Chin. J. Astron. Astrophys. Vol. 1, No. 1, (2001) 29–38
(*http://www.chjaa.org* or *http://chjaa.bao.ac.cn*)

Chinese Journal of
Astronomy and
Astrophysics

Testing the Universality of the TIS Model on Cluster Scales from the X-ray Surface Brightness Profiles

Yu-Ying Zhang [⋆]

Beijing Astronomical Observatory and National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012

Received 2000 November 21; accepted 2001 January 3

Abstract The truncated isothermal sphere (TIS) model has been recently suggested as an alternative for virialized dark halos (Shapiro et al. 1999). Both its profound theoretical motivation and its successful explanations for the galactic rotation curves and the gravitational scaling laws of clusters indicate that the TIS model is a promising candidate among other prevailing models such as the NFW profile and the Burkert profile. This promotes us to re-examine the universality of the TIS model on cluster scales from a different angle. Using an ensemble of X-ray surface brightness profiles of 45 clusters, we test the goodness of fit of the TIS predicted gas distributions to the X-ray data under the assumption of isothermal, hydrostatic equilibrium. Unlike the conventional β model or the NFW/Burkert profile, for which about half of the clusters have the reduced χ^2_ν values smaller than 2, the TIS model fails in the fitting of the X-ray surface brightness profiles of clusters in the sense that 38 out of the 45 clusters show $\chi^2_\nu > 2$. This may constitute a challenge for the universality of the TIS model unless the present analysis is seriously contaminated by other uncertainties including the negligence of non-gravitational heating processes and the unconventional sampling of the X-ray data.

Key words: cosmology: theory — dark matter — galaxies: clusters: general — X-rays: galaxies

1 INTRODUCTION

Typical CDM models turn out to be a great success at explaining the origin and evolution of cosmic structures on large-scales (> 1 Mpc). However, on small scales their predictions seem to be in conflict with a number of independent observations. Among these the disagreement between the cusped central density profile suggested by Navarro, Frenk & White (1995; here after NFW) from high-resolution simulations and the shallow matter cores detected in low surface brightness galaxies, dwarf galaxies and galaxy clusters (e.g., Flores & Primack 1994; de Blok & McGaugh 1997; Tyson, Kochanski & Dell'Antonio 1998) has triggered many investigations both theoretical and observational. A naive speculation is to empirically modify the analytic,

---

[⋆] E-mail: zyy@class2.bao.ac.cn

elegant form of the NFW universal profile. A competing candidate in literature is the Burkert profile (Burkert 1995, 2000; Salucci & Burkert 2000):

$$\rho_{\rm DM}(r) = \frac{\rho_0 r_0^3}{(r + r_0)(r^2 + r_0^2)}, \tag{1}$$

where ρ_0 and r_0 are the central density and the scale length, respectively. Indeed, such a density law resembles an isothermal profile in the inner region with a constant core, while in the outer region the mass profile diverges logarithmically with radius, in agreement with the NFW profile. On galactic scales, the Burkert profile fits fairly well the dark matter halo density distributions of dwarfs and spirals derived from their rotation curves (Kravtsov et al. 1998; Salucci & Burkert 2000). On cluster scales, it has been shown recently that the X-ray surface brightness profiles predicted by Burkert profile can be reasonably approximated by the conventional β model (Wu & Xue 2000). Yet, the physical mechanism behind the Burkert profile remains unclear.

A definite solution to the problem requires a better understanding of the nature of dark matter particles. For instance, if the CDM particles are self-interacting as suggested by Spergel & Steinhardt (2000), the collisional CDM particles may eventually eliminate the central cusp of virialized dark halos. Another less vigorous but helpful approach to the issue is to semi-analytically study the dynamical solutions to the postcollapse of dark halos developed from top-hat perturbation in an expanding universe, incorporated with the well-motivated physical mechanisms (e.g., Fillmore & Goldreich 1984; Bertschinger 1985; Teyssier, Chièze & Alimi 1997; Shapiro, Iliev & Raga 1999; Chiueh & Wu 2000). Regardless of their oversimplifications, these semi-analytic models have successfully predicted the matter distribution and evolution of virialized dark halos that are in overall consistency with observations over a broad mass range from galaxies to rich clusters. Indeed, they complement in some way the sophisticated treatment based on high-resolution numerical simulations. In this paper, we concentrate on the truncated isothermal sphere (TIS) model proposed by Shapiro et al. (1999) from a particular solution of the Lane-Emden equation derived from the postcollapse and virialization of a top-hat density perturbation. The primary reason why we choose the TIS model is that its density profile and resulting rotation curve are essentially indistinguishable from those given by the Burkert profile (Iliev & Shapiro 2000). In this regard, the TIS model could provide a theoretical motivation for the empirical Burkert profile.

