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Anisotropy and Dissipation of Turbulence and Their Effects on Solar Models

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Abstract Based on a dynamic model for turbulent convection, we investigate the effects of dissipation and anisotropy of the turbulence on the convective energy transport. We introduce two time scales to describe the dissipation of the turbulence, and approximate the anisotropy of the turbulence by Rotta's proposal of "return to isotropy". The improved turbulence model results in an equation to determine the temperature gradient in the convection zone, which is of similar form as that of the MLT. We apply the improved MLT to solar models, and find that the increases of the anisotropy and decreases of the dissipation of the turbulence reduce the value of the convection parameter α, because these processes enhance the convective energy transfer rate. Compared with the observed solar p-mode frequencies, it is plausible that the dissipation of the turbulence in the solar convection zone should be fairly strong, while the degree of anisotropy of the turbulence plays a less significant role on the structure of the solar convection zone.

Key words: Sun: interior – turbulent convection — Sun: oscillations

1 INTRODUCTION

The mixing-length theory (MLT) is the most commonly used approach to calculate convective energy transport in stars and other astrophysical situations. Based on the original idea of Prandtl (1952) that turbulent parcels transfer heat in a similar way as molecules of gas do in thermal conduction, the MLT assumes that convection cells, drived by buoyancy, move through a mixing length l and release the heat they carry when they merge with their environment. The most widely adopted formulation of the MLT was developed by Böhm-Vitense (1953, 1958), who took the effect of radiative cooling of the convection cells into account. This modification greatly decreases the efficiency of the convective heat transport in optically thin region, and has been found to give results that agree better with the observations (Cox and Giuli 1968; Kippenhahn and Weigert 1990; Huang and Yu 1998).

Although the MLT is widely applied in various astrophysical problems, its treatment on convective motion is undoubtedly oversimplified. For example, convection eddies have sizes of a wide range. The largest eddies are generated by dynamical instability, and break up into

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smaller and smaller ones to form some kind of energy spectrum of convective motion. The MLT considers in contrast only the averaged eddies and assumes that they are all of the same size of the mixing length l. Canuto and Mazzitelli (1991) developed an approach for an isotropic convection to include the contribution of convection cells of all sizes. It is known also that a fluctuating pressure gradient has the effect of extracting energy from the direction of the buoyancy to drive flows in other directions and so causing the turbulence to develop towards isotropy, but the MLT assumes instead that all eddies move along the direction of the buoyancy. The effects of the tendency of turbulent convection towards isotropy are discussed by Canuto (1993, 1994, 1997) and Xiong et al. (1997). In addition, the MLT usually attributes the dissipation of the turbulence entirely to the annihilation of the eddies and does not consider in detail the balance of the production and dissipation of the turbulence (Gough 1977).

Advanced treatments of convection, which are based on the hydrodynamics of turbulent motion, have been proposed in the past decades. Gough (1977) generalized the MLT to a time-dependent theory, and discussed the MLT as a dimensional solution of a set of linearized equations of hydrodynamics. Xiong (1977, 1989) and Xiong et al. (1997) developed a consistent theory for turbulent convection and found the steady state solution of his time-dependent convection theory to be consistent with the MLT. Kuhfuß (1986) proposed a turbulent convection model that only employs one differential equation to describe the variation of the turbulent kinetic energy. In a series of comprehensive investigations, Canuto proposed more advanced turbulent convection models (Canuto 1992, 1993, 1994, 1997, 1998). Many turbulence properties, such as the entire energy spectrum over all wave-lengths, the generation and dissipation of turbulence, and the isotropization, can then be carefully dealt with in the models, but as a disadvantage many new differential equations which are not easy to solve have to be introduced and incorporated into the problem of stellar evolution. On the other hand, the MLT is simple in form and is still widely used in stellar modelling, for instance in many ingenious solar models. It is therefore desirable to incorporate the recent progress of the turbulent models into the MLT and to find out the effects of the improvements.

In the present paper we investigate the effects of some turbulence properties on the MLT. The turbulence model we used is introduced in Section 2 with some details of discussion on the model approximations employed to close the dynamical equations of turbulence. A mixing-length type solution is found in Section 3 by the use of a quantity f, which is related to the efficiency of the convective energy transport. We apply the improved MLT in solar models in Section 4, and summarize the main conclusions in Section 5.

