivity of gyrosynchrotron radiation propagating in the longitudinal and transverse directions have been calculated for the extraordinary mode (i.e., x-mode) and ordinary mode (o-mode), taking the value of ε_0 corresponding to 10 keV. The numerical results of the emissivity $\eta_{\nu\pm}/BN$ and absorptivity $\kappa_{\nu\pm}B/N$ for $\delta = 3$ are given in Figure 1 for the longitudinal and transverse propagation cases, where the solid lines are for the x-mode and the dotted lines, the o-mode. Figure 1 shows that there are some harmonic structures in the lower harmonics ($s < 10$) for x-mode and o-mode, moreover the harmonic structures are more obvious in the x-mode than in the o-mode.

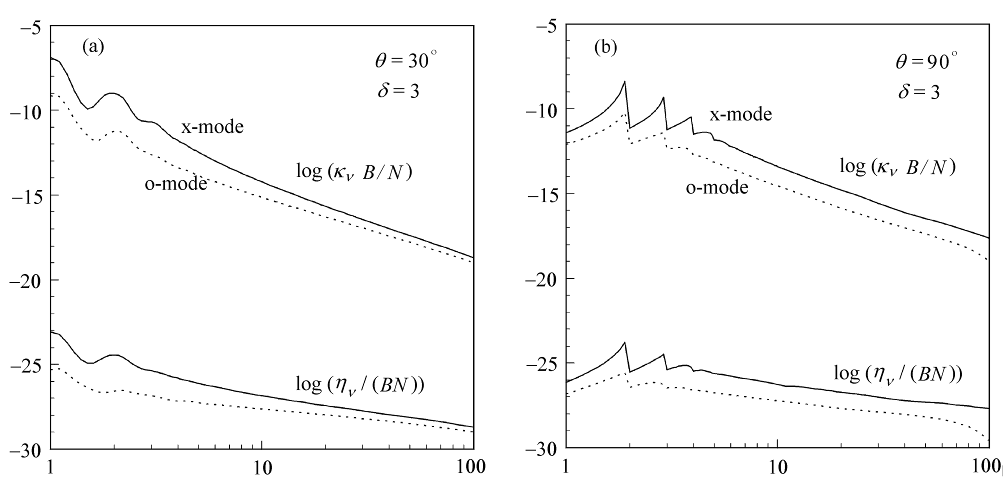

Fig. 1 Emissivities $\eta_{\nu\pm}/BN$ and absorptivities $\kappa_{\nu\pm}B/N$, taking ε_0 as the value corresponding to 10 keV, (a) for $\theta = 30°$, (b) for $\theta = 90°$.

The variations of harmonic structures of emissivity $\eta_{\nu-}/BN$ and absorptivity $\kappa_{\nu-}B/N$ for x-mode with different propagation angle θ are studied from Equations (2)–(5). Figures 2a and

2b show the numerical results of the emissivity $\eta_{\nu-}/BN$ at different propagation angles for $\delta = 3$ and 7, respectively. It is found that the amplitude and number of harmonic structures increase with increasing θ, i.e., there are more rich harmonic structures in the case of the transverse propagation than in the longitudinal case.

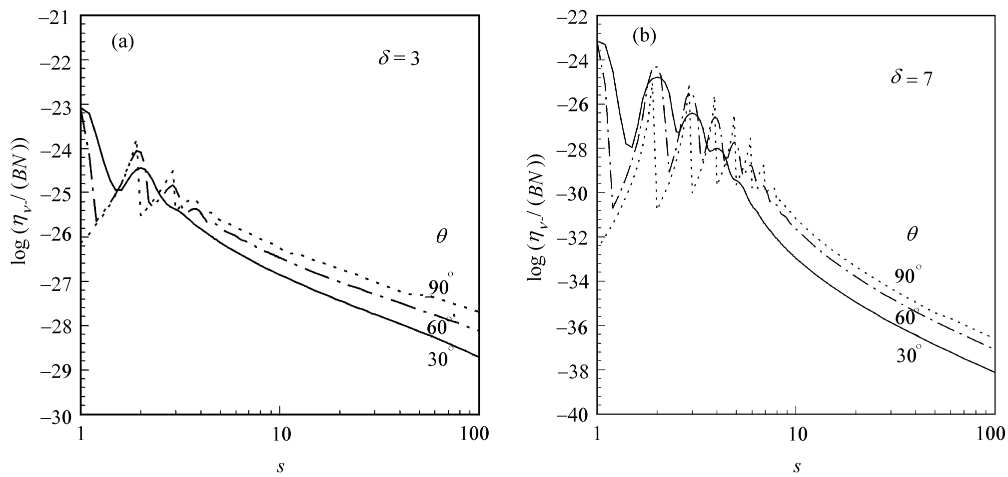

Fig. 2 (a) The variations of the emissivity $\eta_{\nu-}/BN$ with the propagating angle θ for $\delta = 3$. (b) Same as (a) but for $\delta = 7$.

