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INVITED REVIEWS

Solar Activity and Classical Physics

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Abstract This review of solar physics emphasizes several of the more conspicuous scientific puzzles posed by contemporary observational knowledge of the magnetic activity of the Sun. The puzzles emphasize how much classical physics we have yet to learn from the Sun. The physics of solar activity is based on the principles of Newton, Maxwell, Lorentz, Boltzmann, et. al., along with the principles of radiative transfer. In the large, these principles are expressed by magnetohydrodynamics. A brief derivation of the magnetohydrodynamic induction and momentum equations is provided, with a discussion of popular misconceptions.

Key words: Sun: activity – Magnetohydrodynamics – radiative transfer

1 INTRODUCTION

The Sun is close enough to observe in some detail, and it shows that a star is more than the traditional stable self-gravitating thermonuclear body established half a century ago. For the fact is that out of sight beneath the visible surface the outward flow of heat from the thermonuclear core drives hydrodynamics that generates magnetic fields. It is the complicated dynamics of those magnetic fields that produces the modern mysteries of the active Sun. The mysteries are a challenge to classical physics. The mysteries range from the diffusion and dissipation of the fibril magnetic field to such exotic phenomena as faculae and plages, sunspots, ephemeral active regions, and the associated variations in the total brightness of the Sun. The purpose of this brief review (see also Parker 1997) is to list some of the scientific puzzles posed by the observed activity, and then to review the theoretical dynamical principles on which the activity is presumed to be based.

The magnetic fields are generated at some depth in the Sun, but they are buoyant and emerge through the surface in a remarkable intense fibril form (Beckers & Schröter 1968; Livingston & Harvey 1969; Stenflo 1973; Chapman 1973; Berger & Title 1996). The individual fibrils have magnetic fields estimated at 1–2 kilogauss and diameters of the general order of 100 km. There is no indication of significant magnetic fields in the broad regions between the fibrils. Continually deformed by the subsurface convection, these emerging fibril fields perform a variety of astonishing tricks, referred to generally as solar activity. The activity is observed to

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involve bipolar magnetic active regions, sunspots, faculae and plages, prominences (cf. Wiik et al. 1999), coronal mass ejections, ephemeral active regions, X-ray bright points, flares (Svestka 1976; Sylvester & Sylvester 2000) and solar cosmic rays (Litvinenko 2000; Mason, Dwyer & Mazur 2000), the bright filamentary X-ray corona as well as the diffuse X-ray corona, the coronal hole (Golub & Pasachoff 1997), and the solar wind, to mention the major components of the activity (see reviews by Priest 1981, 1982; Low 1996).

Then through coronal expansion and the solar wind, the Sun creates the heliosphere (Parker 1958b, 1963, 1965; Hundhausen 1972; Burlaga 1997; Jokipii, Sonett & Giampapa 1997), extending out to the interstellar gas and galactic magnetic field far beyond Neptune and Pluto. Ions and electrons are accelerated to high velocities at shock transitions in the solar wind both near the Sun and far out in interplanetary space (cf. Reames 1999; Chotoo et al. 2000). The anomalous cosmic rays are evidently created by acceleration at the terminal shock in the supersonic wind (Garcia-Munoz, Mason & Simpson 1973; Fisk, Koslovsky & Ramaty 1974; Pesses, Jokipii & Eichler 1981; Jokipii 1986; Mzur et al. 2000), while the galactic cosmic rays are partially excluded from the heliosphere by the outward sweep of the magnetic fields in the solar wind (Parker 1958a, 1963; Jokipii & Kopriva 1979). Observations of the acceleration and the exclusion are only preliminary at the present time.

There is an immense detailed body of observational information on the diverse aspects of solar activity, increasing rapidly with modern observational technology operating on the ground and in space. Each passing decade brings new views and concepts. Theoretical understanding of these numerous and diverse phenomena has advanced so far as to provide some general ideas of their nature, but there are so many unanswered questions embedded in the theoretical ideas that we cannot say that the physics is properly understood. We must not confuse a description of a phenomenon, e.g. a facula or a sunspot, with a proper understanding in terms of the basic laws of physics. The ultimate question is, how is it that the basic laws of physics compel the Sun to form sunspots, etc.?

Now the magnetic activity of the Sun exhibits the familiar 11-year cyclic variation, as well as an overall waxing and waning over the centuries. For instance the activity was particularly weak during the last half of the 17th century, the so-called Maunder Minimum (1645–1715)(Maunder 1894; Eddy 1973, 1976, 1989; Letfus 2000).

One of the more curious features of solar activity is the variation of the total brightness of the Sun with the general level of the magnetic activity. Another is the increasing agitation of the geomagnetic field by the solar wind throughout the 20th century. This increase parallels the observed increase in sunspot number over the first half of the century. Lockwood, Stamper, and Wild (1999) suggest that the increased agitation implies more than twice as much magnetic flux at the surface of the Sun toward the end of the century than at the beginning (see also Ahluwalia 2000). However, with the complicated emergence of many small active regions and the residual magnetic flux in the network and intranetwork magnetic field (Schrijver & Harvey 1994) it is difficult to know how to interpret the increased total flux.

The challenge is to understand the physics of what is described in so much detail. Magnetohydrodynamics, based on the equations of Newton, Maxwell, and Lorentz, provides a basis for understanding the large-scale motions and deformations of the magnetic fields in the Sun. That is to say, the unknown processes that create the sunspots, the faculae, the fibril state of the field, the ubiquitous relation between magnetic field and coronal heating, and the rapid diffusion of magnetic field throughout the convective zone may be presumed to conform to the partial differential equations of magnetohydrodynamics. However, the partial differential equations do not spell out the nature of the processes. We have to discover the detailed physics for ourselves, working from the best observations available for a clue to the theoretical dynamics. In this way it appears that we may have identified the gross features of the generation of magnetic field in a convective dynamo, rapid reconnection of magnetic fields, the hydrodynamic expansion of the extended corona to supersonic velocity, the spontaneous formation of intense current sheets in the interwoven topology of the magnetic fields (Parker 1994a), the generation of plasma waves and propagating radio waves in shock fronts and in streams of fast electrons, the acceleration of fast particles in shock fronts and in the thin intense current sheets created by rapid magnetic reconnection (Reames 1999; Chotoo et al. 2000), etc. But one needs to check out these concepts with detailed and precise observations. The activity of the Sun makes it clear that the classical physics of Newton, Maxwell, Boltzmann, et al. has more twists and turns than anyone could have imagined (cf. Parker 1994a, 1998, 2000).

Then another curious feature of solar activity is the far reaching effects on terrestrial climate. The long term evolution of the Sun, as well as the decade, century, and millenium variations indicated by geology, ice cores, and ocean bottom cores tell us much about the varying climate of Earth and the curious association with solar activity. In this same vein, contemporary monitoring of the total luminosity and activity of other solar type stars has shown directly the short term variations through which a star like the Sun sometimes passes (cf. Beer et al. 1990; Zhang et al. 1994).

2 MAGNETIC ACTIVITY

Now apart from sunspots, faculae, a rare white light flare, and prominences and the corona projecting beyond the Moon during a total eclipse of he Sun, the activity of the Sun is largely invisible to the classical ground based telescope. The first indication that solar activity is magnetic was Hale's (1908a, b) inference of the 2–3 kilogauss magnetic fields of sunspots from the Zeeman effect in the spectrum.

More recently we have come to appreciate that the sunspot is just the most conspicuous feature of a vast world of magnetic activity. That elusive world was dragged into the spotlight with the development of the electronic magnetograph half a century ago (Babcock & Babcock 1955; Babcock 1958). The astonishing revelation that the magnetic field is composed of intense unresolved fibrils has motivated continuing improvement in the spatial resolution of telescopes and magnetographs. However, for all of the improve telescopic resolution, we can still see only to a limited degree the interactions of the magnetic fibrils coming up through the surface of the Sun (cf. Schrijver et al. 1997; Schrijver & Title 1999; Uralov, Nakajima & Zandanov 2000). Microflares and the X-ray bright points are evidently a consequence of rapid reconnection between interacting fibrils and tiny bipolar fields. One presumes that rapid reconnection converts magnetic free energy into heat and fast particles, jets of gas, shock waves, and plasma and magnetohydrodynamic waves which propagate away and heat the surrounding gas. It is reasonably conjectured that the microflaring is the principal heat source for the expanding coronal hole and solar wind (Martin 1984, 1988; Porter et al. 1987; Porter & Moore 1988; Parker 1991a). However, it is not possible to go beyond these vague statements until observations can provide more explicit and quantitative information.

To continue, we have only preliminary ideas on the formation of the magnetic fibrils (cf.

Spruit 1977; Ferriz-Mas 1996; Blanchflower, Rucklidge & Weiss 1998; Tao, Proctor & Weiss 1998; Wissink et al. 2000). We have no idea to what degree the individual fibrils are subject to convective propulsion beneath the visible surface (Parker 1979b, 1988a) or to what degree the individual fibrils may be twisted (cf. Zhang & Bao 1998; Benevolenskaya 2000). For if they are twisted, the twisting propagates along the fibril at the Alfvén speed and concentrates in the expanded portion of the fibril above the visible surface, producing dynamical instability and, presumably, rapid dissipation of magnetic free energy through reconnection (Parker 1974, 1975a, 1977, 1979c, pp. 175–194, 198–200; Bennett, Roberts & Narain 1999).