The requirement of universality over the entire mass range constitutes a challenge to any theoretically, numerically or empirically motivated density profiles of virialized dark halos. The TIS model has been well tested on galactic scales using the rotation curves of dwarf galaxies and low surface brightness galaxies (Shapiro et al. 1999; Iliev & Shapiro 2000). These authors have also obtained analytically the mass-temperature and radius-temperature scaling laws for clusters which match fairly well the results from numerical simulations by Evrard, Metzler & Navarro (1996). In particular, it has been shown that the TIS model has reproduced the X-ray surface brightness profiles of the simulated clusters but failed to reconcile with the X-ray observations because of the large β value ($\beta = 0.9$) (Shapiro & Iliev 2000). Although extending the observed X-ray surface brightness profiles and excluding the cooling flow regions in the β model fitting or adopting the double β model fitting can moderately raise the β parameter (e.g., Mohr, Mathiesen & Evrard 1999 and MME hereafter; Vikhlinin et al. 1999; Xue & Wu 2000), it is unlikely that the large β discrepancy between the TIS prediction and X-ray observations, if real, can be resolved simply by the employment of different fitting techniques.

In this paper we would like to make a close examination of the universality of the TIS model on cluster scales. To this end, we fit the TIS predicted X-ray surface brightness profiles to an ensemble of X-ray observations of clusters. We will check the goodness of fit and compare it with other models such as the conventional β model, the predictions of the NFW profile and of the Burkert profile. Meanwhile, such an exercise allows us to completely fix the free parameters in the TIS model and identify properties common to the NFW, Burkert and TIS models. One might have thought, before one proceeds to the detailed comparisons, that the TIS model should provide a result similar to the ones predicted by the other models. However, we would like to point out that the situation of the TIS model may be essentially different. This can be easily seen from the density dependence of r^{-2} at large radii, which differs remarkably from the well-known asymptotic behavior of r^{-3} for other models. Throughout this paper, we assume $H_0 = 50$ km s^{-1} Mpc^{-1} and $\Omega_0 = 1$.

2 DENSITY PROFILE OF INTRACLUSTER GAS

By fitting the numerical solution of the Lane-Emden equation for isothermal sphere, Shapiro et al. (1999) provide a good approximation of analytic formula for the density profile of virialized dark halos:

$$\rho_{\rm DM}(r) = \rho_0 \left[\frac{A}{a^2 + (r/r_0)^2} - \frac{B}{b^2 + (r/r_0)^2}\right], \tag{2}$$

where $(A, a^2, B, b^2) = (21.38, 9.08, 19.81, 14.62)$, and ρ_0 and r_0 are the central density and scale length, respectively.

Apart from a finite central core radius, the significant difference of the TIS model from the NFW and Burkert profiles is its asymptotic mass behavior of $M(r) \propto r$ at large radii. In a sense, the TIS can be analogous to the mass distribution derived from the β model for intracluster gas or the King model for cluster galaxies under the isothermal, hydrostatic equilibrium hypothesis. On the other hand, assuming that the gas is in isothermal, hydrostatic equilibrium with the underlying gravitational potential of the cluster described by TIS, we will be able to predict how the intracluster gas is distributed inside clusters. The result can be directly compared with X-ray observations. A similar exercise has been made by Makino, Sasaki & Suto (1998) for the NFW profile and by Wu & Xue (2000) for the Burkert profile. That is, the gas density $n_{\rm gas}$ and temperature T should satisfy the following equation

$$\frac{GM_{\rm DM}(x)}{x^2} = -\frac{kTr_0}{\mu m_p n_{\rm gas}(x)} \frac{dn_{\rm gas}(x)}{dx}, \tag{3}$$

in which we have neglected the self-gravity of gas and approximated the total cluster mass by $M_{\rm DM}$:

$$M_{\rm DM}(r) = 4\pi\rho_0 r_0^3 m(x) \tag{4}$$

$$m(x) = (A - B)x - Aa \arctan\frac{x}{a} + Bb \arctan\frac{x}{b}, \tag{5}$$

where $x = r/r_0$. As a result, the gas number density can be solved analytically

$$\frac{n_{\rm gas}(x)}{n_{\rm gas}(0)} = e^{\frac{\alpha_0 m(x)}{x}} \left(\frac{a^2}{a^2 + x^2}\right)^{\frac{A\alpha_0}{2}} \left(\frac{b^2 + x^2}{b^2}\right)^{\frac{B\alpha_0}{2}}. \tag{6}$$

In Fig. 1 we illustrate the normalized gas profile $n_{\rm gas}/n_{\rm gas}(0)$ for a typical choice of $\alpha_0 = 0.744$ (see below), together with a conventional β model fit with $\beta = 0.59$ and $r_0/r_c = 0.44$. For the purpose of comparison, we have also plotted the typical gas density profiles predicted by the NFW profile and the Burkert profile, respectively. Although there are remarkable differences among the theoretical predictions especially at the central region, the β model seems to provide an acceptable fit to all the three curves within a few tens of scale length (Makino et al. 1998; Wu & Xue 2000). However, unlike the gas distributions predicted by the NFW and Burkert profiles, which become divergent at $r \to \infty$, the TIS predicted gas density profile naturally vanishes at $r \to \infty$. This avoids the introduction of an arbitrary cutoff radius or a subtraction of the gas background.

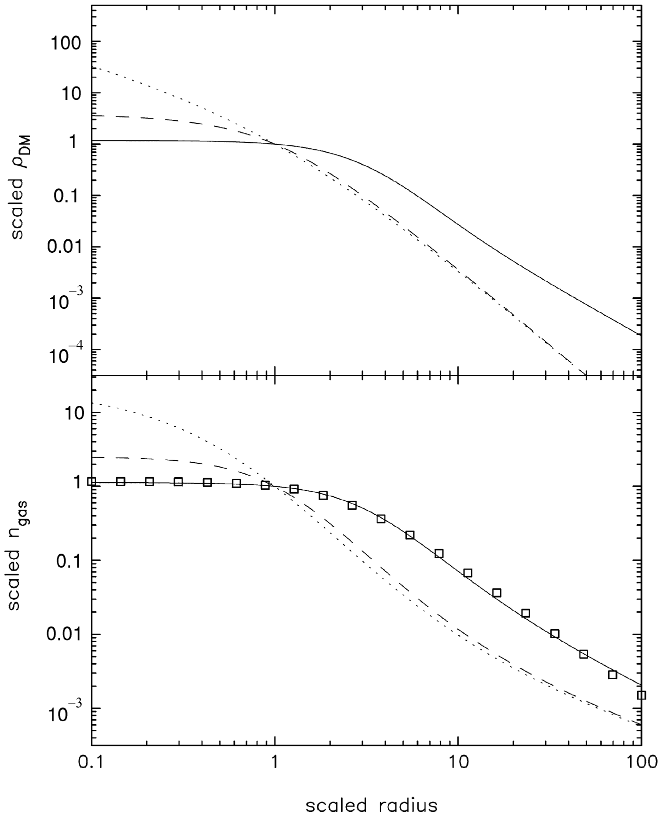

Fig. 1 Upper panel: A comparison of the scaled density profiles of the TIS model (solid line), the NFW profile (dotted line) and the Burkert profile (dashed line). All curves are normalized at the corresponding length scales. Lower panel: The predicted radial gas profiles for TIS, NFW and Burkert models. Superimposed (open cross) is the β model fit to the TIS prediction.