2 TURBULENT CONVECTION MODEL

Dynamical equations for turbulent convection can be written as (see, Hinze 1975; Canuto 1992,1997; Xiong 1989; Xiong et al. 1997)

$$\begin{aligned}\frac{D}{Dt}\overline{u_i'u_j'} &= \frac{1}{\overline{\rho}}\frac{\partial}{\partial x_l}\left(\overline{u_i'\tau_{jl}'}+\overline{u_j'\tau_{il}'}-\overline{\rho}\overline{u_l'u_i'u_j'}-\overline{P'u_j'}\delta_{il}-\overline{P'u_i'}\delta_{jl}\right)-\left(\overline{u_l'u_i'}\frac{\partial\overline{u}_j}{\partial x_l}+\overline{u_l'u_j'}\frac{\partial\overline{u}_i}{\partial x_l}\right)\\ &\quad -\frac{\beta}{\overline{T}}\left(g_i\overline{u_j'T'}+g_j\overline{u_i'T'}\right)-\frac{1}{\overline{\rho}}\left(\overline{\tau_{il}'\frac{\partial u_j'}{\partial x_l}}+\overline{\tau_{jl}'\frac{\partial u_i'}{\partial x_l}}\right)+\frac{1}{\overline{\rho}}\left(\overline{P'\frac{\partial u_j'}{\partial x_i}}+\overline{P'\frac{\partial u_i'}{\partial x_j}}\right), \qquad (1)\end{aligned}$$

$$\frac{D}{Dt}\overline{u_i'T'} = \frac{1}{\overline{\rho}}\frac{\partial}{\partial x_l}\left(\frac{\lambda}{c_P}\overline{u_i'\frac{\partial T'}{\partial x_l}}+\overline{T'\tau_{il}'}-\overline{P'T'}\delta_{il}-\overline{\rho}\overline{u_l'u_i'T'}\right)$$

$$
-\left(\overline{u'_l T'}\frac{\partial \overline{u}_i}{\partial x_l} + \frac{\overline{T}}{c_P}\overline{u'_l u'_i}\frac{\partial \overline{s}}{\partial x_l} + \frac{\beta}{\overline{T}} g_i \overline{T'^2}\right) - \frac{1}{\overline{\rho}}\left(\overline{\tau'_{il}\frac{\partial T'}{\partial x_l}} + \frac{\lambda}{c_P}\overline{\frac{\partial u'_i}{\partial x_l}\frac{\partial T'}{\partial x_l}}\right) + \frac{1}{\overline{\rho}}\overline{P'\frac{\partial T'}{\partial x_i}}, \quad (2)
$$

$$
\frac{D}{Dt}\overline{T'^2} = \frac{1}{\overline{\rho}}\frac{\partial}{\partial x_l}\left(\frac{\lambda}{c_P}\frac{\partial \overline{T'^2}}{\partial x_l} - \overline{\rho}\,\overline{u'_l T'^2}\right) - 2\frac{\overline{T}}{c_P}\frac{\partial \overline{s}}{\partial x_l}\overline{u'_l T'} - 2\frac{\lambda}{\overline{\rho} c_P}\overline{\frac{\partial T'}{\partial x_l}\frac{\partial T'}{\partial x_l}}. \quad (3)
$$

In the above equations, ρ is the density, P the pressure, T the temperature, and they observe an equation of state:

$$
d\ln\rho = \alpha\, d\ln P - \beta\, d\ln T, \quad (4)
$$

where $\alpha = \left(\frac{\partial \ln\rho}{\partial \ln P}\right)_T$ and $\beta = -\left(\frac{\partial \ln\rho}{\partial \ln T}\right)_P$ are the volumetric compression and expansion coefficient, respectively. s is the entropy, c_P the specific heat at constant pressure, and λ the thermal conduction coefficient. u_i and g_i are respectively the velocity and gravitational acceleration along the ith coordinate, and τ_{ij} the components of the viscous stress tensor. The fluid is considered to be composed of a mean field and a fluctuating turbulent perturbation, and a bar over a quantity denotes its mean-flow value while a prime behind denotes its turbulent fluctuation. D/Dt is the time derivative moving along with the fluid, and the Kronecker $\delta_{ij} = 1$ for $i = j$ and $\delta_{ij} = 0$ for $i \neq j$. According to the Boussinesq approximation, we ignore the density fluctuation throughout except in the buoyancy term. Equations (1)-(3) are written in tensor notation of a Cartesian coordinate system, and the Einstein summation convention applies.

It can be noticed that the first term on the right hand side of Eqs. (1)–(3) describes the net flux flowing out of a space volume and should be related to some kind of transport process. This is the nonlocal character of the turbulence, and is found to be important near boundaries of convection zones known as the overshooting effect. In most area of the stellar convection zone, the Reynolds number is so high that the production and dissipation of the turbulence are almost exactly balanced, leaving little flux to flow out of the fluid elements. Here we may thus simply ignore the transport terms. In addition, the time scale for the turbulence to reach steady state is usually much shorter than that of the stellar evolution, and it is reasonable to neglect the time derivative terms on the left hand side of Eqs. (1)–(3). The remaining terms describe the local and steady state of the turbulence.