Then rapid reconnection itself has been studied theoretically in various ideal steady states, usually in the magnetohydrodynamic approximation, in an attempt to understand the extreme rates at which magnetic fields reconnect in nature (c.f. Sweet 1958, 1969; Parker 1957; Petschek & Thorne 1967; Priest 1981, 1982; Soward 1982; Biskamp 1986; Priest & Forbes 1986, 1989; Priest & Lee 1990; Yan, Lee & Priest 1993; Priest & Titov 1996; Craig & Watson 2000, Lozitsky et al. 2000). Most theoretical studies of reconnection have been limited to the two dimensional case, and the recent move to three dimensions, made possible by the increasing power of computers, shows a somewhat different situation (Ugai 1993; Teresawa, Hoshino & Fujimoto 1993; Teresawa 1995). Only recently it has been discovered through numerical simulations that the kinetics of the electrons and ions plays a crucial role in the extremely thin current sheets in the tenuous atmosphere of the Sun, providing theoretical reconnection rates comparable to what is observed (Shay & Drake 1998; Shay et al. 1998). This is an example of the new physics to be learned when the scientist explores beyond the prevailing conventions. This advance is particularly important because of the appearance of reconnection in so many aspects of solar activity.

Now, for all the spectacular displays of magnetic activity that we see in the solar atmosphere (cf. Brueckner & Bartoe 1983; Schrijver & Title 2001), the Sun is very secretive in other respects, e.g. hiding away the origins of the magnetic field far below the visible surface in the rotating turbulent convective zone. We know the basic principles of the large-scale dynamical interaction of gas and field, in the form of magnetohydrodynamics. Yet it is still not clear how the convection creates and maintains the nonuniform rotation of the Sun (Miesch et al. 2000; Durney 2000). Nor is it properly understood how the magnetic fields of the Sun are generated. The basic generation scheme appears to involve the combined effects of the nonuniform rotation and the cyclonic convection, referred to as the $\alpha\omega$-dynamo, but there is no understanding of the rapid diffusion and dissipation of the strong mean magnetic field that is so essential for the operation of the $\alpha\omega$-dynamo.

Then, as already noted, we do not understand why the magnetic field is in the intensely fibril state that appears at the visible surface and is inferred for the deep convective zone (Parker 1984). We do not understand why the magnetic fibrils are swept together and forcibly compressed to form the sunspot (cf. Parker 1979a; Zwaan 1985). Nor are we clear as to why the bipolar magnetic fields of active regions develop the bright faculae and plages that significantly enhance the total brightness of the Sun during periods of high activity (Hoyt et al. 1992; Foukal & Lean 1990; Parker 1995; see also Friis-Christensen & Lassen 1991). In fact we are not clear as to the basic structure of a facula.

We must ask what is the origin of the amazing sandwich structure of the penumbra of a sunspot, made up of alternating vertical slabs of nearly horizontal and steeply inclined magnetic fields (Beckers and Schröter 1968; Wiehr 2000)? Or what is the origin of the small so called ephemeral active regions, distributed widely over the surface of the Sun and varying differently from the large bipolar active regions over the 11-year (22-year) magnetic cycle (Martin & Harvey 1979; Zhang et al. 1999)?

In what ways are the filamentary arches of the X-ray corona heated (cf. Parker 1988b, 1991, 1994a; Sturrock et al. 1990; Feldman et al. 1992; Priest, Parnell & Martin 1994; Deng et al. 2000; Walsh & Galtier 2000; Gomez, Dmitruck & Milano 2000)? How, precisely, is the coronal hole heated so as to produce the fast solar wind (Martin 1984, 1988; Porter et al. 1987; Porter & Moore 1988)? The wind velocities of 600–800 km s^{-1} suggest that a kinetic temperature of $2\times10^6 - 3\times10^6$ K extends many solar radii into space. Then what is the origin of the slow dense solar wind streams? Precisely how does the coronal mass ejection occur (Webb & Hundhausen 1987; Feynman & Martin 1995; Low 1999, 2001; Gibson & Low 2000)? The work on the coronal mass ejection so far indicates a growing deformation of the magnetic field of a bipolar active region into an unstable state, with rapid reconnection cutting loose the ejected magnetic plasmoid. Then going back to the geomagnetic a activity index increasing throughout the 20th century, what was so different about the magnetic fields at the surface of the Sun a century ago?

Now observations of the distant stars show that they emit X-rays, produce dark “starspots”, exhibit cyclic magnetic activity, and have brightness variations in step with the magnetic activity. We infer, then, that they probably exhibit much the same magnetic repertoire as the Sun, if only we were close enough to see it. So the Sun is the scientific gateway to the stars, because the Sun is close enough for detailed study. The detailed observations of the Sun have made it clear that hydrodynamics and magnetohydrodynamics, together with radiative transfer and the strong vertical stratification of the convective zone of a star like the Sun, are too complex to pursue stellar activity by theoretical prowess alone. Observations must point the way and show us what still unimagined concepts are involved in the activity. Only then can the theoretician begin to work out a real understanding from the basic principles of Newton and Maxwell. Thus we look forward to exotic new twists to the application of old classical physical principles. That is the fascination of the science of the Sun and the stars. Their large dimensions provide phenomena that cannot be duplicated in the confines of the terrestrial laboratory (Parker 1985a, 1997).

Helioseismology has already verified the theoretical structure of the solar interior to a remarkable degree of precision (Bahcall & Pinsonneault 1995; Bahcall et al. 1997) and it has determined the peculiar form of the internal nonuniform rotation (Tomczyk, Schou & Thompson 1995; Genovese, Stark & Thompson 1995; Schou et al. 1998; see also Stix 2000; Vanlommel & Cadez 2000). Helioseismology in some of its more sophisticated forms, e.g. time-distance analysis, holds promise of detecting and mapping subsurface flows, and perhaps even subsurface magnetic fields (D’Silva & Duvall 1995; Brüggen & Spruit 2000; Kosovichev & Duvall 2000). Combining this with the observational opportunities for a future ground based 4-m telescope with an adaptive optics system, capable of resolving features down to perhaps 50 km, and with space observations across the entire electromagnetic spectrum, from IR to X-rays, we can expect substantial progress in the coming decades. The Yohkoh, SOHO (Fleck, Domingo & Poland 1995; Berger & Title 1996; Brynildsen et al. 1999), TRACE (Handy et al. 1999; Schrijver et al. 1999; Schrijver & Hurlburt 1999; Schrijver & Title 2001), and ESA spacecraft have already provided spectacular views into the fine structure of the solar atmosphere, where much of the action originates.

3 THE CLASSICAL SUN

Before going into future probing of the mysteries of the Sun, it is well to appreciate what has been accomplished so far in understanding the structure and behavior of a star like the Sun. The "classical" Sun is a self-gravitating ball of gas with a central thermonuclear core, an outflow of heat to the visible surface, a passive photosphere, and a steady luminosity with a thermal spectral distribution determined by the outward decline of temperature through the photosphere. The outflow of heat creates the convective zone in the outer 2/7 of the solar radius R, which at first sight seems merely an annoying detail in constructing theoretical models of the solar interior, but is now recognized as the cause of the aforementioned activity and variability.

The thermonuclear core is at X-ray temperatures ($1\times10^7 - 1.5\times10^7\,\mathrm{K}$) and ten times brighter than a supernova at maximum. Thus we may think of the Sun as a thermonuclear core wrapped in an opaque shroud that leaks only one part in 2×10^{11} of the thermal radiation, providing for us a total luminosity $L = 4\times10^{33}\,\mathrm{ergs\ s^{-1}}$ at a nominal temperature of 5600 K. The neutrinos emitted by the thermonuclear reactions in the core are the one direct observable emission from the deep interior, and their study has led to some fundamental questions about the neutrino rest mass and neutrino oscillations (cf. Bahcall 1999).

There is a slow evolutionary brightening of the Sun as hydrogen in the core converts to helium at a rate of $6\times10^{14}\,\mathrm{g/s}$, for a total of $8\times10^{31}\,\mathrm{g} = 0.04$ solar masses since the Sun formed 4.6×10^9 years ago. The net rate of mass loss to the luminosity is $L/c^2 = 4\times10^{12}\,\mathrm{g/s}$, for a total loss of 3×10^{-4} solar masses since the beginning. This radiative mass loss is to be compared with the smaller mass loss by the contemporary solar wind of about $1\times10^{12}\,\mathrm{g/s}$. It should be no surprise then that the radiation pressure of sunlight is approximately 3×10^3 times greater than the impact pressure of the solar wind.

Now the accumulating helium dilutes the hydrogen in the core, so the thermonuclear burning of the hydrogen cannot maintain the temperature against the unavoidable radiative loss from the surface without contracting and compressing the core. Thus, the Sun as a whole is slowly contracting and growing hotter and brighter. Theoretical models of the solar interior show that the brightness of the early Sun, when it first reached the main sequence, was about 25 percent less that at present, and the brightness is expected to be comparably greater 5×10^9 years from now before the Sun becomes a red giant.