3 APPLICATION TO X-RAY CLUSTERS

In order to link the theoretically predicted radial profile of intracluster gas with X-ray observation, we work with the X-ray surface brightness profiles of clusters in the framework of an optically thin, isothermal plasma emission mechanism, namely,

$$S_{\rm X}(x) \propto \int_x^\infty n_{\rm gas}^2 d\ell, \tag{7}$$

where the integral is performed along the line of sight. Our task is thus to test the goodness of fit of the TIS predicted $S_{\rm X}$ to the X-ray observed $S_{\rm X}$. Similar to the previous work by Xue & Wu (2000) and Wu & Xue (2000), we use the ROSAT PSPC surface brightness profiles of 45 nearby clusters analyzed by Mohr, Mathiesen & Evrard (1999; here after MME). Nevertheless, we exclude the data points in the central region of 0.05 Mpc in each cluster to reduce possible influence of cooling flows, while we keep the same outer radius as that defined by MME. Essentially, the fitting can be classified as two types, and a typical example for each type is shown in Fig. 2(a) and (b), respectively. In the first case, the TIS model results in an excellent fit to the entire data points of the observed $S_{\rm X}$, which is reflected by the significantly reduced χ^2_ν value. In the second case, the fit is apparently not acceptable, indicated by both the large χ^2_ν value and the large dispersion in the residuals. It appears that 7 out of the 45 clusters have the χ^2_ν values smaller than 2, while the majority of the clusters cannot be well fitted by the TIS predicted X-ray surface brightness profiles. This compares with the fractions of 26/45, 25/45 and 22/45 for the similar fits using the β, NFW and Burkert models, respectively (Wu & Xue 2000).

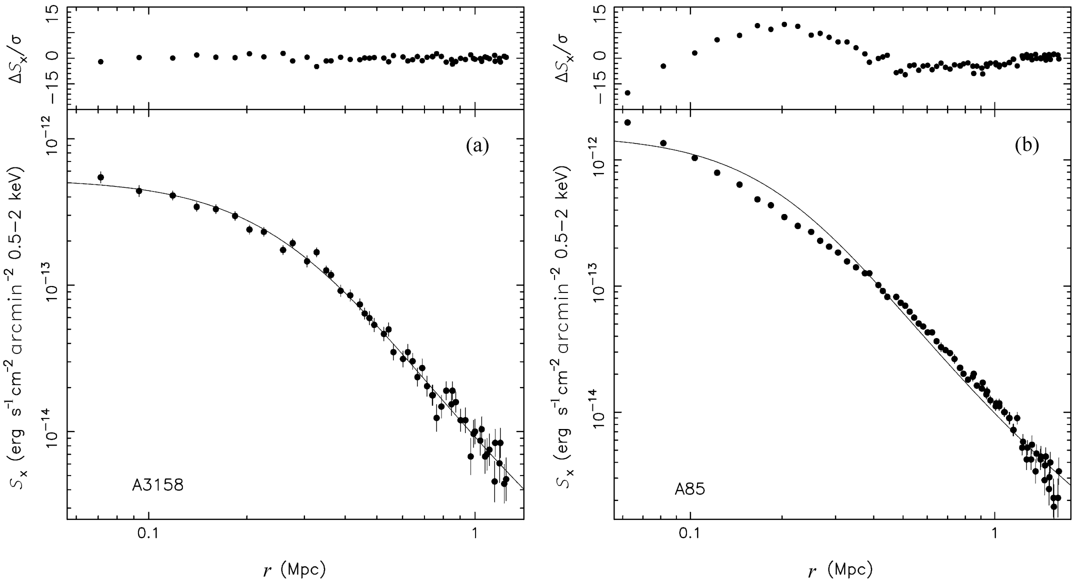

Fig. 2 Two examples of the observed and predicted X-ray surface brightness profiles of clusters. Residuals between the best-fits of the TIS predictions and the X-ray data are displayed in the top panels.

Irrespective of the failure in the χ^2 fit of the TIS prediction to most of the X-ray surface brightness profiles of MME clusters, the above procedure allows us to work out the two free parameters α_0 and r_0 for each cluster. Furthermore, combining with the gas temperature T obtained by X-ray spectral measurement, we will be able to fix the central density ρ_0 in the TIS profile. We take the cooling-flow corrected temperature data from White (2000) where available, and for the remaining clusters we use the X-ray temperatures from the compilation of Wu, Xue & Fang (1999).

