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The Gravitational Effects of a Celestial Body with Magnetic Charge and Moment

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Abstract The gravitational effects (precession of charge-less particles and deflection of light) in the gravitational field of a celestial body with magnetic charge and moment (CM) are investigated. We found that the magnetic charge always weakens the pure Schwarzschild effects, while the magnetic dipole moment deforms the effects in a more complicated way.

Key words: gravitational field – geodesic motion – gravitational effects

The metric of a celestial body with magnetic charge and moment is

$$g_{00} = 1 - \frac{\alpha m}{r} + \frac{k q_m^2}{r^2} + \frac{k\alpha^2 p^2 \cos^2\theta}{r^4}\,, \tag{1}$$

$$g_{11} = -\left(1 - \frac{\alpha m}{r} + \frac{k q_m^2}{r^2} + \frac{k\alpha^2 p^2 \cos^2\theta}{r^4}\right)^{-1}\,, \tag{2}$$

$$g_{22} = -r^2\left(1 - \frac{k\alpha^2 p^2 \cos^2\theta}{r^4}\right)\,, \tag{3}$$

$$g_{33} = -r^2\left(1 - \frac{k\alpha^2 p^2 \cos^2\theta}{r^4}\right)\sin^2\theta\,. \tag{4}$$

The other components being zero, where $k = G/c^4$, $m = GM/c^2$, and q_m, p, M are the magnetic charge, magnetic moment and mass, respectively, and

$$\alpha = \frac{\sqrt{|1 - k q_m^2/m^2|}}{1 - m r^{-1}\left(1 - \sqrt{|1 - k q_m^2/m^2|}\right)}\,. \tag{5}$$

The accelerating effect of the CM field was investigated in Wang & Tang (1986). We will investigate the perihelion effect on charge-less particles and the deflection of light in the CM field.

The geodesic equation of the charge-less particle and photon in the CM field is

$$\frac{d}{d\lambda}\left(g_{\mu\nu}\dot{x}^\nu\right) = \frac{1}{2} g_{\nu\tau,\mu}\dot{x}^\nu \dot{x}^\tau \tag{6}$$

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and it satisfies

$$g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu = \eta \equiv \begin{cases} 0, & \text{(photon)} \\ 1, & \text{(particle)} \end{cases} \tag{7}$$

The CM field is a symmetric, stationary, radiation field, its metric does not contain t and φ, so

$$\frac{d}{d\lambda}\left(g_{0\nu}\dot{x}^\nu\right) = 0\,, \qquad \frac{d}{d\lambda}\left(g_{3\nu}\dot{x}^\nu\right) = 0\,,$$

and we obtain two integrals of motion,

$$g_{00}\dot{t} = -\varepsilon, \qquad g_{33}\dot{\varphi} = h\,. \tag{8}$$

Confining ourselves to the equator, $\theta = \pi/2$, $\dot{\theta} = 0$, from (7), we have

$$g_{00}\dot{t}^2 + g_{11}\dot{r}^2 + g_{33}\dot{\varphi}^2 = \eta\,.$$

Substituting (8) into the above equation, we have

$$g_{11}\dot{r}^2 = \eta - \varepsilon^2/g_{00} - h^2/g_{33}\,. \tag{9}$$

Making the transformation $r = \frac{1}{u'}u' = \frac{du}{d\varphi}$,

$$\dot{r}^2 = \left(\frac{dr}{du}\right)^2 u'^2\dot{\varphi}^2 = \frac{1}{u^4}u'^2\frac{h^2}{g_{33}^2}$$

and putting it into (9), we have

$$u'^2 = \frac{u^4 g_{33}}{g_{11}}\left[(\eta - \frac{g_{33}}{h^2}) - 1\right]\,. \tag{10}$$