The calculation results also show that the amplitude and number of harmonic structures increase with increasing electron energy spectral index δ (see Fig. 3), which means the electrons with softer spectrum produce richer and larger amplitude harmonic structures at lower harmonics than those with a harder spectrum. When $\delta = 7$, the amplitude of the harmonic structures can reach three orders of magnitude for $s = 2$ for the x-mode in the transverse propagation case. The number of the harmonic structures is about the value of the electron energy spectral index δ.

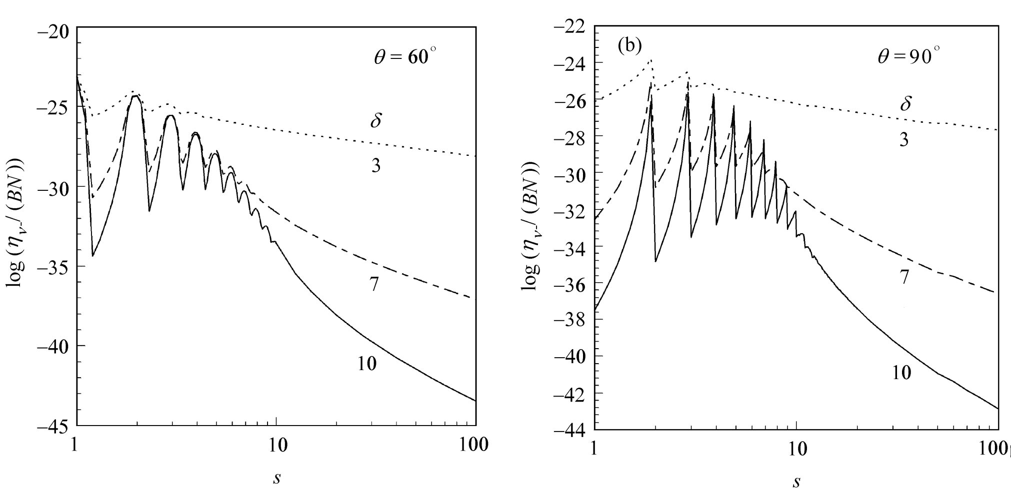

Fig. 3 (a) The variations of the emissivity $\eta_{\nu-}/BN$ with the electron energy spectral index δ for $\theta = 60°$. (b) Same as (a) but for $\theta = 90°$.

The distribution of the harmonic structures with the harmonic number can be obtained from numerical calculations for different values of the propagation angle and electron energy spectral index. So the harmonic structure peaks can also be known accurately. We find that the harmonic numbers of these peaks are integers for small propagation angles and when the propagation angle increases they deviate gradually from integers, but only by a small amount at the lower harmonics (see Table 1).

Table 1 The Distribution of Harmonic Structures with the Harmonic Number at Different Propagation Angles θ for $\delta = 7$

θ	Harmonic number $s = \nu/\nu_B$							
30°	1.0	2.0	3.0	4.0				
60°	1.0	2.0	2.9	3.9	4.9	5.9		
90°	0.9	1.9	2.9	3.9	4.9	5.9	6.9	7.6

3 COMPARISON WITH MASER INSTABILITY

The theory of electron cyclotron and synchrotron maser instabilities was introduced by Wu (1985). The model is based on the resonant wave-particle interaction in the velocity space, hence, electromagnetic waves are amplified close to the harmonics of the local gyrofrequency. The growth rate of the maser instabilities is depressed rapidly with increasing harmonic number, which means the instabilities are only excited at the lower harmonics. In fact, the peak frequencies of the growth rates always deviate from the integer harmonic number; this is caused by the cutoff of the ambient plasma dispersion relation. If the non-resonant wave-particle is taken into consideration, the harmonic peaks of the growth rates are broadened and connected with each other (Huang et al. 1996). The configuration of the growth rates versus frequency is quite similar to the calculations as shown in Table 1 and Figures 1–3.