The second and third terms on the right hand side of Eq. (1) determine the production of turbulence by velocity shearing of the mean-flow field and buoyancy, and the next is the dissipation of the turbulence by molecular viscosity. The last term, the pressure-strain correlation, can be proved to affect only the Reynolds stress components but contributes nothing to the turbulent kinetic energy $k = \frac{1}{2}\overline{u'_m u'_m}$, then its main effect is to rearrange an anisotropic turbulence towards the isotropic state and is therefore called the redistribution term (Hossain and Rodi 1982). Equation (2) is of similar form as Eq. (1), while the term containing pressure fluctuation does not appear in Eq. (3).

It is evident that Eqs. (1)-(3) cannot be solved unless the dissipation and redistribution terms are properly specified in advance. The many approximations adopted here to close the equations are reviewed by Hossain and Rodi (1982). Due to very high Reynolds number, flows develop numerous eddies from very large to very small sizes. The kinetic energy carried by large eddies is transported into small ones through cascade segregation of eddies, and is eventually dissipated into heat at the short wavelength tail where the molecular viscosity dominates the flow. In this process, large eddies generated by the velocity shearing and moving along the direction of the mean flow, or those driven by buoyancy and moving against the direction of gravity, gradually disintegrate into small ones moving more and more equally along all directions. It is therefore reasonable to assume the isotropy of the dissipation term which takes place in the short wavelength regime as (Hinze 1975; Hossain and Rodi 1982; Canuto 1998)

$$\frac{1}{\overline{\rho}}\left(\overline{\tau'_{il}\frac{\partial u'_j}{\partial x_l}}+\overline{\tau'_{jl}\frac{\partial u'_i}{\partial x_l}}\right)=\frac{2}{3}\varepsilon\delta_{ij}, \tag{5}$$

where ε is the dissipation rate and can be approximated by a mixing-length model:

$$\varepsilon=\frac{k^{3/2}}{l} \tag{6}$$

with a mixing length l. The dissipation term in Eq. (2) needs to be considered more carefully. There are two dissipation sources, viscous dissipation and thermal dissipation. Molecular viscosity decreases the fluctuating velocity shearing, while thermal conduction flattens the fluctuating temperature stratification. The combination of these effects restricts the fluctuations of the velocity and temperature, leading to a dissipation of the velocity-temperature correlation. Two time scales are involved here: $\tau_k \sim \frac{k}{\varepsilon}$ is the time that the kinetic energy is exhausted and $\tau_h \sim \frac{\overline{\rho}c_P}{\lambda}\frac{k^3}{\varepsilon^2}$ the time that eddies lose all extra heat to their surroundings. Based on the above arguments we propose the following model for the dissipation term:

$$\varepsilon_t=\frac{1}{\overline{\rho}}\left(\overline{\tau'_{il}\frac{\partial T'}{\partial x_l}}+\frac{\lambda}{c_P}\overline{\frac{\partial u'_i}{\partial x_l}\frac{\partial T'}{\partial x_l}}\right)=C_T\left(\frac{\varepsilon}{k}+\frac{\lambda}{\overline{\rho}c_P}\frac{\varepsilon^2}{k^3}\right)\overline{u'_iT'}. \tag{7}$$

Canuto (1998) found from an analysis of the turbulence energy spectrum that

$$\varepsilon_t=\frac{125+5\left(1+\sigma_t^{-1}\right)P_{\mathrm{t}}}{P_{\mathrm{t}}}\frac{\varepsilon}{k}\overline{u'_iT'}, \tag{8}$$

where $P_{\mathrm{t}}=\frac{\overline{\rho}c_P}{\lambda}\frac{k^2}{\varepsilon}$ is the turbulent Péclet number and σ_t the turbulent Prandtl number. As the time scales involved are closely related to the assumed turbulent energy spectrum, we would like to keep a free parameter C_T in the dissipation term ε_t. Based on similar arguments, it is plausible to assume that the dissipation term of Eq. (3) is proportional to what we have just introduced for Eq. (2) and to model it as

$$\varepsilon_\theta=\frac{\lambda}{\overline{\rho}c_P}\overline{\frac{\partial T'}{\partial x_l}\frac{\partial T'}{\partial x_l}}=C_\theta\left(\frac{\varepsilon}{k}+\frac{\lambda}{\overline{\rho}c_P}\frac{\varepsilon^2}{k^3}\right)\overline{T'^2}, \tag{9}$$

where C_θ is another adjustable constant. Canuto (1998) obtained instead that

$$\varepsilon_\theta=\frac{\frac{875}{16}+\sigma_t^{-1}P_{\mathrm{t}}}{P_{\mathrm{t}}}\frac{\varepsilon}{k}\overline{T'^2}. \tag{10}$$