The diminished brightness of the early Sun raises interesting questions about the early atmosphere of Earth, which is presumed to have had a generally temperate climate through most of its history. The fact is that global climate models indicate that a 3–5 percent decrease in solar brightness today would lead to a gradual freezing of the land and the oceans all the way to the equator in a period of the order of 10^3 years as the oceans yielded up their enormous store of thermal energy. Earth would become a snowball, thereby diminishing the absorption of sunlight and locking into a cold state. Restoring the brightness of the Sun to its normal level would not thaw the planet. It is interesting to note, then, that there is substantial geological evidence that Earth actually did freeze over almost 10^9 years ago as the earlier atmosphere diminished and provided less greenhouse blanketing when the Sun was perhaps 5 percent fainter than at present (Hoffman et al. 1998; Hoffman & Schrag 2000). However, the frozen oceans could not absorb carbon dioxide, so, evidently, Earth was rescued from the deep freeze by the accumulation of volcanic carbon dioxide in the atmosphere.

4 SUNSPOTS AND MAGNETIC FIELDS

In a brief review we cannot describe and discuss the dozens of different observational and theoretical problems posed by the active Sun. So we leave most of the problems to the references cited (see also Parker 1985a, 1997 and references therein). To expound a few sample mysteries, we begin here with the dilemma pose by the sunspot, which is simply a large compressed cluster of fibrils. The example illustrates how each step along the way leads to new questions.

To begin, it is observed that a sunspot forms during the hours and days when fresh magnetic fibrils are emerging in a bipolar active region (cf. Zwaan 1985). During such times the fibrils show a curious tendency to cluster together. The individual cluster grows with the accumulation of hundreds or thousands of fibrils packing closely together at the visible surface. When the diameter of the cluster reaches something of the order of 10^3 km, the cluster appears as a dark pore with a mean field of about 10^3 Gauss, more or less the same as the field strength estimated for the original individual fibril. The formation of the complicated penumbral structure (Beckers & Schröter 1968; Wiehr 2000) begins when the diameter of the growing dark cluster exceeds about 3×10^3 km. However the formation of the penumbra is inhibited in the sector where the magnetic fibrils and other pores are streaming in to join the cluster. The field of the umbra, i.e. the central compact cluster, compresses to $2{\times}10^3 - 3{\times}10^3$ Gauss as the umbra grows, on rare occasions reaching a diameter as large as $5{\times}10^4$ km.

The fact is that the clustering is in direct opposition to the Maxwell stresses in the magnetic fields of the fibrils. The tension along the upward diverging fibrils tends to pull the fibrils apart rather than push them together, and the magnetic pressure, of course, also opposes the compression of the field in the spot.

When fresh magnetic fibrils cease to emerge in the region, the clustering tendency disappears and the fibrils begin to pull away around the edges of the sunspot, until in a few days, or sometimes weeks, the cluster has disintegrated (Zwaan 1985).

Many years ago Meyer, Schmidt, Weiss, and Wilson (1974) pointed out that the only possible explanation for the magnetic clustering is an unseen subsurface converging horizontal flow. We presume that the converging flow feeds a downdraft under the sunspot. The converging flow causes the fibrils to cluster because it drags them along with it and compresses them into the state that we see at the surface as a sunspot.

We have proposed (Parker 1979a) that the individual magnetic fibrils, compressed and close packed at the visible surface, may not be close packed below the surface. Presumably the individual fibrils are more intense and smaller in diameter below the surface, so that there is field-free gas flowing around them. The idea is suggested by the fact that heat flow up through the dark umbra of a sunspot is about a quarter of the normal photospheric value, and is essentially independent of the diameter of the umbra. This would be hard to understand if there were no field-free gas close underneath to convect heat up to the umbra. The peculiar sandwich structure of the penumbra, already mentioned, may be presumed to be an important clue to the dynamics of sunspot formation. Unfortunately, for the moment, we are too ignorant to understand.

It must be understood that these suggested ideas do not constitute a proper scientific explanation. They merely move us to the next level of questions. Why, for instance, should an unseen converging flow and downdraft form when magnetic fibrils are emerging through the surface and cease when fibrils are not emerging? Then, of course, we have to ask why there are magnetic fibrils from which to form the sunspot? That question is broadly related to the convection and nonuniform rotation, addressed in the next section.

5 CONVECTION, NONUNIFORM ROTATION, AND GENERATION OF MAGNETIC FIELDS

As already noted, it has not yet been possible to demonstrate theoretically, or with numerical experiments, etc. that the convection beneath the surface of the Sun drives the nonuniform rotation of the Sun. Briefly, helioseismology shows that the radiative interior of the Sun ($r < 5/7R$) rotates nearly rigidly with a period of about 28 days. The surface of the Sun rotates nonuniformly, with a rotation period of 25 days at the equator, increasing smoothly with increasing latitude to more than 30 days at the poles. Helioseismology shows the surprising fact that the surface rotation extends radially downward through the convective zone to the interface with the radiative zone at $r = 5/7R$, where there is a strong shear layer called the *tachocline*.

The failure to show how the convection leads to the observed nonuniform rotation does not imply that the principles of hydrodynamics are somehow inadequate. For it must be appreciated that it has not yet been possible to run a computer simulation with the full vertical density stratification of the convective zone (a factor of 10^6, or 14 density scale heights, from 0.2 g/cm^3 at the base of the convective zone to 2×10^{-7} g/cm^3 at the photosphere, where the convective cells appear as the granules). Then the convection is turbulent with Reynolds numbers of 10^8 or more. One may wonder if the standard "parameterizing" of the turbulence in terms of an effective eddy viscosity is adequate in so stratified an atmosphere. Finally, there is a vague worry that the magnetic stresses might play a role in the hydrodynamics, although that leads to a higher level of complication in the numerical simulations (see discussion in Cattaneo et al. 1991; Miesch et al. 2000).

This brings us to the next question of the physics of the generation of the cyclic fibril magnetic field of the Sun (Parker 1976a, b). The familiar $\alpha\omega$-dynamo process (Parker 1955, 1957a, 1979c, 1993; Steenbeck, Krause & Rädler 1966; Steenbeck & Krause 1969; Roberts 1972; Stix 1976; Moffatt 1978; Spiegel & Weiss 1980; Krause & Rädler 1980; Soward 1983; Choudhuri 1984; Childress & Gilbert 1995; Tobias 1996; Markiel & Thomas 1999) involves shearing the mean poloidal magnetic field (typically 10 Gauss at the visible surface) to produce a relatively strong toroidal or azimuthal magnetic field. The toroidal field is estimated to have a mean value of at least 3×10^3 Gauss, based on the total magnetic flux of some 4×10^{24} Maxwells, emerging through the visible surface in a long lived activity complex (Gaizauskas et al. 1983). The poloidal flux is then regenerated by the interaction of the cyclonic ($\langle \boldsymbol{v} \cdot \nabla \times \boldsymbol{v} \rangle \neq 0$) convection with the toroidal field. An essential feature of the dynamo is the irreversibility of the process through diffusion of the magnetic fields. In order of magnitude the toroidal magnetic field in the convective zone must diffuse a distance of the order of 40° in latitude, say 3×10^{10} cm, in 10 years (3×10^8 s), equivalent to 1m/s. An effective diffusion coefficient provides a characteristic diffusion distance $(4\eta t)^{1/2}$ in a time t. Hence something of the order of $\eta \sim 10^{12}$ cm^2 s^{-1} is needed. The mixing length concept of turbulence suggests an effective diffusion coefficient of the order of $0.1\lambda v$, where λ is the scale of the dominant eddies with velocity v. Standard models for the convection (cf. Spruit 1974) provide values of $10^{11} - 10^{12}$ cm^2 s^{-1}. So for many years the necessary diffusion seemed to be a natural consequence of the turbulent convection. However, it must be remembered that the concept of turbulent diffusion was developed for the diffusion of scalar fields, e.g. smoke, carried bodily with the fluid without resisting the unlimited deformation and stretching into filaments. With present estimates of the mean azimuthal magnetic field climbing to 3×10^3 Gauss or more, it has become clear that the magnetic field does not submit to the unlimited swirling and stretching. The Maxwell stress in the mean field is as strong as the turbulence, $B^2/4\pi \sim \rho v^2$, so the convection would produce strong waves in the mean field, but it would not swirl and mix the field throughout the region, drawing the field into ever longer, thinner, and more intense filaments.

Perhaps the filamentary or fibril structure of the magnetic field is a clue to the nature of the diffusion. As a matter of habit we are inclined to think of the magnetic fields in the Sun in terms of the mean field, asking why the mean field should exhibit so intense a fibril form at the surface of the Sun. Spruit (1977) has pointed out that the radiative cooling at the surface enhances the concentration of the fibril, but the field is already in a fibril form of perhaps 500 Gauss even as it first emerges (Zwaan 1985). Thus we ask whether the magnetic field is in a fibril form throughout the convective zone (Parker 1984). There is reason to think that it may be. For instance, the local buoyant rise of a bundle of azimuthal field from the base of the convective zone, where we think it is created, forms what is called an Ω-loop - an upward bulge of the field symbolized by the Ω - the intersection of the two legs of the Ω with the visible surface creates the bipolar magnetic regions observed with the magnetograph. The bipolar regions are oriented almost east-west, with the leading polarity at slightly lower latitude than the following polarity. Numerical simulation of the rising flux bundle in the rotating Sun shows that the field strength within the bundle must be in the range $0.5\times10^5 - 1.0\times10^5$ Gauss if the bipolar region is to have the observed small inclination to the azimuthal direction (D'Silva 1993; D'Silva & Choudhuri 1993; Fan, Fisher & DeLuca 1993; Fan Fisher & McClymont 1994; Schüssler et al. 1994; Caligari, Moreno-Insertis & Schüssler 1995). The buoyant rise of a weaker field is overpowered by the Coriolis force, so that the rise is nearly parallel to the spin axis of the Sun, and the Ω-loop emerges through the surface at high latitude and with too much tilt.