Note that the maser instabilities are mainly excited in an extremely magnetized plasma (the ratio between the electron plasma frequency and gyrofrequency is much smaller than one). The growth rates of the extraordinary mode are usually larger than those of the ordinary mode, which is also consistent with the calculation of synchrotron radiation (Fig. 1).

Moreover, the maser instabilities mainly depend on the free energy of the nonthermal electrons perpendicular to the ambient magnetic field; this means the growth rates increase with increasing propagation angle, which agrees with the result in Fig. 2. On the other hand, the maser instabilities are usually excited by weakly or moderately relativistic electrons (several tens or hundreds keV), which is also consistent with the result in Figures 1–3.

4 DISCUSSION

The classical radiations (such as gyrosynchrotron or bremsstrahlung) are based on thermodynamic and radiation equilibria, while using the Kirchhoff's law to calculate the Einstein coefficients (Ginzburg & Zheleznyakov 1958). These assumptions are only available for the static or the quasi-static evolution of radiations due to the energization of the charged particles. When the wave-particle interaction is taken into consideration, the nonthermal electrons not only contribute to the direct emissions, but also to the plasma instabilities, which depend on both Maxwell and Vlasov equations. If the wave-particle interaction is neglected, or if the distribution of nonthermal electrons does not change with time, the theory for plasma instabilities is reduced to the Maxwell equations. Note that the instabilities may not provide any new radiation mechanism, but only describe the growth of waves from an initial intensity that depends on the ambient radiations. That is the intrinsic relation between gyroresonance/gyrosynchrotron radiations and cyclotron/synchrotron maser instabilities. From this point of view, it is easy to understand the similarity pointed out in Section 3.

Moreover, the efficiency of energy transfer from the charged particles to the waves is much higher in plasma instabilities than in the classical radiations. For example, only a small part of the ambient electrons is accelerated in solar flares, and the density of the nonthermal electrons is less than one thousandth that of the ambient electrons. Hence, the radiations of the nonthermal particles may be neglected in comparison with the ambient particles, but the nonthermal component may excite the maser instabilities and amplify the ambient radiations on time scale as short as seconds to milliseconds.

The electron cyclotron maser instability is based on the Maxwell and Vlasov equations. If the wave-particle interaction is neglected, this model is reduced to the case of gyrosynchrotron radiation. Therefore, the nonthermal electrons may play two roles: to produce gyrosynchrotron radiation, and to excite cyclotron or synchrotron maser instability, hence to amplify the ambient or initial radiation. So there is a relation between the classical gyrosynchrotron and cyclotron maser instability radiation. In fact the burst radiation during the impulsive or decay phases of flares may be frequently associated with plasma instabilities, especially for some fine time structures. Both of the classical radiation and plasma instability theories and their relations should be studied to explain the complicated solar burst process. This is the first aim of the study of this paper.

On the other hand it is noted that the diagnosis of solar coronal plasma parameters is still an open question, especially for burst regions. Recently some ideas were presented about the magnetic field diagnosis of burst regions, using a set of burst spectral parameters at optically thin part based on the synchrotron and gyrosynchrotron radiation theory in quasi-longitudinal propagation (Zhao 1993; Zhou 1994; Zhou & Karlicky 1994; Kucera et al. 1994; Zhou 1997) and the quasi-transverse propagation one (Zhou & Wang 2000). From electron cyclotron maser theory the plasma parameters can also be estimated using observational characters of spike emissions (Huang et al. 2000). If the harmonic structures described in this paper can be observed, they may also be used to diagnose the magnetic field strength of burst regions, especially for some limb events, because the harmonic structures are more obvious in the transverse propagation case than that in the longitudinal case. It is the second purpose of this paper. Diagnoses may be carried out in different scales (active regions, burst regions, and the sources of some fine structures) with different mechanisms (gyroresonance, gyrosynchrotron, and cyclotron maser). The different diagnostic results may be checked with each other from the intrinsic relations between these mechanisms. Moreover, it possibly enables us to reconstruct the 3-D distribution and evolution of solar coronal plasma parameters from the radio observations with high spatial and spectral resolution (such as OVRO).

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