For the redistribution term containing the pressure-strain correlation in Eq. (1), Rotta's proposal of "return to isotropy" is adopted (Rotta 1951):

$$\frac{1}{\overline{\rho}}\left(\overline{P'\frac{\partial u'_j}{\partial x_i}}+\overline{P'\frac{\partial u'_i}{\partial x_j}}\right)=-C_k\frac{\varepsilon}{k}\left(\overline{u'_iu'_j}-\frac{2}{3}k\delta_{ij}\right), \tag{11}$$

where C_k is the last free parameter we have to introduce. Under the assumptions introduced above, the Reynolds stresses can be evaluated easily as follows:

$$C_k \frac{\varepsilon}{k}\left(\overline{u_i'u_j'} - \frac{2}{3}k\delta_{ij}\right) = -\left(\overline{u_l'u_i'}\frac{\partial \overline{u}_j}{\partial x_l} + \overline{u_l'u_j'}\frac{\partial \overline{u}_i}{\partial x_l}\right) - \frac{\beta}{\overline{\overline{T}}}\left(g_i\overline{u_j'T'} + g_j\overline{u_i'T'}\right) - \frac{2}{3}\varepsilon\delta_{ij}. \quad (12)$$

Taking the contraction over the index i and j we obtain:

$$-\overline{u_l'u_m'}\frac{\partial \overline{u}_m}{\partial x_l} - \frac{\beta}{\overline{\overline{T}}}g_l\overline{u_l'T'} = \varepsilon. \quad (13)$$

This equation indicates that the generation and dissipation of the turbulence are locally balanced with each other. For flows with very high Reynolds numbers, this condition is essentially satisfied. By using Eq.(13), the Reynolds stress can be expressed as

$$\overline{u_i'u_j'} = \frac{2}{3}k\delta_{ij} - \frac{1}{C_k}\frac{k}{\varepsilon}\left[\overline{u_l'u_i'}\frac{\partial \overline{u}_j}{\partial x_l} + \overline{u_l'u_j'}\frac{\partial \overline{u}_i}{\partial x_l} - \frac{2}{3}\overline{u_l'u_m'}\frac{\partial \overline{u}_m}{\partial x_l}\delta_{ij} + \frac{\beta}{\overline{\overline{T}}}\left(g_i\overline{u_j'T'} + g_j\overline{u_i'T'} - \frac{2}{3}g_l\overline{u_l'T'}\delta_{ij}\right)\right]. \quad (14)$$

It may be noted that in a convection region with zero mean-flow velocity, the non-diagonal components are all zero, while the diagonal component along the gravitational acceleration direction is different from the other two diagonal components. This result clearly reveals the anisotropic character of the buoyancy-driven convection. On the other hand, the redistribution term containing pressure-temperature correlation in Eq. (2) contributes to the dissipation and can be incorporated into the dissipation term.

3 IMPROVED MLT SOLUTION

Applying the above equations to a stellar convection zone, and noting that the mean-flow velocity is zero while all other physical quantities have only a radial component and are only functions of the radius, we obtain the basic equations for determining the state of the turbulence in the convection zone. With the help of a new parameter D defined as:

$$D = \frac{2}{3} + \frac{4}{3C_k} \qquad \begin{cases} C_k \to \infty & \text{for isotropic turbulence}\,, \\ C_k \text{ is finite} & \text{for anisotropic turbulence}\,. \end{cases} \quad (15)$$

We obtain:

$$\frac{\varepsilon^3}{k^3} + \frac{\lambda}{\overline{\rho}l^2c_P}\left(2\frac{\varepsilon^2}{k^2} - \frac{D}{C_T}\frac{\beta g_r}{c_P}\frac{\partial \overline{s}}{\partial r}\right) + \left[\left(\frac{\lambda}{\overline{\rho}l^2c_P}\right)^2 - \left(\frac{D}{C_T} + \frac{1}{C_TC_\theta}\right)\frac{\beta g_r}{c_P}\frac{\partial \overline{s}}{\partial r}\right]\frac{\varepsilon}{k} = 0. \quad (16)$$

The total energy flux is the sum of the radiative and convective fluxes (Cox and Giuli 1968; Kippenhahn and Weigert 1990; Huang and Yu 1998):

$$F_{\mathrm{R}} + F_{\mathrm{C}} = -\frac{\overline{\rho}\overline{T}g_r\lambda}{\overline{P}}\nabla + \overline{\rho}c_P\overline{u_r'T'} = -\frac{\overline{\rho}\overline{T}g_r\lambda}{\overline{P}}\nabla_r, \quad (17)$$

where the temperature gradient $\nabla = \frac{d\ln\overline{T}}{d\ln\overline{P}}$. Defining a quantity f as

$$f = \frac{\nabla - \nabla_s}{\nabla_r - \nabla_s}, \quad (18)$$

where $\nabla_s = \left(\frac{\partial \ln \overline{T}}{\partial \ln \overline{P}}\right)_s = \frac{\overline{P}\beta}{\overline{\rho}\overline{T}c_P}$ is the adiabatic temperature gradient, we find

$$\begin{aligned} 2C_T\frac{\varepsilon^2}{k^2} \; &+ \; \left[\frac{\lambda C_T}{\overline{\rho} l^2 c_P} - \left(D + \frac{1}{C_\theta}\right)\frac{\overline{\rho}^2 g_r^2 \beta l^2 c_P}{\lambda \overline{P}}\left(\nabla_r - \nabla_s\right) f\right]\frac{\varepsilon}{k} \\ &- \; \frac{\overline{\rho} g_r^2 \beta}{\overline{P}}\left(\nabla_r - \nabla_s\right)\left[Df - C_T(1-f)\right] = 0\,. \end{aligned} \tag{19}$$

