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The Figure and Dynamical Parameters of Io Inferred from Internal Structure Models

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Abstract The recent Galileo spacecraft explored Jupiter and its satellite system and provided us with new geodetic data. In order to discuss the dynamical parameters and secular tidal effect of Io, the theory of synchronous satellite is described in detail. Using the new geodetic data of Io, two sets of Io's internal structure models are constructed based on the asthenosphere assumption. The liberation parameters α, β, γ and dynamical flattening H are calculated for the models of Io. A comparison of Io with the Moon indicates that they are quite different in many characteristics in spite of the fact that they are approximately equal in mass and size and that they both orbit synchronously.

Key words: Io—synchronous orbit—dynamical parameters

1 INTRODUCTION

Even before the great visit of the Voyager spacecraft to Jupiter, Peale et al. (1979) proposed that Io should be intensely heated by tides. They described Io as a largely molten body with a thin, rigid outer shell overlying a fluid interior and suggested that Io has evolved to such a structure as a consequence of tidal heating. They also predicted that widespread and recurrent surface volcanism would occur. The most spectacular discovery of the Voyager mission is the existence of active volcanoes, and the volcanic material rose to heights of several hundred kilometers above the surface of Io (Morabito et al. 1979; Smith et al. 1979). This phenomenon identified tidal heating to be a potentially important energy source (Yorder 1979) and supported Peale et al.'s view. The 1980s saw the emergence of two important assumptions regarding the internal structure, what Ross et al. (1990) described as the deep mantle model and asthenosphere model. In the deep mantle model, an elastic lithosphere up to tens of kilometers thick is supposed to cover a hot, low-Q, and near-solid mantle (Ross & Schubert 1986; Ojakangas & Stevenson 1986; Segatz et al. 1988; Fischer & Spohn 1990). In the alternative asthenosphere model, first proposed by Schubert et al. (1981), the mantle is decoupled from the lithosphere by a thin and partially molten asthenosphere. Comparing the calculated topography with the observed result, Ross et al. (1990) considered the asthenosphere model to be the better one. In the 1990s, the Galileo spacecraft re-explored Jupiter and its satellites, gathered more precise geodetic data (Anderson et al. 1996). A magnetic signature was discovered at Io (Kivelson et al. 1996).

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In this paper, on the assumption of the asthenosphere model and with reference to the results regarding the terrestrial planets and the Moon (Zhang 1994; Zhang & Zhang 1995), we use the new geodetic data of Io (see Table 1) to construct a series of models for the internal structure of Io and calculate the dynamical parameters, which are then compared with those of the Moon.

Table 1 Basic Parameters of Io

Parameter	Unit	Value	Reference
R	km	1821.3	Davies et al. (1996)
GM	km^3s^{-2}	5959.91	Anderson et al.(1996)
ω	rad s^{-1}	4.1109×10^{-5}	Anderson et al.(1996)
C_{22}	10^{-6}	559	Anderson et al.(1996)
$\frac{I}{MR^2}$		0.371∼0.385	Anderson et al.(1996)
$\overline{\rho}$	g cm^{-3}	3.5294	Anderson et al.(1996)

In Table 1, R is the average radius, GM the product of the gravitational constant G and the mass M, ω the angular velocity, C_{22} the second sectorial Stokes coefficient, $\overline{\rho}$ the mean density, and $\frac{I}{MR^2}$ the dimensionless mean moment-of-inertia.

2 BASIC EQUATIONS AND RESULTS

2.1 The External Gravitational Field of Synchronously Orbiting Satellites

Io, like the Moon, orbits synchronously, i.e. its orbiting period is equal to its rotational period. It then follows that its angular velocity ω is equal to the mean motion n. Based on the theory of synchronous satellites (Burša 1989), Io's external potential V can be expressed as a sum of three terms,

$$W = V + Q + V_t \,, \tag{1}$$

where V stands for the gravitational potential,

$$V = \frac{GM}{R_e}\left[\frac{R_e}{r} + \sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{R_e}{r}\right)^{n+1}(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)P_{nm}(\sin m\varphi)\right], \tag{2}$$