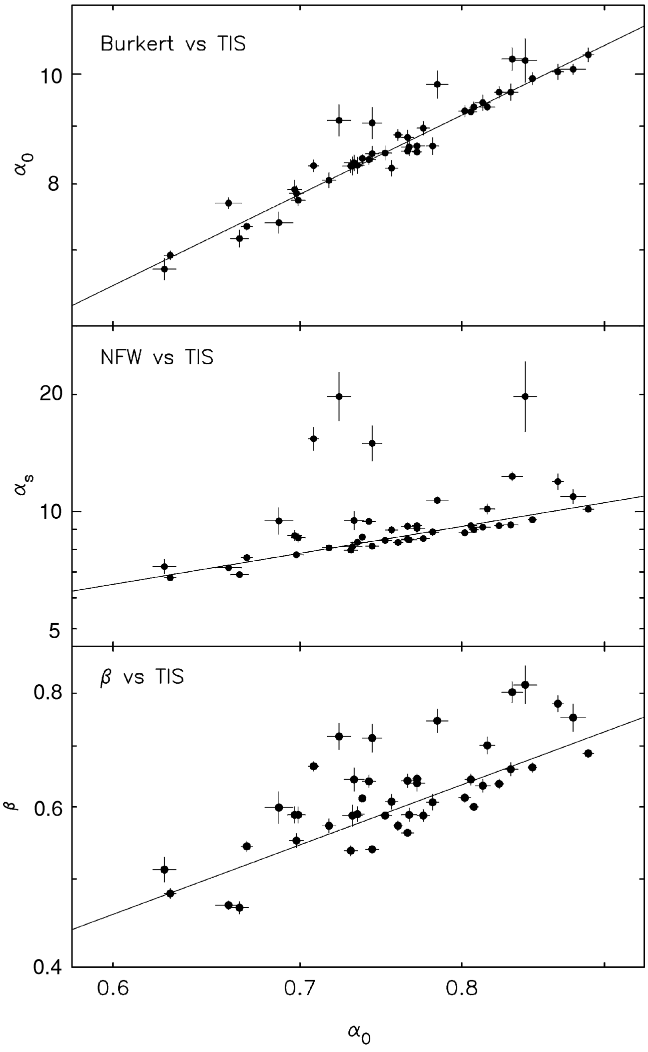

Fig. 3 Correlations between α_0 in the gas profile predicted by TIS model and the power-law index of gas distribution described by β model or predicted by NFW profile and Burkert profile, respectively.

4 COMPARISON WITH OTHER MODELS

Unlike the conventional β model or the NFW or Burkert profiles, the TIS model may become questionable in terms of the goodness of fit to the X-ray surface brightness profiles alone. This may throw doubt on the speculation that this model could act as a theoretical motivation for the Burkert profile (Iliev & Shapiro 2000). However, reasonable caution should be exercised on whether or not the X-ray surface brightness profiles of the MME clusters can be used to definitely rule out the TIS model because MME binned their S_X using photon counts rather than the conventional concentric rings of equal width. Such a 'super-resolution' method may result in rather a large value of χ^2_ν. Recall that nearly half of the clusters can be fitted by none of the existing models such as the β model, the double β model, and the predictions by NFW/Burkert profiles (MME; Xue & Wu 2000; Wu & Xue 2000). On the other hand, if we leave the goodness of fit aside, the power-law index α_0, the central density ρ_0 and the core radius r_0 in the TIS model are strongly correlated with the corresponding quantities in other models (see Fig. 3–Fig. 5), in which all the parameters for the other models are taken from Wu & Xue (2000). The best-fit power-law relations and the linear fits, along with the 68% confidence limits, are summarized in Table 1.