The roots of this quadratic equation are

$$\frac{\varepsilon}{k} = \frac{D + \frac{1}{C_\theta}}{4C_T}\frac{\lambda P_{\mathrm{e}}^2}{\overline{\rho} l^2 c_P}\left[f - \frac{C_T P_{\mathrm{e}}^{-2}}{D + \frac{1}{C_\theta}} \pm \sqrt{\left(f - \frac{C_T P_{\mathrm{e}}^{-2}}{D + \frac{1}{C_\theta}}\right)^2 + \frac{8C_T P_{\mathrm{e}}^{-2}}{\left(D + \frac{1}{C_\theta}\right)^2}\left[Df - C_T\left(1-f\right)\right]}\,\right], \tag{20}$$

where

$$P_{\mathrm{e}} = -\frac{\overline{\rho} g_r l^2 c_P}{\lambda}\sqrt{\frac{\overline{\rho}\beta}{\overline{P}}\left(\nabla_r - \nabla_s\right)}. \tag{21}$$

Only the solution with the positive sign before the square root can ensure a positive value for ε/k and so is the solution we are looking for; the other solution should be discarded. The dimensionless quantity P_{e} is of definite physical meaning: it measures the relative efficiency of convection to conduction for heat transfer in the convective region. Substituting the right solution for ε/k into Eq. (17), we obtain

$$\left(f - \frac{C_T P_{\mathrm{e}}^{-2}}{D + \frac{1}{C_\theta}} + \sqrt{\left(f - \frac{C_T P_{\mathrm{e}}^{-2}}{D + \frac{1}{C_\theta}}\right)^2 + \frac{8C_T P_{\mathrm{e}}^{-2}}{\left(D + \frac{1}{C_\theta}\right)^2}\left[Df - C_T\left(1-f\right)\right]}\,\right)^3 = \frac{64 C_T^3 P_{\mathrm{e}}^{-4}}{\left(D + \frac{1}{C_\theta}\right)^3}\left(1-f\right). \tag{22}$$

Now we arrive at an equation for f, which is of a similar form as that of the standard MLT (Kippenhahn 1962; Cox and Giuli 1968).

Figures 1–3 compares the solutions of Eq. (22) for different parameters with that of the MLT, with a new variable x defined by

$$x = \sqrt{\frac{C_T}{8D}}\frac{1}{P_{\mathrm{e}}}. \tag{23}$$

We shall call hereafter the solutions of Eq. (22) as IMLT as against the solution of the MLT. It may be noted in Fig. 1 that the IMLT leads to a smaller f than the MLT. With increasing C_θ, the f increases but soon approaches its upper limit, which is still below the MLT solution. Also, the IMLT gives a larger convective luminosity, and a smaller temperature gradient, than does the MLT. All the solutions converge to the MLT solution for large values of x, while at the small end of x the IMLT approaches proportionately faster the adiabatic solution than does the MLT. As a result, the IMLT results in a much larger adiabatic convection region than does the MLT.

With increasing C_T, the transition region of f is seen in Fig. 2 to move quickly and regularly towards small values of x, and the IMLT can result in a larger f than does the MLT. For small values of C_T, f increases more steeply with increasing x, and the IMLT leads to a larger adiabatic convection region than does the MLT. When C_T is large enough, however, the variation of f slows down and the IMLT gives a very extended nonadiabatic convection region. Note in particular that f is much more sensitive to C_T than to C_θ.