Q, the potential of the centrifugal force,

$$Q = \frac{1}{3}\frac{GM}{R_e}\left(\frac{r}{R_e}\right)^2 q[1 - P_{20}(\sin\varphi)], \tag{3}$$

and V_t, the tidal potential,

$$V_t = \frac{GM_J}{a}\Big(\frac{r}{a}\Big)^2\Big[-\frac{1}{2}P_{20}(\sin\varphi) + \frac{1}{4}P_{22}(\sin\varphi)\cos 2\lambda\Big], \tag{4}$$

where R_e and a denote the mean equatorial radius of Jupiter and mean orbital semi-major axis of Io, M_J the mass of Jupiter, C_{nm} and S_{nm} the Stokes parameters, $P_{nm}(\sin\varphi)$ the associated Legendre function, respectively, q a small parameter and defined as

$$q = \frac{\omega^2 R_e^3}{GM} = \frac{n^2 R_e^3}{GM} = \frac{GM_J}{GM}\left(\frac{R_e}{a}\right)^3. \tag{5}$$

Assuming that the initial figure of Io is ideally spherical (at $\omega = 0$), the perturbing potential ΔW due to the rotational and tidal deformation can be expressed as

$$\Delta W = k_s \Delta Q + k_t V_t, \tag{6}$$

where k_s is the secular Love number introduced by Munk and MacDonald (1960) as a measure of the body-yield-to-centrifugal deformation in the course of development during the whole evolution history of the satellite, k_t is the analogous parameter to k_s, the "secular tidal Love number", for describing the secular tidal deformation, and ΔQ is the perturbing part of the potential Q of the centrifugal force,

$$\Delta Q = Q - \frac{1}{3}\omega^2 r^2 = -\frac{1}{3}\frac{GM}{r}\left(\frac{R_e}{r}\right)^{-3} q P_{20}(\sin\varphi). \tag{7}$$

On the other hand, the perturbing potential ΔW can be expressed in a different form, as follows,

$$\Delta W = \frac{GM}{r}\left(\frac{R_e}{r}\right)^2 [-\Delta J_2 P_{20}(\sin\varphi) + \Delta C_{22} P_{22}(\sin\varphi)\cos 2\lambda], \tag{8}$$

where ΔJ_2 and ΔC_{22} are the rotational and tidal deformation of the zonal C_{20} and the sectorial C_{22} Stokes parameters, respectively. If the initial figure is the ideal sphere (at $\omega = 0$), then

$$\begin{aligned} \Delta J_2 &= J_2 = \frac{C - \frac{1}{2}(A+B)}{MR_e^2}, \\ \Delta C_{22} &= C_{22} = \frac{B-A}{4MR_e^2}, \end{aligned} \tag{9}$$

where $C > B > A$ are the three principal moments of inertia of Io.

At the boundary, (6) and (8) should be identical. By solving the first (Dirichlet's) boundary-value problem for a sphere (radius R), with $r = R = R_e$, the parameters J_2 and C_{22} can be expressed as functions of k_s and k_t,

$$\begin{aligned} J_2 &= \frac{1}{3} q (k_s + \frac{3}{2} k_t), \\ C_{22} &= \frac{1}{4} q k_t\,. \end{aligned} \tag{10}$$

Assuming that the body-yield-to-centrifugal deformation is equal to the body-yield-to-tidal deformation, $k_t = k_s$, Eq. (10) then yields

$$k_s = k_t = \frac{4C_{22}}{q}, \tag{11}$$

and

$$J_2 = \frac{10}{3} C_{22}\,. \tag{12}$$

It can be deduced that J_2 is dependent on C_{22} for synchronously orbiting satellites. Eq. (12) was treated as a constraint by Anderson et al. (1996). They analyzed the external gravitational field of Io obtained by the Galileo spacecraft. Given C_{22} and q, Io's dimensionless axial moment of inertia $\frac{C}{MR_e^2}$ follows from the expression (cf. Burša 1994),

$$\frac{C}{MR_e^2} = \frac{2}{3}\Big[1 - \frac{2}{5}\left(\frac{4-k_s}{1+k_s}\right)^{1/2}\Big]\,. \tag{13}$$

2.2 Internal Structure Models of Io

Schubert et al. (1981) first proposed the asthenosphere model, which consists of four regions: a core, a mantle, an asthenosphere and an outer shell. The mean thickness of the lithosphere, i.e. the outer shell, is about 35 km, and the density is about 2.7 g cm^{-3}, while the thickness of the asthenosphere is at least around 50 km, but no more than a few hundred kilometers, and its density is about 2.9 g cm^{-3}.