It could be assumed that the mean field at the bottom of the convective zone is $0.5\times10^5 - 1.0\times10^5$ Gauss, but the total stresses in so strong a mean field raise questions about the ability of the fluid motions to generate the field. A more conservative position is to suppose that the azimuthal magnetic field is in some sort of fibril form, with the $0.5\times10^5 - 1.0\times10^5$ Gauss representing the field within each separate fibril (Parker 1994b). If the mean field is near the minimum estimate of 3×10^3 Gauss, the filling factor for the fibrils is 0.06, thereby gaining a factor of about 16 over the assumption that the mean field is so strong.

As a working hypothesis, then, suppose that the magnetic field is in a fibril state throughout the convective zone. This suggests that instead of working with mean fields in the dynamo equations, the individual fibril may be the basic magnetic entity. An individual magnetic fibril is caught up in the convection, carried along by the aerodynamic drag of the local convection. A circular cylinder with a radius a experiences a transverse aerodynamic drag $\rho v^2 a C_D/2$ per unit length in a fluid of density ρ with transverse velocity v, where C_D is the drag coefficient, of the order of 0.3 for large Reynolds numbers. We might expect a nearly circular cross section if the fibril is strongly twisted, but that might also mean that the fibril is kinky. It also would mean that the twisting would propagate along the fibril as a torsional Alfvén wave to the most expanded portion of the fibril, presumably at the apex of any Ω-loop or bulge (Parker 1974, 1975a, 1977, 1983).

On the other hand, if the fibril is not strongly twisted, its cross section would not be circular. That is a problem in itself. One can imagine a shredded ribbon billowing like a sail in the transverse wind and providing a much larger drag than a simple circular cylinder. So we leave the problem here with the statement that the individual fibrils tend to be carried along more or less with the large-scale convection, not unlike the continuous field.

One would expect to find rapid reconnection where one fibril meets another nonparallel fibril. That is to say, a broad array of fibrils may be pulled against and around another broad array, and the two would interpenetrate through reconnection between the individual fibrils. The reconnection would progress across the individual fibril in a very short time as a consequence of the small thickness of each fibril and the strong field within each fibril. Simple estimates suggest that the effective mean rate of interpenetration of nonparallel regions of fibril fields in the convective zone may proceed at the rate of the order of 1 m/s required for an effective solar dynamo. That is to say, we observe the regeneration of the large-scale mean solar magnetic field because the field is made up of many individual fibrils. One can imagine, too, that the magnetic fragments cut loose from the multiple reconnections between individual fibrils may be a source of the very small-scale magnetic elements observed on the Sun.

There is an extensive literature on the dynamics of fibrils, individually and collectively (Parker 1979b, 1979c, Chap. 10, 1982, 1985b; Tsinganos 1979), but there is nothing yet that puts the physics of fibril behavior together to simulate a dynamo and the general magnetic fields of the Sun. The basic idea would be that the identity of the individual fibril is maintained in opposition to the turbulent intermixing of field and field-free fluid by the continual lengthening and thinning of the fibril carried about in the convection and nonuniform rotation (Vishniac 1995a, b). The principles of the $\alpha\omega$-dynamo remain qualitatively unchanged, the essential difference being only that the diffusion arises through the fibril form of the field rather than from turbulent mixing. Unfortunately any estimate of the effective diffusion rates begins with the rate of rapid reconnection of nonparallel fibrils. We know so little about the structure of the individual fibrils and, indeed, about the various modes of rapid reconnection itself, that the problem is not tractable beyond rough estimates at the present time.

Needless to say, the problem posed by the contemporary convection, nonuniform rotation, and field generation has an additional aspect. That is to say, eventually it will be necessary to understand why the Sun drifts through phases of greatly reduced activity, e.g. the Maunder Minimum.

6 DYNAMICAL RAPID RECONNECTION

Scientific understanding of the sunspot, the coronal mass ejection, the spicules, prominences, flares, and coronal heating are in various stages of development, with the old sunspot problem probably the least understood of all. It should be emphasized that rapid reconnection of magnetic field, driven by the Maxwell stresses, plays a key role in almost every aspect of solar activity. The coronal mass ejection appears to involve rapid reconnection to cut it loose from the Sun (cf. Feynman & Martin 1995; Low 1996, 1999, 2001; Andrews, Wang & Wu 1999; Innes et al. 1999; Ivanov et al. 1999; Lyons & Somnet 1999; Plunkett et al. 2000; Gibson & Low 2000). A spicule may represent the upward ejecta from a microflare at some low altitude (cf. Sterling 2000), and flares of all sizes are, of course, the purest example of rapid reconnection.

The coronal hole, whose expansion produces the high speed solar wind, appears to be heated largely by the dissipation of waves produced by the microflaring created by the interaction of small magnetic bipoles and individual fibrils emerging in the supergranules and being swept into the boundaries between supergranules (Martin 1984, 1988; Porter & Moore 1988). The X-ray emitting coronal filaments appear to be heated by microflaring at their feet as well as by very small flares-nanoflares- along their length (Parker 1988b, 1994). All of these ideas need observational study at sufficiently high resolution to provide quantitative assessment of the reconnection rates and total energy release. On the theoretical side, there remains much to be done in developing the mathematical relations between the field line topology and the formation of tangential discontinuities (current sheets) in the magnetic field. The optical analogy relates the formation of tangential discontinuities to inhomogeneities in the magnitude of the field (Parker 1994). However, it remains yet to develop the general theory for the variations in field magnitude in terms of the topology of the field lines. The essential point for the Sun is that magnetic reconnection occurs at the surfaces of tangential discontinuity. Then the reconnection rates under a variety of initial conditions and boundary conditions need to be assessed, taking into account the appropriate kinetic effects (Shay & Drake 1998; Shay et al. 1998). The conditions for reconnection between fibrils necessarily becomes involved with the dynamical state of the fibrils themselves, about which very little is known at the present time because the diameter of the individual fibril lies below the limit of resolution of existing telescopes. This all ties in closely with the theoretical and observational studies of flares, microflares, and nanoflares in the various complex settings presented by the Sun.

7 THEORETICAL DYNAMICS

It is essential to have an appreciation of the basic theoretical principles describing the large-scale dynamics of the active plasmas and magnetic fields in the Sun. Popular notions concerning the role of electric currents in the dynamics often are careless of the basic laws of Newton and Maxwell, leading to serious misconceptions. An example is given in Section 14. We show here a simple derivation of the hydrodynamic and magnetohydrodynamic equations from the basic principles of Newton, Maxwell, and Lorentz, illustrating their universal applicability to the large-scale dynamics of gases and magnetic fields, regardless of the degree of ionization of the gas and the frequency of interparticle collisions. As we shall see, the basic dynamical variables are the bulk plasma velocity $\boldsymbol{u}$ and the magnetic field $\boldsymbol{B}$, along with the plasma pressure p_{ij} , the plasma density ρ, and whatever gravitational field may be present.

It should be understood that the discussion is restricted to the nonrelativistic case, neglecting terms second order in u/c compared to one. Denote the electric and magnetic fields by $\boldsymbol{E}$ and $\boldsymbol{B}$, respectively, in the laboratory frame of reference. In the local frame of reference of the plasma, moving with velocity $\boldsymbol{u}$ relative to the laboratory, the fields are $\boldsymbol{E}'$ and $\boldsymbol{B}'$, respectively, related to $\boldsymbol{E}$ and $\boldsymbol{B}$ by the nonrelativistic Lorentz transformations,

$$\boldsymbol{E}' = \boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}/c\,, \tag{1}$$

$$\boldsymbol{B}' = \boldsymbol{B} - \boldsymbol{u} \times \boldsymbol{E}/c\,. \tag{2}$$

Imagine, then, that we are sitting in chairs in the laboratory in the presence of the geomagnetic field $\boldsymbol{B}$. Assume that the air in the laboratory has been swept clear of electrostatic charges and the walls of the laboratory are sheathed with copper. Consider the question, “Is there an electric field in the laboratory?” The obvious answer is negative. However, if $\boldsymbol{E} = 0$ in the laboratory, it flows that $\boldsymbol{E}' = \boldsymbol{u} \times \boldsymbol{B}/c$ in any reference frame moving with velocity $\boldsymbol{u}$ relative to the laboratory. Thus, in fact, there are electric fields all about us in the laboratory in each of the infinitely many different moving frames of reference. We avoid these fields only as long as we remain motionless in our chairs. If we move, thereby entering another frame of reference, we experience the magnetic field associated with that frame of reference. There is a frame of reference in which any arbitrary $\boldsymbol{E}'$ exists, subject only to the restrictions that $\boldsymbol{E}' \cdot \boldsymbol{B} = 0$ and $|\boldsymbol{E}'| < |\boldsymbol{B}|$. In fact, there are infinitely many such frames of reference, all with different velocities parallel to $\boldsymbol{B}$. Similarly, for any arbitrary $\boldsymbol{E}$ in the laboratory (subject to the same restrictions as $\boldsymbol{E}'$ above) there are infinitely many moving frames of reference in which $\boldsymbol{E}' = 0$.

