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Orbit Determination Using Satellite-to-Satellite Tracking Data

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Abstract Satellite-to-Satellite Tracking (SST) data can be used to determine the orbits of spacecraft in two ways. One is combined orbit determination, which combines SST data with ground-based tracking data and exploits the enhanced tracking geometry. The other is the autonomous orbit determination, which uses only SST. The latter only fits some particular circumstances since it suffers the rank defect problem in other circumstances. The proof of this statement is presented. The nature of the problem is also investigated in order to find an effective solution. Several methods of solution are discussed. The feasibility of the methods is demonstrated by their application to a simulation.

Key words: Celestial mechanics: orbit determination — Methods: miscellaneous

1 INTRODUCTION

The tracking arc-length should be increased in order to improve the accuracy in orbit determination of LEO (Low Earth Orbit) satellites. The local ground-based tracking network does not provide sufficient orbit coverage for the user satellites. The most promising method is to use high orbiting satellites, such as GPS and TDRS, as trackers to observe the user satellites. For example, two geosynchronous satellites could cover more than 85% of the orbit of any given user satellite. These high orbiting satellites form the space-based network which can partially replace the ground-based network. Similar to GPS satellites, the precise TRDS orbits are routinely derived from the observations by the ground stations. They can also be improved simultaneously with the user satellite trajectories. This method treats the high orbiting satellites as 'roving ground stations' whose station coordinates are to be estimated. It combines the SST data with the ground-based tracking data and takes advantages of the extended geometry.

Besides the global navigation satellite system and the high geosynchronous satellite constellation, there is a local navigation satellite system (Wang 1999). With increasing application of satellites, LEO satellite constellations are flourishing. Since the number of satellites in one constellation is rather large, it is not desirable to observe all the satellites of a constellation by

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the ground-based network. An alternative method is to let the ground stations observe only one or a small number of satellites in the given constellation, and to measure the others based on these. Instead of the high orbiting satellites, the satellites with the ground stations tracking are treated as trackers. The theory of this method is similar to the case mentioned above where the user satellites are observed by the space-based network.

In all the preceding tracking methods, the orbit of the user spacecraft suffers from the inaccuracies of the tracking satellite trajectories just as the uncertainties of the ground station coordinates. It could be improved by estimating the tracker orbits and the user orbits simultaneously, namely by the combined orbit determination of the two kinds of satellites. The ground-based tracking data must be included; otherwise the procedure of orbit determination would be failing. Particularly, the autonomous orbit determination has to be used since no support of the ground stations could be provided under certain circumstances. However, it cannot be realized in principle since it leads to a rank defect problem of the normal matrix in the LS (least square) estimate procedure. In regard of orbit geometry, it results in an uncertainty of the orbit called orbit overall drift. In the following sections, a theoretical analysis of two methods using the SST data is given first. The cause of the rank defect problem occurring in the autonomous orbit determination case is sought in order to find an effective solution. Next, several methods of solution are discussed and their feasibility is demonstrated by their application to simulation.

2 THEORETICAL ANALYSIS

For convenience in the analysis, we consider a system which includes one ground station, one user spacecraft (user) and one tracking satellite (tracker). All the measurements between them are range data. In the Earth Inertial Coordinates (c.f. Fig. 1), ρ_s is the range between the tracker S_1 and the user S. ρ_e is the range between the tracker and the ground station A. They satisfy the following two equations.

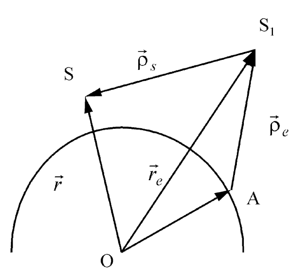

Fig. 1 Geometry of two satellites and the ground station

$$\rho_s = \rho_s(\boldsymbol{r}, \boldsymbol{r}_e) = |\boldsymbol{r} - \boldsymbol{r}_e| \,, \tag{1}$$

$$\rho_e = \rho_e(\boldsymbol{r}_e, \boldsymbol{R}) = |\boldsymbol{r}_e - \boldsymbol{R}| \,, \tag{2}$$

where $\boldsymbol{R}$ is the geocentric location of the ground station, called the vector of the station coordinates.