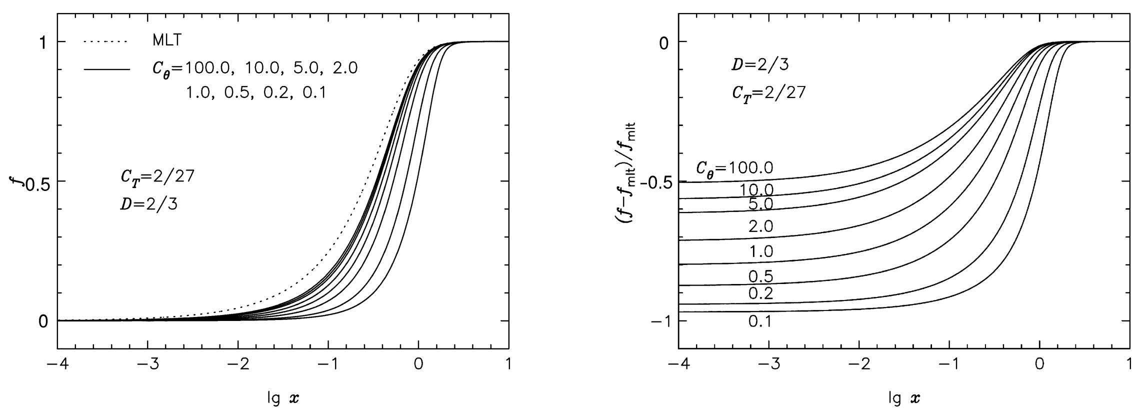

Fig. 1 Comparisons of the IMLT (solid lines) under different values of C_θ with the solution of the MLT (dotted line). Curves from left to right are respectively for $C_\theta = 100, 10, 5, 2, 1, 0.5, 0.2, 0.1$. The solutions are displayed in the left panel while the relative differences are shown in the right panel of the figure.

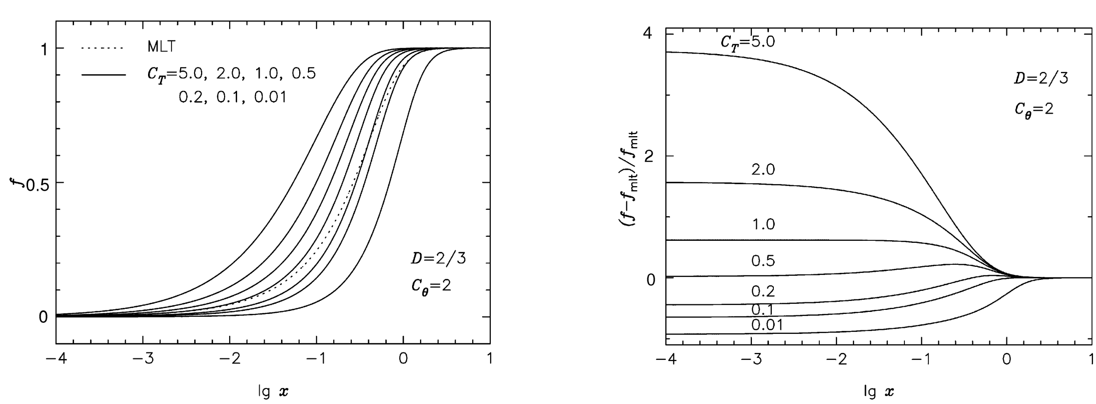

Fig. 2 Comparisons of the IMLT (solid lines) under different values of C_T with the solution of the MLT (dotted line). Curves from left to right are respectively for $C_T = 5, 2, 1, 0.5, 0.2, 0.1, 0.01$.

For the parameter D of the degree of anisotropy of the turbulence, there is a lower limit of 2/3 with which the IMLT results in an f along with other parameters shown in Fig. 3 that is quite close to that of the MLT. An increase of D makes the transition region of f move slowly to larger values of x, while the dependence of the run of f on D is the least among the three considered parameters. However, we note with interest that, under proper combinations of the parameters, it is possible for the IMLT to be below the MLT solution for small values of x and above for large values of x.

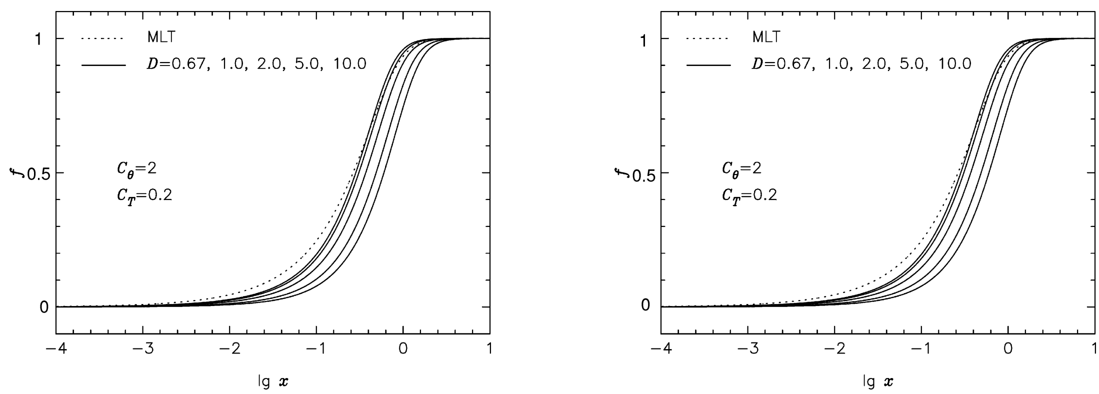

Fig. 3 Comparisons of the IMLT (solid lines) under different values of D with the solution of the MLT (dotted line). Curves from left to right are respectively for $D = 0.67, 1, 2, 5, 10$.