Now consider a gas that is at least partially, if not wholly, ionized, so that the gas freely conducts electricity. This conducting gas contains a magnetic field $\boldsymbol{B}$ and is in a dynamical state with characteristic velocity u with a large characteristic scale L. The characteristic dynamical time is then L/u. Ampere’s law tells us that there are electric currents present, with a weak current density j of the order of $cB/4L$ for large L. It follows that there must be a weak electric field $\boldsymbol{E}'$ in the local frame of reference of the moving gas (with local velocity $\boldsymbol{u}(\boldsymbol{r}, t)$ at position $(\boldsymbol{r}, t)$) to maintain the current in opposition to the slight frictional drag (resistivity) of the gas. Restricting attention to gases that conduct well enough that $\boldsymbol{E}'$ is small compared to $\boldsymbol{u} \times \boldsymbol{B}/c$, the Lorentz transformation giving $\boldsymbol{E}'$ reduces to $\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}/c \cong 0$, or

$$\boldsymbol{E} = -\boldsymbol{u} \times \boldsymbol{B}/c\,, \tag{3}$$

in the laboratory frame. That is to say, an electric field appears in the laboratory because there is no significant electric field in the frame of reference of the moving gas. Substituting this expression for $\boldsymbol{E}$ into Faraday’s induction equation,

$$\partial \boldsymbol{B}/\partial t = -c\nabla \times \boldsymbol{E}\,,$$

the result is the familiar magnetohydrodynamic induction equation

$$\partial \boldsymbol{B}/\partial t = \nabla \times (\boldsymbol{u} \times \boldsymbol{B})\,. \tag{4}$$

This induction equation asserts that the magnetic field $\boldsymbol{B}$ is carried bodily with the fluid velocity $\boldsymbol{u}$. The magnetic energy density is $B^2/8\pi$ and the Poynting vector, giving the electromagnetic energy flux, is

$$c\boldsymbol{E} \times \boldsymbol{B}/4\pi = \boldsymbol{u}_\perp B^2/4\pi\,,$$

where $\boldsymbol{u}_\perp$ is the bulk velocity of the fluid perpendicular to $\boldsymbol{B}$ and $B^2/4\pi$ is the magnetic enthalpy density. The essential point is that the magnetic field is a stress system with its own substantial energy and, hence, its own small inertial mass and gravitational source density, all swept along with the moving gas. This is the basis for magnetohydrodynamics, which follows on the large scale for any gas or fluid with no significant electrical insulating properties. The basic point is that $\boldsymbol{E}'$ is small. The fluid may be anything from a collisionless plasma to a dense partially ionized gas. Exceptions can be found in extreme circumstances, of course, where small scales appear in the midst of large scales, e.g. in the thin intense current sheets where regions of nonparallel magnetic field press together, providing rapid reconnection and sometimes significant electric fields parallel to $\boldsymbol{B}$ (cf. Parker 1972, 1994; Schindler, Hess & Birn1991; Shay & Drake 1998). To understand where such things may occur, consider the stresses in the magneic field.

The stress in the magnetic field is described by the Maxwell stress tensor for the magnetic field, $M_{jk} = -\delta_{jk}B^2/8\pi + B_jB_k/4\pi$, where the first term represents an isotropic pressure $B^2/8\pi$ and the second term represents the tension $B^2/4\pi$ along the magnetic field. The electric stresses are small $O(v^2/c^2)$ compared to the magnetic stresses and can be neglected. The force exerted by the magnetic field on the fluid is $\partial M_{jk}/\partial x_k$, which can be written $(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}/4\pi$ or $-\nabla B^2/8\pi + (\boldsymbol{B} \cdot \nabla)\boldsymbol{B}/4\pi$. Thus, static equilibrium of the magnetic field alone is described by $\partial M_{jk}/\partial x_k = 0$, implying that $\nabla \times \boldsymbol{B} = \alpha \boldsymbol{B}$, where α is an arbitrary function of position, subject only to the condition ($\boldsymbol{B} \cdot \nabla\alpha = 0$) that α is constant along each field line. Thus the structure of α is intimately related to the topology of the field lines.

Now the field lines extending across a given region are swirled and twisted about each other by whatever convective flows may be present, so the twisting and interwinding of the field lines can take most any arbitrary form. However, note that it is not possible to tie knots without introducing resistive diffusion and reconnection. The curious fact is that for almost all field line topologies, i.e. twisting and interweaving, static equilibrium of the field introduces internal surfaces of tangential discontinuity (current sheets) across which the field magnitude is continuous but the field direction is discontinuous (Parker 1994). This remarkable phenomenon arises from the properties of the Maxwell stress tensor such that the equilibrium equation $\nabla \times \boldsymbol{B} = \alpha \boldsymbol{B}$ possesses mixed characteristics. The curl of the equilibrium equation yields $\nabla^2 \boldsymbol{B} + \alpha^2 \boldsymbol{B} = \boldsymbol{B} \times \nabla\alpha$, where the Laplace operator indicates two families of complex characteristics (cf. Parker 1979c, 1994). That in itself would indicate an elliptic equation, which admits of no internal surfaces of discontinuity. However, the divergence of the equilibrium equation yields the condition $\boldsymbol{B} \cdot \nabla\alpha = 0$, already noted, from which it is obvious that the field lines represent a family of real characteristics. Hence the solutions to $\nabla \times \boldsymbol{B} = \alpha \boldsymbol{B}$ may have discontinuities from one field line of flux surface to the next, unlike the familiar solutions to purely elliptic equations. Surfaces of tangential discontinuity are the rule in arbitrary field topologies, and the discontinuities extend along flux surfaces defined by the field lines.

Within any given flux surface the field lines project as if they were light rays propagating through an index of refraction proportional to the field magnitude B, referred to as the *optical analogy* (Parker 1991b, 1994). Thus the field lines refract around regions of enhanced magnetic pressure $B^2/8\pi$. Avoiding regions of enhanced magnetic pressure creates gaps in the flux surface, thereby allowing the regions of continuous field on either side to come into contact through the gap. Since otherwise separate regions of field are generally not parallel, their contact through the gap creates a surface of tangential discontinuity.

The importance of the spontaneous formation of surfaces of tangential discontinuity in the slowly churning magnetic fields in the tenuous atmosphere of the Sun lies in the initiation of rapid reconnection across the incipient surfaces. The result is to produce small flares – microflares and nanoflares – throughout any large-scale field subject to continual convective deformation.

8 MAGNETOHYDRODYNAMICS AND DISSIPATION

The next step in developing the theory is to work out the small but nonvanishing electric field $\boldsymbol{E}'(\boldsymbol{j})$ necessary to drive the small current density $\boldsymbol{j}$ required by Ampere. The induction equation is

$$\partial \boldsymbol{B}/\partial t = \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) - c\nabla \times \boldsymbol{E}'(\boldsymbol{j})\,. \tag{5}$$

The additional term represents the diffusion and dissipation of the magnetic field by the frictional drag on the electric current.

In the simple case that the gas is so dense that the mean free path is less than the electron cyclotron radius, the scalar Ohm's law is an adequate approximation, with $\boldsymbol{j} = \sigma \boldsymbol{E}'$ in terms of the electrical conductivity $\sigma(\sigma \sim 2 \times 10^7 T^{3/2}/\mathrm{s}$ in ionized hydrogen). Then with Ampere's law $\boldsymbol{j} = (c/4\pi)\nabla \times \boldsymbol{B}$, we have

$$\begin{aligned} \partial \boldsymbol{B}/\partial t &= \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) - \nabla \times (\eta \nabla \times \boldsymbol{B}) \\ &= \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B} - \nabla \eta \times (\nabla \times \boldsymbol{B})\,, \end{aligned}$$

where the resistive diffusion coefficient η is equal to $c^2/4\pi\sigma$.

In other situations the dissipation term can be more complicated. For instance, in a slightly ionized gas, e.g. the terrestrial ionosphere or the solar photosphere, it is necessary to consider the bulk velocity of the neutral gas, the ions, and the electrons separately, with collisional friction between each of the three components. Eliminating the ion and electron bulk velocities in terms of the current density and mutual drag, and obtaining an expression for $\boldsymbol{E}$ in terms of the velocity $\boldsymbol{v}$ of the neutral gas, the induction equation can be written (Parker 1966),

$$\partial \boldsymbol{b}/\partial t = \nabla \times (\boldsymbol{v} \times \boldsymbol{b}) - \nabla \times \{\eta \nabla \times \boldsymbol{b} + \alpha(\nabla \times \boldsymbol{b}) \times \boldsymbol{b} - \beta[(\nabla \times \boldsymbol{b}) \times \boldsymbol{b}] \times \boldsymbol{b}\}\,,$$

where $\boldsymbol{b}$ is the dimensionless magnetic field $\boldsymbol{B}/B$ expressed in terms of the characteristic field strength B. With N neutral atoms per unit volume and n electrons and n singly charged ions per unit volume ($n \ll N$), the coefficients η, α, and β are expressed in terms of the collision time τ_I for an ion moving among the neutral atoms, the collision time τ_E for an electron moving among the neutral atoms, and the electron-ion collision time τ.

$$\begin{aligned} \eta &= \frac{c^2}{4\pi n e^2}\left[\frac{(M/\tau_I)(m/\tau_E)}{M/\tau_I + m/\tau_E} + \frac{m}{\tau}\right] \\ &\cong \frac{c^2}{4\pi\sigma}\,. \end{aligned}$$

The Hall coefficient is

$$\alpha = \frac{cB(M/\tau_I - m/\tau_E)}{4\pi n e(M/\tau_I + m/\tau_E)}\,.$$

The Pedersen resistive diffusion coefficient is

$$\beta = \frac{B^2}{4\pi n(M/\tau_I + m/\tau_E)}\,.$$

In order of magnitude,

$$\frac{\alpha}{\eta} \cong \Omega_E \tau_E, \qquad \frac{\beta}{\eta} \cong \Omega_E \tau_E \Omega_I \tau_I,$$

where $\Omega_I = eB/Mc$ and $\Omega_E = eB/mc$ represent the characteristic ion and electron cyclotron frequencies, respectively. Note that the Hall effect is nondissipative and becomes important at low collision frequencies, $\Omega_E \tau_E > 1$. (For further discussion see Parker 1996.) The essential point is that the magnetic field is carried bodily with the bulk velocity $\boldsymbol{v}$ of the slightly ionized gas in most astronomical situations.