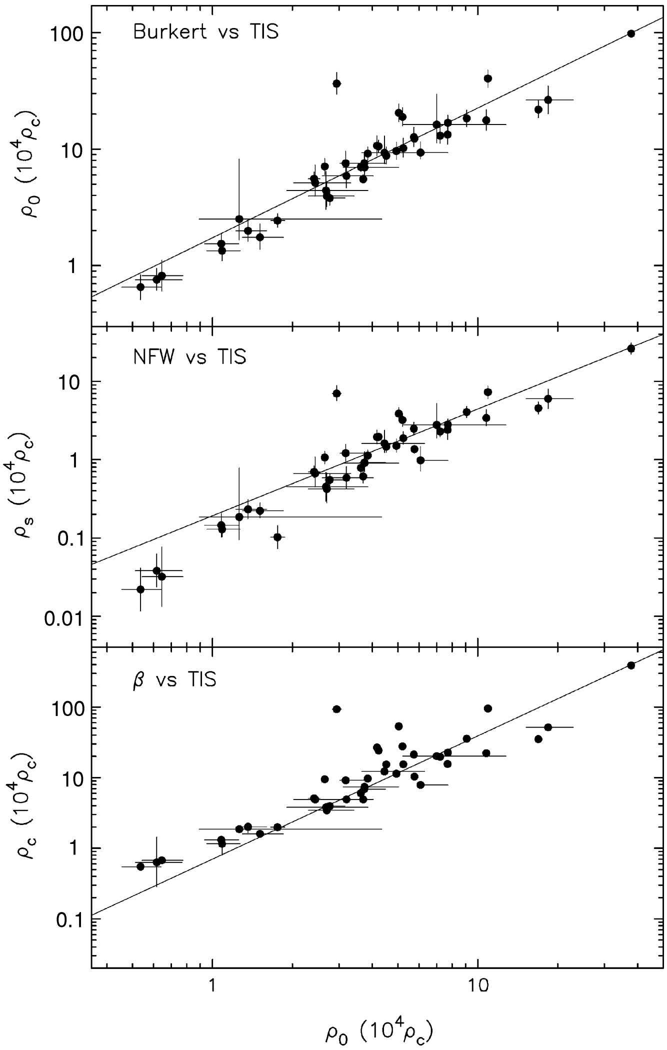

Fig. 4 The same as Fig.3 but for correlations between characteristic density parameters

Table 1 Best-fit Correlations[a]

TIS		β model[b]	NFW[c]	Burkert[d]
α_0	A	0.077 ± 0.009	-0.905 ± 0.021	-0.900 ± 0.009
	B	0.879 ± 0.036	0.840 ± 0.023	0.832 ± 0.010
	C	1.229 ± 0.105	0.082 ± 0.014	0.087 ± 0.003
ρ_0	A	0.088 ± 0.033	0.526 ± 0.009	-0.215 ± 0.015
	B	0.573 ± 0.013	0.733 ± 0.022	0.898 ± 0.011
	C	0.502 ± 0.268	5.224 ± 4.996	0.542 ± 0.172
r_0	A	-0.590 ± 0.006	-0.933 ± 0.004	-0.545 ± 0.007
	B	0.603 ± 0.009	0.621 ± 0.011	0.783 ± 0.011
	C	0.632 ± 0.341	0.158 ± 0.072	0.423 ± 0.121

[a] The correlations are from χ^2 fits of the form $Y = 10^A X^B$ or $Y = CX$;

[b] The power-law index, length scale and central density are β, r_c and $\rho_0 = 9\beta kT/4\pi G\mu m_p r_c^2$;

[c] The power-law index, length scale and characteristic density are α, r_s and ρ_s;

[d] The power-law index, core scale and central density are α_0, r_0 and ρ_0.