The tracker ephemeris is routinely derived from the ground tracking and the user orbit is subsequently determined from the SST data. The tracker is treated just as a roving ground station. Its position, $\boldsymbol{r}_e$ in Equation (1), is a known function of t and can be computed from the tracking by the ground-based network based on Equation (2). This conventional method combines the tracking of the space-based network with the tracking of the ground-based network to improve the user orbit. There is no essential difference from the conventional method using only the ground-based network for orbit determination. It is limited by the inaccuracy of the tracker ephemeris due to the weakness in the ground-based tracking geometry. This limitation could be ameliorated by another method which estimates the tracker ephemeris and the user orbit simultaneously. This method also combines measurement of the space-based network with the ground-based network tracking and computes the orbits based on Equation (1) and Equation (2). While it could improve the accuracy, it is not essentially different from the above method where the tracker orbit is not estimated simultaneously. Both methods depend on ground-based tracking. If only Equation (1) is used, i.e., without any tracking data from the ground-based network, then there will be an essential difference.

To specify the problem concisely, only the orbit state vector is improved. Other relevant dynamical parameters and the body geometrical parameters are not estimated. Choose the state vector as

$$X = \begin{pmatrix} \sigma \\ \sigma_1 \end{pmatrix} . \tag{3}$$

Where σ and σ_1 are the orbit elements of the user and the tracker respectively. They are defined as

$$\sigma = \begin{pmatrix} a \\ e \\ i \\ \Omega \\ \omega \\ M \end{pmatrix} , \qquad \sigma_1 = \begin{pmatrix} a_1 \\ e_1 \\ i_1 \\ \Omega_1 \\ \omega_1 \\ M_1 \end{pmatrix} , \tag{4}$$

where a, e, i, Ω, ω and M are the Kepler orbit elements. The estimated state vector X at time t_0 is denoted by $X_0\,(\sigma_0, \sigma_{10})$. We propose to use the orbit elements as the state vector rather than the position $\boldsymbol{r}$ and the velocity $\dot{\boldsymbol{r}}$. This way will bring out more clearly the nature of rank defect in autonomous orbit determination from SST data. It also contributes to finding the solution to the rank defect problem. It also has certain advantages in the calculation of the orbit determination (Liu & Zhang 1999).

Since the measurements are range data, the observational vector can be written as

$$Y = \begin{pmatrix} \rho_s \\ \rho_e \end{pmatrix} . \tag{5}$$

The different sampling times, the SST and the ground-based tracking data, are denoted by t and t_e, respectively. The general forms of the state equations and observation-state relationship are linearized about a reference solution. The linear equation (condition equation) of the orbit determination has the form

$$y = Bx + V , \tag{6}$$

where V is the random observational noise, y is the observation residual and x is the correction vector to the estimated state X_0. They can be expressed as

$$y = Y_o - Y_c = \begin{pmatrix} (\rho_s)_o - (\rho_s)_c \\ (\rho_e)_o - (\rho_e)_c \end{pmatrix} , \tag{7}$$

$$x = X_0 - X_0^* , \tag{8}$$

where $(\rho_s)_o$, $(\rho_e)_o$ and $(\rho_s)_c$, $(\rho_e)_c$ are the values of observation and theoretical calculation corresponding to the reference orbit X_0^* respectively. Whether the problem is rank defective or not depends on the property of matrix B,

$$B = \begin{pmatrix} B_s \\ B_e \end{pmatrix} = \begin{pmatrix} \left(\frac{\partial \rho_s}{\partial(\boldsymbol{r}, \boldsymbol{r}_e)}\right) \left(\frac{\partial(\boldsymbol{r}, \boldsymbol{r}_e)}{\partial X}\right) \left(\frac{\partial X}{\partial X_0}\right) \\ \left(\frac{\partial \rho_e}{\partial(\boldsymbol{r}_e, \boldsymbol{R})}\right) \left(\frac{\partial(\boldsymbol{r}_e, \boldsymbol{R})}{\partial X}\right) \left(\frac{\partial X}{\partial X_0}\right) \end{pmatrix} . \tag{9}$$