9 MOMENTUM EQUATION

Next consider the equations for conservation of particles and for conservation of momentum. As we shall see, the hydrodynamic terms in the magnetohydrodynamic momentum equation are nothing more than the inventory of the bulk momentum flux in Newton's equation of motion. Hence they apply universally, regardless of the state of the gas.

Consider a plasma composed of $N(x_k, t)$ electrons and singly charged ions per unit volume. The scale of variation is L, which is large compared to the cyclotron radius of any of the particles. It is assumed that N is sufficiently large that NL^3 is an extremely large number, because we intend to treat the dynamics on a statistical basis on a small scale $\lambda(\ll L)$, requiring that there is a large number of particles in the small volume $V = \lambda^3$, i.e. $N\lambda^3 \gg 1$. Thus the mean number density N is well defined everywhere throughout the gas, with very little deviation with time from the local statistical mean.

Denote the velocity of each individual particle (ion or electron) by v_j and write $v_j = u_j + w_j$, where u_j is the mean bulk velocity and w_j is the thermal velocity, relative to the moving mean bulk of the plasma. Thus the sum over all particles in the small volume V at time t is

$$\Sigma M v_j + \Sigma m v_j = NV(M+m)u_j \,,$$

where the first sum is over the ions in V, indicated by the ion mass M, and the second sum is over the electrons in V, indicated by the electron mass m. This summation convention is used throughout this section. It follows that the sum over the thermal velocities w_j vanishes,

$$\Sigma M w_j + \Sigma m w_j = 0 \,.$$

This is essentially the definition of the thermal velocity. For the weak electric currents in the large-scale plasma the momentum of the electric current, involving the net streaming of the electrons relative to the ions, can be neglected. Hence

$$\Sigma M w_j = \Sigma m w_j = 0 \,.$$

Then, given that the sum over the thermal momenta of the particles in the small volume V is zero, it follows that there is no net thermal flux of particles across any surface cutting across V.

Consider the time rate of change of the number NV of particles in V. The thermal velocities contribute no net outflow across the surface S of V, so the net outflow is the result only of the bulk velocity u_j. The outward flux of particles is given by Nu_j integrated over S. Applying Gauss's theorem, this is equal to the divergence of Nu_j integrated over V. Hence

$$\frac{\partial}{\partial t}[NV(M+m)] = -\int \mathrm{d}V \frac{\partial}{\partial x_j}[N(M+m)u_j] = -V(M+m)\frac{\partial}{\partial x_j} N u_j$$

in the mean. The net result is

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial X_k} N u_k = 0, \qquad \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial X_k} \rho u_k = 0\,,$$

where $\rho = N(M+m)$ is the density.

Consider the time rate of change of the well defined mean momentum $VN(M+m)u_j$ in V. The momentum varies as a consequence of the total Lorentz force $V\partial M_{jk}/\partial x_k$ and whatever other force VF_j, e.g. gravity, there may be. The total momentum also varies because of the momentum carried inward or outward across the surface S by the individual particles. The momentum of an ion, for instance, is $M(u_j + w_j)$ and the velocity with which this is transported in the k-direction is $(u_k + w_K)$. Thus the momentum flux is the sum of $M(u_j+w_j)(u_k+w_k)$ over unit area. The cross product terms $u_j w_k$ and $w_j u_k$ sum to zero over any surface of V, leaving only the momentum flux $N(M+m)u_j u_k$ of the bulk motion and the momentum flux density $(\Sigma M w_j w_k + \Sigma m w_j w_k)/V$ of the thermal motions. The mean outward flux of momentum is given by the integral of these two fluxes over the surface S, which can be written as the volume integral of the divergence, upon employing Gauss's theorem. For the mean velocity u_j the result is

$$\int \mathrm{d}V(M+m)\frac{\partial}{\partial x_k} N u_j u_k = V\frac{\partial}{\partial x_k}\rho u_j u_k\,.$$

For the thermal velocities w_j the result is

$$\frac{\partial}{\partial x_k}\Sigma(M w_j w_k + m w_j w_k) = V\frac{\partial}{\partial x_k} p_{jk}\,,$$

where the sum is over all particles in V and the pressure tensor is defined as

$$p_{jk} = (\Sigma M w_j w_k + \Sigma m w_j w_k)/V\,.$$

Putting these contributions together, it follows that the time rate of change of the mean momentum density $N(M+m)u_j u_k$ is given by

$$\frac{\partial}{\partial t}\rho u_j = -\frac{\partial p_{jk}}{\partial x_k} - \frac{\partial R_{jk}}{\partial x_k} + \frac{\partial M_{jk}}{\partial x_k} + F_j\,,$$

where $R_{jk} = \rho u_j u_k$ is the Reynolds stress tensor, representing the momentum transport or stress transmitted by the bulk motion. Multiplying the equation for conservation of mass by u_j and subtracting from the momentum equation provides the familiar form

$$\rho\left(\frac{\partial u_j}{\partial t} + u_k\frac{\partial u_j}{\partial x_k}\right) = \frac{\partial p_{jk}}{\partial x_k} + \frac{\partial M_{jk}}{\partial x_k} + F_j\,.$$

There are several comments in order here. As already noted, the derivation of the momentum equation is carried out only to lowest order, which does not include such subtleties as viscosity. The purpose has been to illustrate the universal character of the basic momentum equation. The derivation stands without regard for whether the particles collide with one another. The implicit assumption is only that is large compared to the Debye radius and the effective collision radius of any of the particles. Note, too, that the derivation could have included additional species of particle, e.g. a neutral atoms. When the degree of ionization is slight, the bulk velocity is essentially the velocity of the neutral component in both the induction and momentum equations. The electrons and ions are tied to the neutral gas through collisions. The Maxwell stresses are exerted on the electrons and ions, of course. With sufficiently low gas densities, the electrons and ions may be pushed through the neutral gas with significant speeds. The effect is sometimes called ambipolar diffusion (cf. Parker 1979c, p. 45).

The momentum equation shows the dynamical interplay of the Maxwell stress M_{jk} and the Reynolds stress R_{jk}. In particular, note the symmetry of the Maxwell stress, which is quadratic in B_j, and the Reynolds stress, which is quadratic in u_j. The induction equation is a linear relation between B_j and u_j. It is apparent, then, that B_j and u_j are equal partners in the dynamics.

10 EQUATION OF STATE

Consider the calculation of the pressure tensor p_{jk}, which may involve both the variation of the gas density ρ through the equation for conservation of mass, and the variation of temperature, perhaps through heat transfer by conduction and radiation.

As a practical matter the incompressible flow $\partial u_j/\partial x_j = 0$ may sometimes be an adequate approximation. When expansion and contraction are an essential part of the dynamics, it may be adequate to approximate the pressure tensor as being isotropic, with $p_{jk} = p\delta_{jk}$, representing p as a simple function of ρ, e.g. the polytrope model, $p(\rho) \sim \rho^\alpha$. In tenuous gases, where collisions and/or plasma instabilities may not be adequate to maintain an approximation to isotropy, the pressures parallel and perpendicular to the magnetic field must be distinguished. The pressure tensor p_{jk} is simply related to $p_\parallel$ and $p_\perp$, as is shown in the next section. In the absence of significant scattering of the thermal motions, $p_\parallel$ and $p_\perp$ vary adiabatically and independently of each other. In many cases the Chew-Goldberger-Low approximation (Chew, Goldberger & Low 1956; Bittencourt 1986, pp.314–319) may be adequate. The result is expressible in terms of two invariant quantities, which may be expressed as

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{p_\perp}{NB}\right) = 0\,,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{B^2 p_\parallel}{N^3}\right) = 0\,.$$

These invariants are based on the idea that the particles are localized to some degree both along and across the magnetic field. Thus if l represents the scale of the localization along $\boldsymbol{B}$, the parallel thermal velocity $w_\parallel$ is subject to the longitudinal invariant $lw_\parallel$. The perpendicular thermal velocity is dictated by the transverse invariant, $w_\perp^2/B$. The cross sectional area A of a flux tube defines a characteristic volume Al with NAl invariant to conserve particles and NB invariant to conserve magnetic flux. Noting that $p_\parallel$ is proportional to $Nw_\parallel^2$ and $p_\perp$ is proportional to $Nw_\perp^2/2$, the quantities l and A can be eliminated, with the foregoing result.