For $\theta = \pi/2$, the metric becomes

$$\begin{aligned} g_{00} &= 1 - 2mu + Qu^2\,, \\ g_{11} &= -\left(1 - 2mu + Qu^2 + pu^4/2\right)^{-1}\,, \\ g_{22} &= g_{33} = -u^{-2}\,, \end{aligned}$$

where, $Q = kq_m^2, P = k\alpha^2p^2, \alpha \approx \sqrt{|1 - Qm^{-2}|}$, so (10) becomes

$$u' \approx \frac{\varepsilon^2 - \eta}{h^2} + \frac{\alpha\eta m}{h^2}u - u^2 - \frac{\eta Q}{h^2}u^2 + \alpha m u^3 + \left(\frac{\varepsilon^2 - \eta}{2h^2}P - Q\right)u^4 + \frac{\varepsilon^2}{h^2}mPu^5 - \frac{1}{2}P\left(\frac{\varepsilon^2}{h^2}Q + 1\right)u^6\,.$$

Differentiating the above equation with respect to φ, we have

$$u'' + u - \eta\frac{m}{h^2} = -\eta\frac{Q}{h^2}u + 3mu^2 + \left(\frac{\varepsilon^2 - \eta}{h^2}P - 2Q\right)u^3 + \frac{5\varepsilon^2}{h^2}mPu^4 - \frac{3}{2}P\left(\frac{\varepsilon^2}{h^2}Q + 1\right)u^5\,. \tag{11}$$

(1) Effect on the Pericenter of a Charge-less Particle

For the particle, $\eta = 1$, and (11) becomes

$$u'' + u - \frac{m}{h^2} = -\frac{Q}{h^2}u + 3mu^2 + Au^3 + Bu^4 - Cu^5\,, \tag{12}$$

where $A \equiv \frac{\varepsilon^2-1}{h^2}P - 2Q$, $B \equiv \frac{5\varepsilon^2}{\alpha h^2}mP$, $C \equiv \frac{3}{2}P\left(\frac{\varepsilon^2}{h^2}Q + 1\right)$. When the right hand side of (12) is zero, we have

$$u = \frac{m}{h^2}\left(1 + e\cos\varphi\right) .$$

Consider the right hand side of (12) as small quantity, and use perturbation method to solve the equation:

$$u'' + u = \frac{m}{h^2}\left[1 + eR\cos\varphi + S\left(\varphi\right)\right] . \tag{13}$$

where $S\left(\varphi\right)$ is a constant term, and

$$R = \frac{1}{h^2}\left[-Q + 6m^2 + \frac{3Am^2}{4h^2}\left(4 + e^2\right) + \frac{Bm^2}{h^4}\left(4 + 3e^2\right) - \frac{5Cm^4}{8h^6}\left(8 + 12e^2 + e^4\right)\right] .$$

Substituting the value of A, B, C into the above equation, we obtain

$$R = \frac{6m^2}{h^2}\left(1 - C_1Q + C_2P - C_3PQ\right) ,$$

where

$$\begin{aligned} C_1 &= \tfrac{1}{6m^2} + \tfrac{4+e^2}{4h^2} , \\ C_2 &= \tfrac{5m^2\varepsilon^2}{32h^8} , \\ C_3 &= \tfrac{1}{12h^4}\left[\tfrac{3}{2}(\varepsilon^2 - 1)\left(4 + e^2\right) + \tfrac{5m^2}{h^2}\left(4\varepsilon^2 + 3e^2\varepsilon^2 - 3 - 9e^2/2 - 3e^4/8\right)\right] . \end{aligned}$$

The solution of (13) is

$$u = \frac{m}{h^2}\left[1 + e(\cos\varphi + \frac{R}{2}\varphi\sin\varphi) + S'(\varphi)\right] , \tag{14}$$

where $S'\left(\varphi\right)$ is constant. So after the particle has gone once round, the pericenter angle becomes

$$\Delta = \pi R = \frac{6\pi m^2}{h^2}\left(1 - C_1Q + C_2P - C_3PQ\right) . \tag{15}$$

Discussion: The term $\frac{6\pi m}{h^2}$ comes from the Schwarzschild mass; the other terms show that the magnetic charge makes the pericenter angle smaller; while the effect of the magnetic moment is not clear cut.