For each observation, the three partial derivative matrices in the right-hand side of Equation (9) represent one a (2×6) vector, one a (6×12) vector and one a (12×12) vector respectively. So matrix B is a (2×12) vector. If the total number of samplings is $k = (k_1 + k_2)$, matrix B is a $(k \times 12)$ vector. Only if one sampling of each tracking type is considered, can we know the property of matrix B and the nature of the rank defect problem. Omitting the derivation, the final form of matrix B can be written as

$$B = \begin{pmatrix} \frac{1}{(\rho_s)_1} b_{1,j} \\ \frac{1}{(\rho_e)_1} b_{2,j} \end{pmatrix} , \qquad (j = 1, 2, \cdots, 12) , \tag{10}$$

where the main terms of each components can be expressed as

$$\begin{cases} b_{1,1} = (\boldsymbol{r} - \boldsymbol{r}_e)(\frac{\boldsymbol{r}}{a}) + (\boldsymbol{r} - \boldsymbol{r}_e)(\frac{\dot{\boldsymbol{r}}}{n})(-\frac{3n}{2a}\Delta t) , \\ b_{1,2} = (\boldsymbol{r} - \boldsymbol{r}_e)(H\boldsymbol{r} + K\dot{\boldsymbol{r}}) , \\ b_{1,3} = (\boldsymbol{r} - \boldsymbol{r}_e)(\frac{z}{\sin i}\hat{\boldsymbol{R}}) , \\ b_{1,4} = (\boldsymbol{r} - \boldsymbol{r}_e)\boldsymbol{\Omega} , \\ b_{1,5} = (\boldsymbol{r} - \boldsymbol{r}_e)(\hat{\boldsymbol{R}} \times \boldsymbol{r}) , \\ b_{1,6} = (\boldsymbol{r} - \boldsymbol{r}_e)(\frac{\dot{\boldsymbol{r}}}{n}) . \end{cases} \tag{11}$$

$$\begin{cases} b_{1,7} = -(\boldsymbol{r} - \boldsymbol{r}_e)(\frac{\boldsymbol{r_e}}{n_1}) - (\boldsymbol{r} - \boldsymbol{r}_e)(\frac{\dot{\boldsymbol{r}}_e}{n_1})(-\frac{3n}{2a}\Delta t) , \\ b_{1,8} = -(\boldsymbol{r} - \boldsymbol{r}_e)(H_1\boldsymbol{r}_e + K_1\dot{\boldsymbol{r}}) , \\ b_{1,9} = -(\boldsymbol{r} - \boldsymbol{r}_e)(\frac{z_e}{\sin i_1}\hat{\mathrm{R}}_1) , \\ b_{1,10} = -(\boldsymbol{r} - \boldsymbol{r}_e)\boldsymbol{\Omega}_1 , \\ b_{1,11} = -(\boldsymbol{r} - \boldsymbol{r}_e)(\hat{\boldsymbol{R}}_1 \times \boldsymbol{r}_e) , \\ b_{1,12} = -(\boldsymbol{r} - \boldsymbol{r}_e)(\frac{\dot{\boldsymbol{r}}_e}{n_1}) . \end{cases} \tag{12}$$