Needless to say, there are cases where more precise calculations of $p_\parallel$ and $p_\perp$ may be required, perhaps treating the kinetics of the electrons and ions in quantitative detailed. The rapid reconnection of two opposite magnetic fields is an important case in point (Shay & Drake 1998; see also, Schindler, Hesse & Birn 1991; Foukal & Hinata 1991).

11 NEWTON, MAXWELL, AND AMPERE

The electrically conducting, electrically neutral gas, wholly or partly ionized, in the presence of a magnetic field behaves like a classical fluid subjected to the Maxwell stresses of the magnetic field. Observed dynamical behavior of the gases at the Sun, in interplanetary space, and in the terrestrial magnetosphere appear to be compatible with this general fluid behavior. Exceptions are the rapid dissipation and reconnection in the thin intense current sheets between regions of nonparallel magnetic field. However, there is a basic question that has sometimes caused concern in theoretical models, and that is whether the requirements of Ampere's law are properly satisfied. What guarantee is there that the electric current, calculated from some generalized Ohm's law, for instance, will adjust itself so as to satisfy Ampere. The answer is, of course, that Ampere's law is a fundamental law of nature, part of Maxwell's equation. It will take care of itself and if we have calculated the current correctly from the basic principles of physics, we do not need to concern ourselves. There is no way that we can contradict Ampere's law. What is more, electric charge is automatically conserved ($\nabla \cdot \boldsymbol{j} = 0$) because there is no vector function $\boldsymbol{B}(\boldsymbol{r})$ whose curl fails to have vanishing divergence.

On the other hand, in view of the concern sometimes expressed over this point, it is not without interest to see how nature takes care of its own fundamental laws. Write Maxwell's equation

$$\frac{\partial \boldsymbol{E}}{\partial t} = c\nabla \times \boldsymbol{B} - 4\pi \boldsymbol{j}\,,$$

which reduces to Ampere's law in the present nonrelativistic case,

$$c\nabla \times \boldsymbol{B} = 4\pi \boldsymbol{j}\,.$$

The electric current $\boldsymbol{j}$ depends upon the Newtonian mechanics of the electron and ion motions. Turning attention to the simple case of a collisionless plasma the free motions parallel to $\boldsymbol{B}$ quickly adjust to the requirements of Ampere's law. For if the current is inadequate, there is immediately a $\partial \boldsymbol{E}/\partial t$ in the direction of insufficient $\boldsymbol{j}$. Both ions and electrons are accelerated so as to augment $\boldsymbol{j}$, thereby bringing $\boldsymbol{j}$ into compliance with Ampere's law. The characteristic time in which this occurs is the plasma period, $(m/Ne^2)^{1/2}$, for the electrons.

Consider the current perpendicular to $\boldsymbol{B}$. The foregoing remarks apply initially, but within the cyclotron period $2\pi/\Omega$, where $\Omega = eB/mc$, the particle motion is deflected toward the direction perpendicular to both $\boldsymbol{B}$ and $\boldsymbol{E}$. This is the origin of the electric drift velocity $\boldsymbol{u}_D = c\boldsymbol{E} \times \boldsymbol{B}/B^2$ of both the electrons and ions, which provides bulk flow but no current. A continuing growth of $\boldsymbol{E}$ will keep some current flowing in the direction necessary to satisfy Ampere' law, but with each increase in $\boldsymbol{E}$ the current dies out again in the cyclotron period. It looks like a runaway situation, with $\boldsymbol{E}$ growing without bound. The electric drift velocity would then grow without bound. On the other hand, this interesting situation is not to be found in nature, so far as modern observations are aware. And where would the energy come from if it did occur?

It is evident, then, that the Newtonian mechanics of the ions and electrons perpendicular to $\boldsymbol{B}$ must somehow automatically supply the electric current required by Ampere. The idea is easily tested for a collisionless plasma and magnetic field with characteristic scale L large compared to the characteristic cyclotron radius R of the ions and electrons. Use the guiding center approximation to describe the motion of the individual particles of mass m and charge q. Then sum over the electron and ion motions to obtain the net electric current (Parker 1957b). Taking into account the various geometric factors, it can then be shown that

$$j_\perp = \frac{c}{8\pi p_m}\boldsymbol{B} \times \left\{\nabla p_\perp + \left(\frac{p_\parallel - p_\perp}{p_m}\right)\frac{(\boldsymbol{B}\cdot\nabla)\boldsymbol{B}}{8\pi} + \rho\frac{\mathrm{d}u_D}{\mathrm{d}t}\right\} ,$$

where $p_m \equiv B^2/8\pi$. Then $p_\perp \equiv \Sigma m w_\perp^2/2$ represents the pressure of the perpendicular thermal velocities $w_\perp$ and $p_\parallel \equiv \Sigma m w_\parallel^2$ is the pressure of the parallel thermal velocities $w_\parallel$. The summations are over all particles in a unit volume. For simplicity we assume singly charged ions, with m representing the mass of each individual particle, whatever that may be.

Now the perpendicular component of $\nabla \times \boldsymbol{B}$ can be written as

$$(\nabla \times \boldsymbol{B})_\perp = \frac{\boldsymbol{B}}{B^2} \times \left[-\frac{1}{2}\nabla B^2 + (\boldsymbol{B}\cdot\nabla)\boldsymbol{B}\right] ,$$

with the result that Ampere's law becomes

$$\rho\frac{\mathrm{d}\boldsymbol{u}_D}{\mathrm{d}t} = -\nabla_\perp(p_\perp + p_m) + \frac{[(\boldsymbol{B}\cdot\nabla)\boldsymbol{B}]_\perp}{4\pi}\left(1 + \frac{p_\perp - p_\parallel}{2p_m}\right) .$$

This is, of course, the perpendicular component of the Newtonian momentum equation for a plasma with pressures $p_\parallel$ and $p_\perp$. The appropriate form of p_{jk} is obvious by inspection (see also Bittencourt 1986, pp.312–314). The calculation shows that, if the bulk motion $\boldsymbol{u}_D$ is described by Newton's equation of motion, then the magnetic field deforms in such a way that the Newtonian thermal motions of the ions and electrons automatically provide the current required by Ampere's law. That is to say, Ampere's law implies Newton's equations of motion for the individual particles. This is hardly surprising, of course, when we recall the derivation of Poynting's theorem, showing the precise analogy between the momentum of particles and the electromagnetic momentum density $\boldsymbol{E}\times\boldsymbol{B}/4\pi c$ and the electromagnetic energy flux $c\boldsymbol{E}\times\boldsymbol{B}/4\pi$. The analogy would not exist for some other equation of motion.

For the record, note that the bulk flow $\boldsymbol{u}_\parallel$ parallel to $\boldsymbol{B}$ is described by

$$\rho\left(\frac{\partial u_\parallel}{\partial t} + u_\parallel\frac{\partial u_\parallel}{\partial s}\right) = -\frac{\partial p_\parallel}{\partial s} + \frac{p_\parallel - p_\perp}{B}\frac{\partial B}{\partial s} ,$$

where s denotes distance measured along $\boldsymbol{B}$.

12 POPULAR MISCONCEPTIONS

A few words are in order on some of the popular misconceptions. For instance, it is sometimes stated that the electric current is the cause of the magnetic field, and, therefore, is the more fundamental physical variable. Then, evidently, based on the notion that $\boldsymbol{E}$ drives $\boldsymbol{j}$, the two variables $(\boldsymbol{j}, \boldsymbol{E})$ are employed in discussion of the dynamics. The difficulty is that this $(\boldsymbol{j}, \boldsymbol{E})$ paradigm, based on a false premise, leads to no practical field equations. As we have shown, the dynamical equations come out in terms of the bulk velocity $\boldsymbol{u}$ and the magnetic field $\boldsymbol{B}$. These field equations in $(\boldsymbol{u}, \boldsymbol{B})$ can, of course, be rewritten in terms of $(\boldsymbol{j}, \boldsymbol{E})$ using the Biot-Savart integral

$$\boldsymbol{B}(\boldsymbol{r}, t) = \frac{1}{c}\int \frac{\mathrm{d}^3\boldsymbol{r}'\boldsymbol{j}(\boldsymbol{r}', t) \times (\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3}$$

to replace $\boldsymbol{B}$ and the electric drift velocity expression $\boldsymbol{u}_D = c\boldsymbol{E} \times \boldsymbol{B}/B^2$ to eliminate $\boldsymbol{u}_\perp$. However, the resulting global nonlinear integro-differential equation in terms of $(\boldsymbol{j}, \boldsymbol{E})$ is so unwieldy as to be of little use.

In fact $\boldsymbol{j}$ is not the cause of $\boldsymbol{B}$. The concept of *cause* and *effect* is well defined in physics. The cause is the prime mover, introducing the momentum and energy that brings about the effect. Ampere's law relates $\boldsymbol{j}$ and $\boldsymbol{B}$ but makes no distinction as to which is the cause of the other.

One is accustomed in the physics laboratory to produce a magnetic field by applying an *emf* to a coil of wire so as to drive an electric current I along the wire, thereby creating a magnetic field. The *emf* and the current are clearly the cause of the magnetic field. But suppose that, once the field is established by the current, the *emf* is shorted out of the circuit with a length of wire, thereby removing the *emf* from the circuit. There remains the current I flowing around the coil of wire (with inductance L and resistance R). The magnetic field has an energy density $B^2/8\pi$ and the total magnetic energy is given by $LI^2/2$. The electric circuit equation is $RI + LdI/dt = 0$, from which it follows that the current decays in proportion to $\exp(-Rt/L)$. During this phase it is the energy of the magnetic field that drives the current, so the field is the cause of the current.