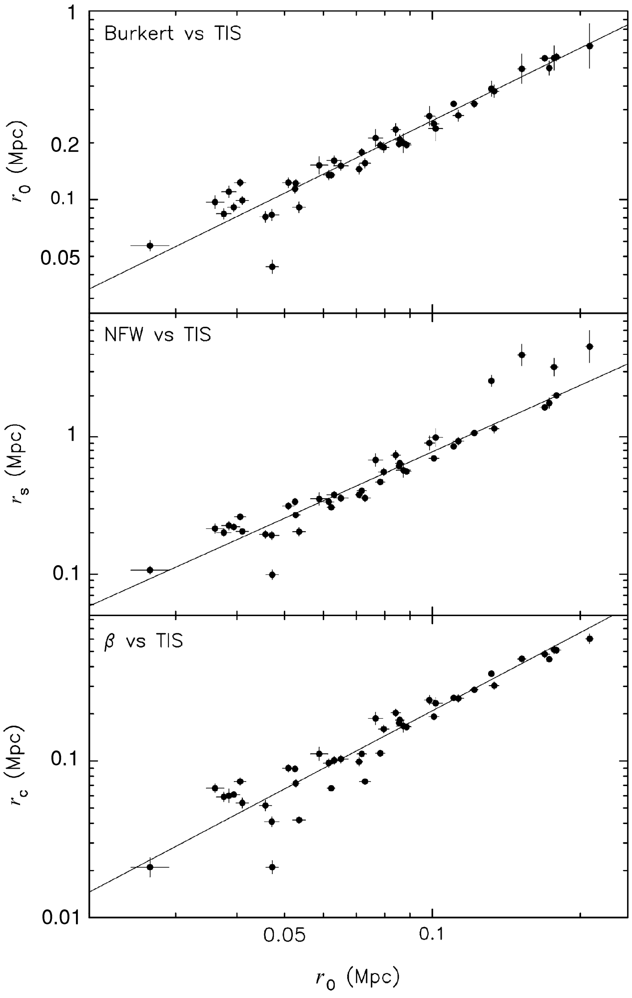

Fig. 5 The same as Fig.3 but for correlations between length scales

Perhaps, the most interesting result is the core radius distribution of the TIS model obtained from the fittings of the 45 clusters (Fig.6). Recall that the study of gravitational lensing by clusters of galaxies has placed a tight constraint on the core sizes of dark matter distributions, which should not exceed $\sim$ 100 kpc (Hammer 1991; Wu & Hammer 1993; Grossman & Saha 1994; Miralda-Escudé 1995; Tyson et al. 1998; etc.). It appears that the histogram of the resulting core radii is peaked at r_0 = 0.04–0.06 Mpc, and 76% of the clusters have their core radii smaller 0.1 Mpc. This result is in fairly good agreement with the lensing analysis when the possible selection effect is taken into account in the detection of strong lensing events. In particular, the core radii of all the clusters in the list are well within 0.21 Mpc. In contrast, the gas core radii span a broad range from 0.01 to 0.6 Mpc with a mean value of $r_c \approx 0.2$ Mpc. In other words, the dark matter distribution in the framework of the TIS model is more concentrated than the gas profile, a result favored by gravitational lensing and numerical simulations.

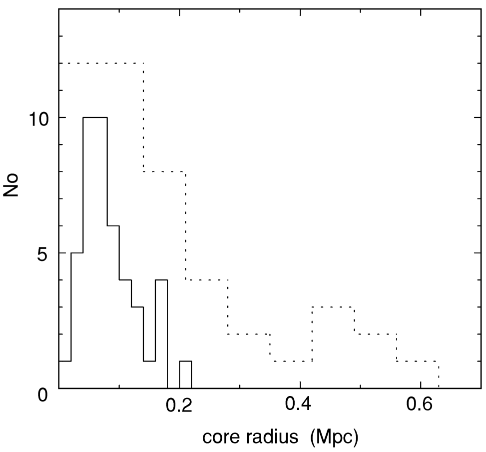

Fig. 6 Histogram of dark matter cores of 45 clusters derived from the TIS model (solid line). Also plotted is the distribution of gas radii fitted by the β model (dotted line) for comparison.

5 DISCUSSION AND CONCLUSIONS

Compared to the other empirically and theoretically proposed models, the TIS model does not return an equally good goodness of fit to the X-ray surface brightness profiles of clusters, contrary to our intuition at the outset. The possible reasons are as follows: Firstly, the X-ray cluster sample we have adopted may be inappropriate for such an analysis because of the unconventional way of binning photon counts, which may account for the large values of χ^2_ν, whereas the conventional way of measuring the X-ray surface brightness profile gives rise to too few data points to make the fitting. Secondly, the gas distribution may be contaminated by non-gravitational heating processes in the early phase of cluster formation. Energy injection into intracluster gas from galactic winds and AGNs would produce a shallower gas profile (e.g., David et al. 1990; Ponman, Cannon & Navarro 1999; Lloyd-David, Ponman & Cannon 2000). If we do not include this effect in the theoretical prediction of the gas density profile in the framework of hydrostatic equilibrium, it may be difficult to reproduce the the observed X-ray surface brightness profiles of clusters. Thirdly, the TIS model simply fails in the universality test on cluster scales, although it results naturally from the Lane-Emden equation for the postcollapse and virialization of a top-hat density perturbation and has successfully explained the rotation curves on galactic scales and the gravitational scaling laws on cluster scales (Shapiro & Iliev 2000).