4 APPLICATIONS TO SOLAR MODELS

We have calculated four solar models, MS1, MS2, MS3, and MS4, with different choices of the turbulent parameters C_T, C_θ, and D. These models were computed by an evolution code described by Sienliewicz in 1997. The nuclear reaction rates are those used in Bahcall and Pinsonneault (1995). The OPAL equation of state from Lawrence Livermore National Laboratory is adopted (Rogers, Swenson and Iglesias 1996). The opacity data for stellar matter consist of two data sets: the OPAL opacities GN93hz series are used in high temperature region (Iglesias and Rogers 1996), while the opacities from Alexander and Ferguson (1994) that include the contributions of molecules and grains are used in low temperature region. The Schwarzschild criterion is used to determine the boundaries of the convection zone (Cox and Giuli 1968; Huang and Yu 1998). Inside the convection zone, Eq. (22) is used to calculate the convective energy transfer rate, in comparison with the standard solar model (SSM) that employs the standard mixing-length theory. The main results are summarized in Table 1. It can be seen that the solar models resulted from the improved MLT are not significantly different from the standard solar model (SSM). The most marked difference appears in the convection parameter α, which is the ratio of the mixing length to local pressure scale height. The use of the improved MLT greatly decreases the value of the convection parameter α. When D=1.0, C_T=3.0, and C_θ=1.25, α is found to be 0.977, which is much smaller than the standard solar model value of 1.638. Increasing the degree of anisotropy of the turbulence can result in more efficient convective energy transfer rate, and we find correspondingly that the convection parameter α decreases to 0.708 when D is increased to 2.0. Similarly, decreasing the dissipation of the turbulence can also enhance the convective energy transport, and we find that decreasing C_T to 1.5 can further reduce the convection parameter α to 0.433. These results indicate that the approximations introduced in the present turbulence model work consistently and are reasonable improvements to the standard MLT.

The frequencies of the p-mode oscillations for spherical harmonic index $l = 0, 1, 2, 3$ are computed based on an adiabatic oscillation code described by Li (1992). Higher spherical degree modes are seriously influenced by the nonadiabatic effects, and are not considered in the present investigation. Figure 4 shows the differences between these frequencies and the observed values by Libbrecht et al. (1990). It may be noted that the dotted lines that represent the results of MS3 almost coincide with the solid lines that represent those of the SSM, while

Table 1 Properties of the Solar Models with the Improved Mixing-Length Theory

Model	SSM	MS1	MS2	MS3	MS4
D		1.0	2.0	2.0	1.0
C_T		3.0	3.0	1.5	10.0
C_θ		1.25	1.25	1.25	1.25
α	1.63765	0.97685	0.70807	0.43299	2.35054
X_0	0.704898	0.7049	0.7049	0.704898	0.7049
X_c	0.348480	0.348473	0.348473	0.348482	0.348470
$T_c(\times 10^6\mathrm{K})$	15.69	15.69	15.69	15.69	15.69
$\rho_c(\mathrm{g/cm^3})$	153.2	153.2	153.2	153.2	153.2
$M_{bc}(M_\odot)$	0.979	0.979	0.979	0.979	0.979
$R_{bc}(R_\odot)$	0.724	0.724	0.724	0.724	0.724
$T_{bc}(\times 10^6\mathrm{K})$	2.122	2.121	2.121	2.122	2.122