It is proposed that about two-thirds of tidal heating takes place in the asthenosphere. The density of the mantle is about 3.3 g cm^{-3}. As for the core, two models are considered in this paper, i.e. Fe-FeS core (5.15 g cm^{-3}) and Fe core (8.00 g cm^{-3}). (c.f. Anderson et al. 1996). By solving the Emden equation, the density distribution within the core and mantle can be deduced and hence the size of the core and $\frac{I}{MR_e^2}$. Our procedure to construct Io's internal structure model is as follows.

Assuming that Io satisfies the condition of hydrostatic equilibrium within the mantle and core,

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{4\pi G\rho(r)}{r}\int_0^r r^2\rho(r)\mathrm{d}r, \tag{14}$$

the Emden equation can be derived (cf. Zhang & Zhang 1995),

$$\frac{1}{r^2\rho}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\rho^{b-2}\frac{\mathrm{d}\rho}{\mathrm{d}r}\right) + \frac{4\pi G}{K_0}\rho_u^b = 0, \tag{15}$$

where K_0 and b are known positive constants with different values for the mantle and core (cf. Zhang 2000), and ρ_u denotes the value of the density at $p = 0$.

Taking sets of input parameters that satisfy the observed constraints in $\bar{\rho}$ and $\frac{I}{MR^2}$, we solve Eq. (15) for the density distribution $\rho(r)$ for both the mantle and core. A unique solution is unavailable due to insufficient constraints. Eight typical solutions are picked out and shown in Table 2. The first five (Io-1∼5) refer to an Fe-FeS core, while the last three (Io-6∼8) refer to a pure Fe core.

In Table 2, ρ_c denotes the central density, $p(0)$ the central pressure, r_c/R the ratio of the core's radius to the average radius, M_c/M the ratio of the core's mass to the total mass, ds_2 the asthenospheric thickness (the adopted value of the lithospheric thickness is 35 km), ρ_1 the density of the lithosphere, ρ_2 the density of the asthenosphere, and $\overline{\rho}_m$ the average density of the mantle. In our calculations, ρ_c and ds_2 are regarded as fixed, and $p(0)$, ρ_1 and ρ_2 are regarded as adjustable parameters. The calculated values of $\frac{I}{MR^2}$ are in the range between 0.371 and 0.385.

Table 2 Eight Models of Io

Model	ρ_c g cm^{-3}	$p(0)$ kbar	r_c/R	M_c/M	$\overline{\rho}_m$ g cm^{-3}	ds_2 km	ρ_2 g cm^{-3}	ρ_1 g cm^{-3}	$\frac{I}{MR^2}$
Io-1	5.15	75.2	0.51	0.1922	3.5075	165	2.900	2.684	0.3707
Io-2	5.15	75.2	0.51	0.1922	3.4563	125	2.900	2.676	0.3721
Io-3	5.15	75.0	0.50	0.1812	3.4284	85	2.900	2.685	0.3743
Io-4	5.15	73.0	0.46	0.1412	3.4513	85	3.100	2.750	0.3791
Io-5	5.15	67.0	0.37	0.0736	3.5478	85	3.155	2.700	0.3846
Io-6	8.00	78.0	0.29	0.0550	3.5400	85	2.980	2.724	0.3799
Io-7	8.00	85.3	0.33	0.0809	3.4979	85	2.900	2.590	0.3744
Io-8	8.00	93.5	0.37	0.1138	3.4155	85	2.830	2.710	0.3691

From Table 2 it can be seen that:

(1) For the first three models (Io-1∼3), whose values of the density of the core are the same, r_c/R and $p(0)$ vary weakly with ds_2; however, $\frac{I}{MR^2}$ increases and $\overline{\rho}_m$ decreases with decreasing ds_2.

(2) For fixed ρ_c and ds_2, r_c/R increases and $\frac{I}{MR^2}$ and $\overline{\rho}_m$ decreases as $p(0)$ increases.

(3) For the model with a Fe-FeS core, the dimensionless moment of inertia $\frac{I}{MR^2}$ can reach 0.3846, while for the model with a pure Fe core, $\frac{I}{MR^2}$ is no greater than 0.3799.