$$\begin{cases} b_{2,1} = -(\boldsymbol{r}_e - \boldsymbol{R})(\frac{\boldsymbol{r_e}}{n_1}) - (\boldsymbol{r} - \boldsymbol{R})(\frac{\dot{\boldsymbol{r}}_e}{n_1})(-\frac{3n}{2a}\Delta t) , \\ b_{2,2} = -(\boldsymbol{r} - \boldsymbol{R})(H_1\boldsymbol{r}_e + K_1\dot{\boldsymbol{r}}) , \\ b_{2,3} = -(\boldsymbol{r} - \boldsymbol{R})(\frac{z_e}{\sin i_1}\hat{\boldsymbol{R}}_1) , \\ b_{2,4} = -(\boldsymbol{r} - \boldsymbol{R})\boldsymbol{\Omega}_1 , \\ b_{2,5} = -(\boldsymbol{r} - \boldsymbol{R})(\hat{\boldsymbol{R}}_1 \times \boldsymbol{r}) , \\ b_{2,6} = -(\boldsymbol{r} - \boldsymbol{R})(\frac{\dot{\boldsymbol{r}}_e}{n_1}) . \end{cases} \tag{13}$$

$$b_{2,7} = b_{2,8} = \cdots = b_{2,12} = 0 . \tag{14}$$

On Equations (11)–(14), the quantities are defined as

$$\Delta t = t - t_0 , \qquad \Delta t_e = t_e - t_0 , \tag{15}$$

$$\begin{cases} H = -\frac{a}{p}(\cos E + e) , K = \frac{\sin E}{n}\left(1 + \frac{r}{p}\right) , \\ p = a\left(1 - e^2\right) , n = \sqrt{\mu}a^{-\frac{3}{2}} , \mu = GM_e , \end{cases} \tag{16}$$

$$\boldsymbol{\Omega} = \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix} , \qquad \hat{\boldsymbol{R}} = \begin{pmatrix} \sin i \sin \Omega \\ -\sin i \cos \Omega \\ \cos i \end{pmatrix} , \tag{17}$$

where E is the eccentric anomaly, GM_e is the gravity constant of the earth. All the quantities with the subscript '1' correspond to the tracker, expressions that are similar to those of the user are not listed again.

The property of matrix B can be seen from one pair of $(b_{1,4}, b_{1,10})$ and $(b_{2,4}, b_{2,10})$. Using the expression of $\mathbf{\Omega}$ (and $\mathbf{\Omega}_1$) on Equation (15) , they can be written as

$$\begin{cases} b_{1,4} = x_e y - y_e x, & b_{1,10} = -(x_e y - y_e x) = -b_{1,4}\,, \\ b_{2,4} = X y_e - Y x_e, & b_{2,10} = 0 \neq -b_{2,4}\,. \end{cases} \tag{18}$$

Hence the absolute values of two ranks in matrix B are equal if there is no ground-based tracking data. Therefore, the matrix is not full rank since the determinant of the normal matrix $B^T B$ in Equation (6) will satisfies $\left|B^T B\right| = 0$. As a result the orbits cannot be determined. This method is the autonomous orbit determination using only SST data. But if ground-based tracking of the tracker is available, then matrix B will not be rank defective, and we have the combined orbit determination using both SST data and ground-based tracking data. Here the orbit of the tracker is estimated together with that of the user. Note that under a certain circumstance there are similar problems on $b_{1,3}$ and $b_{1,9}$ which are related to the inclination i. Another form of expressions of $b_{1,3}$ and $b_{1,9}$ is

$$\begin{cases} b_{1,3} = (x z_e - z x_e)\sin\Omega - (y z_e - z y_e)\cos\Omega\,, \\ b_{1,9} = -(x z_e - z x_e)\sin\Omega_1 + (y z_e - z y_e)\cos\Omega_1\,. \end{cases} \tag{19}$$

When $\Omega = \Omega_1$ or $\Omega = \Omega_1 + 180°$, we have

$$b_{1,9} = -b_{1,3}, \quad \text{or} \quad b_{1,9} = b_{1,3}\,, \tag{20}$$

whereas $b_{2,3}$ and $b_{2,9}$ have no such a problem.

3 SOLUTION

From the behavior of matrix B regarding rank defect, we learn that the cause of the problem in the autonomous orbit determination using SST data is the uncertainty of the orbit plane, or what is called the overall drift of orbit. To use the position vector and the velocity vector as the state vector will make it difficult to show the cause and will not help to find an effective solution to the problem.