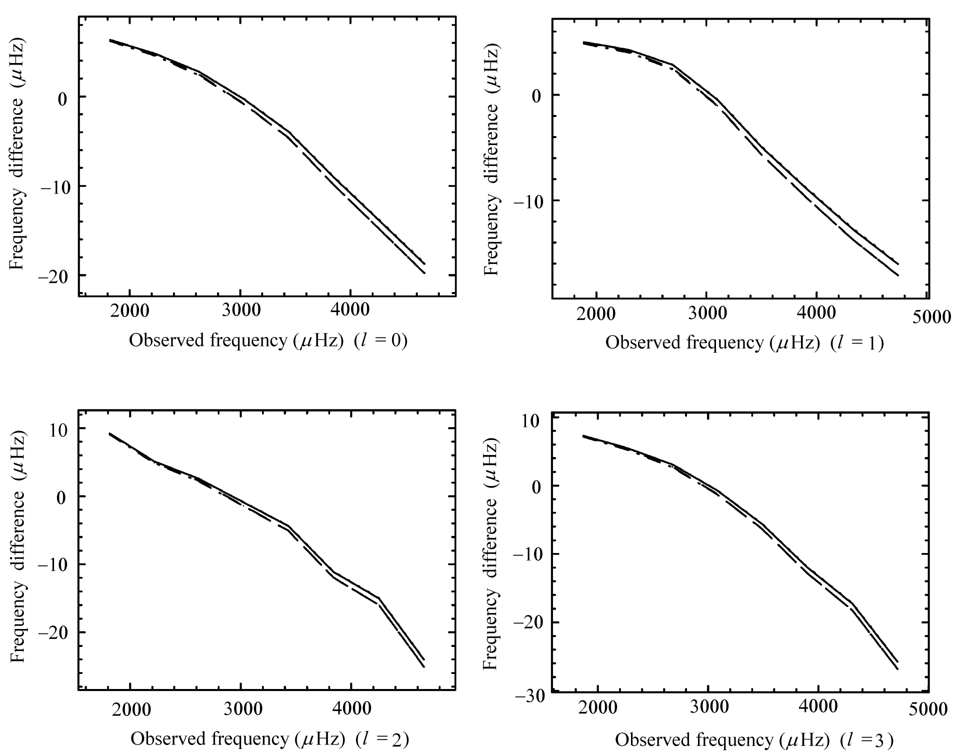

Fig. 4 Comparisons of the differences of the observed and calculated solar p-mode frequencies for the considered solar models.

the results of the MS1 and MS2 are almost identical and systematically 1μHz lower than that of the standard solar model. Compared with the observations, MS1 and MS2 seem to give noticeable improvement over the SSM in the low frequency section, while in the high frequency part the nonadiabatic effects play a more important role, and direct comparison between the adiabatic and observed frequencies is inappropriate. This result implies that the dissipation of the turbulence in the solar convection zone should be fairly strong, while the degree of anisotropy of the turbulence plays a less significant role in the structure of the solar convection zone.

In Fig. 5 we compare the difference in the sound speed between IMLT and the SSM. As expected, the sound speed is different only in the convection zone. We can see that increasing the anisotropy of the turbulence D reduces the sound speed in the convection zone. This is a direct result of the temperature depression in the convection zone due to the enhancement of the convective energy transport rate. Decreasing the dissipation of the turbulence produces a similar effect, and we can find that the influence of the turbulent dissipation is very much greater than that of the turbulent anisotropy.

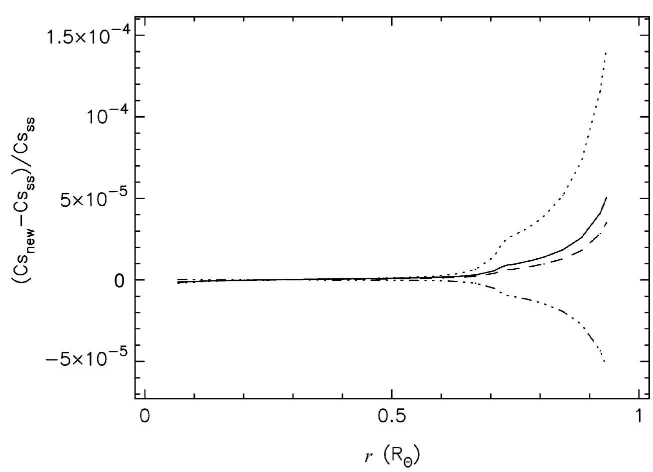

Fig. 5 Comparisons of the differences of the sound speed for the considered models with respect to the standard solar model.

5 SUMMARY

The mixing-length theory is widely used in the construction of ingenious solar models. However many important physical processes are not properly included in the MLT, such as the dissipation and anisotropy of the turbulence. In the present paper we investigate these effects on the solar models. The turbulent motion in the solar convection zone is described by an improved dynamic model. We made careful considerations on the dissipation of the turbulence, and introduced two time scales to describe the effects of turbulent viscosity and thermal conduction. The anisotropy of the turbulence is approximated by Rotta's proposal of "return to isotropy". The improved turbulence model results in an equation to determine the temperature gradient in the convection zone, which is of similar form to that of the MLT. We computed some solar models based on the improved turbulence model. It is found that an increase of the anisotropy and a decrease of the dissipation reduce the value of the convection parameter α, because these processes enhance the convective energy transfer rate. This result implies that the approximations introduced in our turbulence model work consistently and give physically reasonable results. Compared with the observed solar p-mode frequencies, it is plausible that the dissipation of the turbulence in the solar convection zone should be fairly strong, while the degree of anisotropy of the turbulence plays a less significant role on the structure of the solar convection zone. Increasing the convective energy transfer rate by increasing the anisotropy or decreasing the dissipation of the turbulence will reduce the sound speed in the solar convection zone. But it should be noted that the differences in sound speed between the IMLT models and the SSM are at most 10^{-4}, which are well below the present level of helioseismic determinations (about 1%) and other physical uncertainties (such as EOS). In addition, our treatment is still a local theory. Turbulent transport processes may significantly change the profiles of kinetic energy and dissipation of the turbulence in the solar convection zone. Interested readers may refer to Xiong (1989) and Canuto (1998).

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