Consider, then, the electric current in a broad plasma permeated by a large-scale magnetic field. Under astrophysical conditions there is no externally applied *emf* to drive the current. Working in the local frame of reference of the plasma moving with velocity $\boldsymbol{u}(\boldsymbol{r}, t)$, we again denote the electric field by $\boldsymbol{E}'(\boldsymbol{r}, t)$, equal to $\boldsymbol{E}(\boldsymbol{r}, t) + \boldsymbol{v}(\boldsymbol{r}, t) \times \boldsymbol{B}(\boldsymbol{r}, t)/c$ in terms of the electric field $\boldsymbol{E}(\boldsymbol{r}, t)$ and magnetic field $\boldsymbol{B}(\boldsymbol{r}, t)$ in the laboratory. We neglect terms second order in v/c compared to one. Now, whatever the state of the plasma or partially ionized gas, there is some frictional impediment to the flow of the electrons relative to the ions. Thus, the electric field $\boldsymbol{E}'$ must continually push the electrons to keep the current going. The rate at which $\boldsymbol{E}'$ does work on $\boldsymbol{j}$ is $\boldsymbol{E}' \cdot \boldsymbol{j}$. Because of the frictional or resistive impediment, however slight, $\boldsymbol{E}' \cdot \boldsymbol{j} > 0$. That is to say, in the large scale, where time variations are slow, the electron inertia plays no significant role, and the energy flow is at all times from the magnetic field to the current. So the magnetic field is the cause of the electric current. The current is a passive quantity, transmitting no stresses and involving no significant kinetic energy. Given the magnetic field, the current follows from Ampere's law.

Now there is no divine prohibition against working with $(\boldsymbol{j}, \boldsymbol{E})$ if one wishes to do so. However, it is essential that the $(\boldsymbol{j}, \boldsymbol{E})$ paradigm be used in the context of Newton and Maxwell, with the attendant difficulties already noted. Unfortunately this has not been universally appreciated. For instance, it is sometimes asserted that the electric current in a dynamical plasma can be properly described by an electric circuit analog, in the form of a fixed circuit made up of one or more current loops with suitable resistances, inductances, and capacitances. The linear ordinary differential equations describing the currents in the loops are more easily solved than the correct nonlinear partial differential equations deduced from Newton and Maxwell. So the idea has become widely popular, producing some astonishing theoretical conclusions. As an example, it has been asserted that, if the current were suddenly blocked by a local large increase in resistivity (perhaps as a result of an electric double layer or the excitation of plasma turbulence), then the immense equivalent inductance of the circuit would produce an enormous voltage drop across the resistive region (Alfvén & Carlquist 1967). Such a voltage would easily produce cosmic rays in solar flares, etc. It is shown with a specific example in Section 14 that no such effect occurs, because the current is immediately rerouted rather than being cut off.

13 ELECTRIC CIRCUIT ANALOG

Unfortunately the analog circuit equations do not apply to the currents flowing in a dynamical plasma. The connectivity of the analog circuit is fixed, whereas the current required by Ampere's law in the moving plasma may change its connectivity as the field deforms over time. More universal is the fact that the current in the analog circuit is subject to the electric field $\boldsymbol{E}$ in the laboratory frame of reference, whereas the current $\boldsymbol{j}$ in the plasma sees only the electric field $\boldsymbol{E}'$ in the frame of reference of the moving plasma. So the circumstances are qualitatively different.

If the plasma is constrained to be at rest, so that $\boldsymbol{v} = 0$ and $\boldsymbol{E}' = \boldsymbol{E}$, it is possible to construct a proper electric circuit analog. In that case the evolution of the magnetic field is described by the appropriate diffusion equation, written $\partial \boldsymbol{B}/\partial t = \eta \nabla^2 \boldsymbol{B}$ for the simple case of a uniform scalar resistivity. Once the solution $\boldsymbol{B}(\boldsymbol{r}, t)$ is worked out, the total current I is available from Ampere's law, and the magnetic energy is given by the volume integral of $B^2/8\pi$ and is equal to $LI^2/2$ in terms of the inductance L. The rate of decline of the magnetic energy is equal to I^2R, so that both L and R can be worked out. One finds, however, that both L and R are usually functions of t for the simple reason that the characteristic scale varies as $(4\eta t)^{1/2}$. In a single current loop the current declines as

$$I(t) = I(0) \exp \left[- \int_0^t \mathrm{d}\tau R(\tau)/L(\tau) \right] .$$

As an exercise the reader may find it amusing to work out the analog circuits for a slab of uniform magnetic field, a uniform rod of field, and a column of azimuthal field. The asymptotic decline of these fields has the form $t^{-1/2}$, t^{-1}, and t^{-2}, respectively. The essential point is that the analog can be constructed correctly only after the complete solution $\boldsymbol{B}(\boldsymbol{r}, t)$ is known, from which the precise values of $R(t)$ and $L(t)$ are then deduced.

When the plasma is allowed to move, then $\boldsymbol{E}' \neq \boldsymbol{E}$ and the circuit analog is not applicable. A single example suffices.

14 CURRENT BLOCKAGE

Consider the uniform magnetic field B in the z-direction in an infinite pace filled with a uniform, incompressible, inviscid, infinitely conducting ($\eta = 0$) fluid. Then let the region $-a < y < +a$ be sheared by the fluid velocity kzU in the x-direction for a time τ, producing the x-component of field $B_x = kBU\tau$ in $-a < y < +a$, with $B_x = 0$ elsewhere. Thus, there are current sheets at $y = \pm a$, with current density $J = ckBU\tau/4\pi$ in the positive z-direction at $y = +a$ and in the negative z-direction at $y = -a$.

At some later time, designated $t = 0$ for convenience, the electrical conductivity in the thin slab $-h < z < +h$ is switched off, so that J is blocked at $z = \pm h$. The immediate effect in the upper half space ($y > h$) is to divert J into the negative y-direction across the surface $z = h$. In the lower half space J is diverted into the positive y-direction across the surface $z = -h$. This diversion of J is associated with the free propagation and departure (at the speed c) of B_x (in $-a < y < +a$, $-h < z < +h$) away through the nonconducting slab,

$-h < z < +h$. Subsequently $(t > 0) B_x = 0$ on $z = \pm h$. The Maxwell stress $M_{xz} = B_x B/4\pi$ is interrupted at $z = \pm h$, and the Lorentz force $\partial M_{xz}/\partial z$ is exerted on the fluid. The initial B_x becomes $b(z,t)$ in the x-direction in $z^2 > h^2$ and the fluid is accelerated to the velocity $v(z,t)$ in the x-direction. The magnetohydrodynamic induction equation is $\partial b/\partial t = B\partial v/\partial z$ and the momentum equation is $\partial v/\partial t = (B/4\pi\rho)\partial b/\partial z$, with the fluid pressure varying as $p + b^2/8\pi = constant$. Eliminating v from the two equations yields the familiar wave equation

$$\frac{\partial^2 b}{\partial t^2} - C^2 \frac{\partial^2 b}{\partial z^2} = 0\,,$$

where the Alfvén speed is $C = B/(4\pi\rho)^{1/2}$. Given the initial field B_x and the vanishing of b on $z = +h$, the appropriate solution in $-a < 0 < +a, z > +h$, is $b(z,t) = BkU\tau S(z - h - Ct)$. Here $S(X)$ is the unit step function, equal to $+1$ for $X > 0$ and zero for $X < 0$. The fluid motion is $v = CkU\tau[1 - S(z - h - Ct)]$ in the x-direction. The fluid is accelerated to the velocity $CkU\tau$ by M_{xz} as the wave front $z - h = Ct$ sweeps by and the x-component of the field falls to zero. Behind the wave front, the field is just B in the z-direction and the fluid moves toward positive x with the uniform velocity $CkU\tau$. In the lower half space the motion is in the opposite direction, of course. Thus blocking the electric current at $z = \pm h$ has merely the effect of reconnecting the currents across the propagating wave front and converting the energy of b into the kinetic energy of the fluid velocity v.

The electric field $\boldsymbol{E} = -\boldsymbol{v} \times \boldsymbol{B}/c$ in the region of moving fluid becomes $E_y = BkU\tau(C/c)$ in the y-direction for $z \geq h$. The electrostatic potential on the surface $z = h$ is $-yE_y$. Thus, if we assume that there is zero potential difference across the resistive slab $-h < z < +h$ at $y = 0$, the potential difference is $2aE_y$, or $2aBkU\tau(C/c)$ at $y = +a$. The potential difference is determined entirely by local conditions and has nothing to do with the unlimited potential difference between $z = -h$ and $z = +h$ imagined by the electric circuit analog with the unlimited inductance of an infinitely long electric current path.

The essential point is that the electric circuit analog is deceptive and, as commonly conceived, provides false results. There remains the possibility, of course, of solving the dynamical problem with the equations deduced from Newton, Maxwell, and Lorentz, and then constructing, after the fact, some sort of electric circuit analog that exhibits the same time dependence. But nothing scientific is to be gained by such an exercise.

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