(2) Deflection of Light

For the photon, $\eta = 0$ and (11) reduces to

$$u'' + u = 3mu^2 + A'u^3 + Bu^4 - Cu^5 , \tag{16}$$

where $A' \equiv \frac{\varepsilon^2}{h^2}P - 2Q$. We shall again use the perturbation method to solve the equation. Its first approximation is $u = \sin\varphi/R_0$. Put it into (16) and we have

$$u'' + u = \lambda_0 + \lambda_1\sin\varphi - \lambda_2\cos 2\varphi - \lambda_3\sin 3\varphi + \lambda_4\cos 4\varphi - \lambda_5\sin 5\varphi ,$$

where

$$\lambda_0 = \frac{3m}{2R_0^2} + \frac{3B}{8R_0^4}, \qquad \lambda_1 = \frac{3A'}{2R_0^3} + \frac{5C}{8R_0},$$

$$\lambda_2 = \frac{3m}{2R_0^2} + \frac{B}{2R_0^4}\,, \qquad \lambda_3 = \frac{3A'}{4R_0^3} + \frac{5C}{16R_0^5}\,,$$
$$\lambda_4 = \frac{B}{8R_0^4}\,, \qquad \lambda_5 = \frac{C}{16R_0^5}\,.$$

So the second approximation is

$$u = \sin\varphi/R_0 + \lambda_0 - \frac{\lambda_1}{2}\varphi\cos\varphi + \frac{\lambda_2}{3}\cos 2\varphi + \frac{\lambda_3}{8}\sin 3\varphi - \frac{\lambda_4}{15}\cos 4\varphi + \frac{\lambda_5}{24}\sin 5\varphi\,.$$

Writing $\varphi_1 = -\delta_1$ and $\varphi_2 = \pi + \delta_2$, $u = 0$ in the above equation and we have

$$\delta_1 = \frac{\lambda_0 + \lambda_2/3 - \lambda_4/15}{1/R_0 - \lambda_1/2 + 3\lambda_3/8 + 5\lambda_5/24}, \quad \delta_2 = \frac{\lambda_0 + \pi\lambda_1/2 + \lambda_2/3 - \lambda_4/15}{1/R_0 - \lambda_1/2 + 3\lambda_3/8 + 5\lambda_5/24}\,.$$

So the total deflection is

$$\delta = \delta_1 + \delta_2 \approx 2R_0\left(\lambda_0 + \lambda_2/3 - \lambda_4/15 + \pi\lambda_1/4\right)\,.$$

Putting the values of $\lambda_i (i = 0, 1, 2, 4,)$ and A', B, C into the above equation, we have

$$\delta = \frac{4m}{R_0}\left(1 - d_1 Q + d_2 P - d_3 PQ\right)\,, \tag{17}$$

where

$$d_1 = \frac{3\pi}{16mR_0}\,, \qquad d_2 = \frac{15\pi\varepsilon^2}{128mR_0^3h^2}\,, \qquad d_3 = \frac{2\varepsilon^2}{3R_0^3h^2} + \frac{3\pi}{32mR_0}\left(\frac{\varepsilon^2}{h^2} - \frac{5}{4R_0^2}\right)\,.$$

For the photon, $h \to \infty, \varepsilon \to \infty, \varepsilon/h \approx 1/R_0$, so we have

$$d_1 = \frac{3\pi}{16mR_0}, d_2 = \frac{1}{R_0^3}\left(\frac{2}{3R_0} - \frac{3\pi}{128m}\right), d_3 = \frac{15\pi}{128mR_0^5}.$$

Discussion: In (17), the term $\frac{4m}{R_0}$ comes from the Schwarzschild mass; the magnetic charge makes the deflection of light smaller; when there is no magnetic moment, there is no deflection for $Q = 16mR_0/3\pi$, and light is not attracted, rather, it is repelled for $Q > 16mR_0/3\pi$ —this result is different from the Schwarzschild case. Exceptionally when $d > 0\left(\frac{2m}{R_0} > \frac{9\pi}{128} \approx 0.22\right)$, we cannot decide whether the magnetic moment will make the deflection of light smaller or not, because there is the interaction term of magnetic charge and moment. If $2m/R_0 < 0.22$, it is obvious that the magnetic moment makes deflection of light smaller. If there are no magnetic charges, then similarly, light is repelled.

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