Using biased estimators could solve the conventional unobservable determination problems to a certain extent (Cicci 1988). But it does not work well in the rank defect problem occurring in the autonomous orbit determination. Since the problem concerns the orbit plane, it can be solved through giving a priori information of (i, Ω) that are obtained by some other methods. Moreover, (i, Ω) are the two elements that tend to be predicted more accurately than the other orbit elements. They suffer less from the atmosphere model error than other elements since the effects of the atmosphere on them are mainly due to its rotation.

4 SIMULATION AND RESULT

The procedure of orbit determination for the user spacecraft was simulated in two cases. One was to use a high altitude satellite as the tracker. The other was to use another satellite in the same constellation as the tracker. Both cases showed autonomous orbit determination without ground-based tracking data to be infeasible. There is no substantial improvement even if we use biased estimator (the ridge-type estimator) discussed in paper (Cicci 1988). The method with a priori information of (i, Ω) of the tracker orbit does work effectively. Considering the predicting accuracy that can generally be obtained, the errors of (i, Ω) were given as $\Delta(i, \Omega) = 10^{-5}$.

Here we give the result of the latter case with two LEO satellites. The orbit elements of their benchmark trajectories are given in Table 1. Instead of semimajor axis a, the period T is used as an orbit element.

Table 1 Benchmark Trajectories of Two LEO Satellites

	Period (min)	Eccentricity	Inclination (degree)	Longitude of the ascending node (degree)	Longitude of perihelion (degree)	Mean anomaly (degree)
Tracker	100.0	0.001	50.0	50.0	50.0	0.0
User	100.0	0.001	50.0	110.0	50.0	0.0

Both SST data and the ground-based tracking data are included in the range measurements. The random error of range measurement is 10 m. Ten passes are simulated, including 585 points of SST data and 67 points of ground-based tracking data. Three types of LS estimators are applied. The improved trajectories of various methods are compared with the benchmark orbits. The orbit differences are indicated in Table 2.

Table 2 Orbit Differences

Data	SST Data and the Ground-based Tracking Data		SST data
LS Estimator Type	Conventional	Fixed i and Ω of the Tracker	Given a priori i and Ω of the Tracker
RMS	0.1115×10^{-5}	0.1113×10^{-5}	0.1780×10^{-5}
Δa_1	-0.1444×10^{-9}	-0.9114×10^{-8}	-0.5272×10^{-7}
Δe_1	-0.5834×10^{-7}	-0.1088×10^{-5}	0.7133×10^{-6}
Δi_1	-0.3173×10^{-6}	0.1000×10^{-4}	0.2148×10^{-4}
$\Delta\Omega_1$	0.3427×10^{-6}	0.1000×10^{-4}	0.1939×10^{-4}
$\Delta(\omega+M)_1$	-0.4346×10^{-7}	-0.1884×10^{-4}	0.2546×10^{-4}
Δa	0.5875×10^{-9}	-0.6817×10^{-7}	0.1590×10^{-7}
Δe	-0.1420×10^{-6}	-0.2298×10^{-6}	0.1787×10^{-5}
Δi	-0.2138×10^{-6}	0.1737×10^{-4}	-0.5172×10^{-5}
$\Delta\Omega$	0.4571×10^{-6}	-0.3513×10^{-5}	0.1551×10^{-4}
$\Delta(\omega+M)$	-0.2318×10^{-6}	0.2011×10^{-5}	0.3383×10^{-4}

In Table 2, RMS and Δa_1, Δa are in units of the earth's equatorial radius a_e. The angular residuals are in radians.

From the results presented in Table 2, we are inclined to conclude that the method combining SST data with the ground-based tracking data is the most effective method. Another two methods, least square estimators at fixed or given a priori values of i and Ω of the tracker, also lead to sound results. Since it is not difficult to predict accurate 'a priori values' of (i, Ω), as already mentioned, the methods are also feasible. The autonomous orbit determination without any information of (i, Ω) could not be realized since the procedure of orbit determination either did not converge or led to bad results.

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