The TIS model has an advantage over the NFW and Burkert profiles if we leave the problem aside that only a small fraction of clusters have passed the χ_ν test: the TIS core radii deduced from the X-ray surface brightness profiles are significantly smaller than the X-ray core radii, with a mean value of $r_0 \approx 0.07$ Mpc. This satisfies the constraint set by a number of independent studies from strong gravitational lensing (arc thinness, arc number statistics, modeling of arcs, etc.). Recall that the NFW profile shows an uncomfortable cusp at the central region, while the core radii in the Burkert profile determined from the fitting of the X-ray surface brightness profiles of clusters are unreasonably large ($r_0 \approx 0.2$ Mpc) (Xue & Wu 2001). So, as compared with these two models, the TIS model provides a reasonable, inner soft core in between.

Overall, it is premature to rule out the TIS model on the bias of the fitting of the theoretically predicted gas distribution to the X-ray observations, although about 84% of the X-ray clusters in the MME sample do not pass the χ^2_ν test. Actually, the two free parameters in the TIS model are strongly correlated with the corresponding parameters in other prevailing models (the β model, the NFW and Burkert profiles). Considering the fact that about half of the X-ray clusters have also failed in the χ^2_ν test for other models, we conclude that all these proposed models are indistinguishable at present. Since the intracluster gas can be disturbed by non-gravitational heating processes and dynamical activities (e.g., cooling flows, merging, etc.), use of the X-ray observations may provide a good estimate of the parameters for any proposed dark halo models but may not be an ideal tool for the purpose of precise calibrations.

Acknowledgments This work was supported by the National Natural Science Foundation of China, under Grant 19725311, and the Ministry of Science and Technology of China, under Grant NKBRSF G19990754.

References

Bertschinger E., 1985, ApJS, 58, 39
Burkert A., 1995, ApJ, 447, L25
Burkert A., 2000, ApJ, 534, L143
Chiueh T., Wu X.-P., 2000, A&A, 353, 822
David L. P., Arnaud K. A., Forman W., Jones C., 1990, ApJ, 356, 32
de Blok W. J. G., McGaugh S. S., 1997, MNRAS, 290, 533
Evrard A. E., Metzler C. A., Navarro J. F., 1996, ApJ, 469, 494
Fillmore J. A., Goldreich P., 1984, ApJ, 281, 1
Flores R. A., Primack J. R., 1994, ApJ, 427, L1
Grossman S. A., Saha P., 1994, ApJ, 431, 74
Hammer F., 1991, ApJ, 383, 66
Iliev I., T., Shapiro P. R., 2000 ApJ, submitted
Kravtsov A. V., Klypin A. A., Bullock J. S., Primack J. R., 1998, ApJ, 502, 48
Lloyd-Davies E. J., Ponman T. J., Cannon D. B., 2000, MNRAS, 315, 689
Makino N., Sasaki S., Suto Y., 1998, ApJ, 497, 555
Miralda-Escudé J., 1995, ApJ, 438, 514
Mohr J. J., Mathiesen B., Evrard A. E., 1999, ApJ, 517, 627 (MME)
Navarro J. F., Frenk C. S., White S. D. M., 1995, MNRAS, 275, 720 (NFW)
Ponman T. J., Cannon D. B., Navarro J. F., 1999, Nature, 397, 135
Salucci P., Burkert A., 2000, ApJ, 537, L9
Shapiro P. R, Iliev I. T., 2000, astro-ph/0003428
Shapiro P. R, Iliev I. T. Raga A. C., 1999, MNRAS, 307, 203
Spergel D. N., Steinhardt P. J., 2000, Phys. Rev. Lett., 84, 3760
Teyssier R., Chièze, J.-P., Alimi J.-M., 1997, ApJ, 480, 36
Tyson J. A., Kochanski G. P., Dell'Antonio I. P., 1998, ApJ, 498, L107
Vikhlinin A., Forman W., Jones C., 1999, ApJ, 525, 47
White D. A., 2000, MNRAS, 312, 663
Wu X.-P., Hammer F., 1993, MNRAS, 262, 197
Wu X.-P., Xue Y.-J., 2000, ApJ, 542, 578
Wu X.-P., Xue Y.-J., Fang L.-Z., 1999, ApJ, 524, 22
Xue Y.-J., Wu X.-P., 2000, MNRAS, 318, 715
Xue Y.-J., Wu X.-P., 2001, Int. J. Mod. Phy., in press