2.3 Dynamical Parameters of Io

Under the condition of hydrostatic equilibrium, for any given density distribution $\rho(r)$, it is easy to obtain the hydrostatic flattening $e(r)$ of the internal equipotential surface by solving the Clairaut equation (cf. Zhang 1997). The flattening $e_s \equiv e(R)$ is the flattening of the surface of the rotational ellipsoid.

If not distorted by the secular tidal deformation, the second zonal Stokes parameter $J_2^{(0)}$ under rotational hydrostatic equilibrium can be expressed as (Burša 1984)

$$J_2^{(0)} = \frac{2}{3}e_s - \frac{1}{3}q\,. \tag{16}$$

In fact, Io is deformed by tides, therefore J_2 is not identical to $J_2^{(0)}$. The difference between J_2 and $J_2^{(0)}$

$$\delta J_2 = J_2 - J_2^{(0)} \tag{17}$$

reflects the secular effect of the tides. Values of $J_2^{(0)}$ and δJ_2 are given in Table 3 for the four chosen models.

Since Io is assumed to be in tidal and rotational equilibrium, it is a triaxial ellipsoid whose liberation parameters (α, β and γ) and dynamical flattening H are

$$\left.\begin{aligned} \alpha &= \tfrac{C-B}{A}, \\ \beta &= \tfrac{C-A}{B}, \\ \gamma &= \tfrac{B-A}{C}, \\ H &= \tfrac{C-\frac{A+B}{2}}{C}. \end{aligned}\right\} \tag{18}$$

These parameters, calculated from the models, are displayed in Table 3, where the corresponding values for the Moon are also listed (cf. Zhang 1984).

Table 3 Dynamical Parameters of Four Io Models and Moon

	Io-1	Io-3	Io-5	Io-6	Moon
$\frac{I}{MR^2}$	0.3707	0.3743	0.3846	0.3799	0.3904
$e_s(10^{-3})$	1.918	1.943	2.021	1.986	0.0221
$J_2^{(0)}(10^{-3})$	0.7079	0.7246	0.7766	0.7532	0.008644
$\delta J_2(10^{-3})$	1.1551	1.1384	1.0864	1.1098	0.19354⋆
$\alpha(10^{-3})$	2.026	2.006	1.952	1.977	0.4037
$\beta(10^{-3})$	8.057	7.980	7.766	7.862	0.6317
$\gamma(10^{-3})$	6.081	5.974	5.814	5.886	0.2280
$H(10^{-3})$	5.025	4.977	4.844	4.904	0.5167

⋆ δJ_2 is the nonhydrostatic value of the Moon. J_2 of the Moon is 2.02151×10^{-4}; and C_{22} of the Moon is 2.2302×10^{-5}.

3 COMPARISON OF Io WITH THE MOON

The density and size of Io are approximately equal to those of the Moon, and both satellites orbit synchronously. However, they are quite different in several aspects, as is shown in Table 3. From a comparison of Io with the Moon, it is deduced that:

(1) J_2 of Io (10^{-3}) is larger than that of the Moon (10^{-4}) by one order of magnitude. Io is in hydrostatic equilibrium ($J_2 = \frac{10}{3}C_{22}$), but the Moon deviates from hydrostatic equilibrium ($J_2 \neq \frac{10}{3}C_{22}$).

(2) For the Moon, the nonhydrostatic component is about two orders of magnitude higher than the hydrostatic component (cf. Zhang 1994). However, for Io, $J_2^{(0)}$ is of the same order of magnitude as δJ_2.

(3) The liberation parameters and dynamical flattening of Io exceed those of the Moon by one order of magnitude. Among Io's three liberation parameters, α is the smallest ($\alpha < \gamma < \beta$), while for the Moon, γ is the smallest ($\gamma < \alpha < \beta$).

(4) The dimensionless moment of inertia of Io is less than that of the Moon. Our study suggests that the relative radius of Io's core lies between 0.29 and 0.51. Thus, It may possess a large core, which is favorable for it to have a magnetic field.

4 CONCLUSIVE REMARKS

In this paper, several models of Io are constructed, similar to those of Anderson et al. (1996). Our results indicate that δJ_2 of Io is of the same order of magnitude as J_2 itself, which suggests that Io is strongly affected by tides and may have a large core, with a relative radius of 0.51. Although the mass and size of Io are similar to those of the Moon, they are still quite different in many aspects.

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