Statistical properties of fast radio bursts elucidate their origins: magnetars are favoured over gamma-ray bursts

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\textbf{Abstract} Fast radio bursts (FRBs) are extremely strong radio flares lasting several milliseconds, most of which come from unidentified objects at a cosmological distance. They can be apparently repeating or not. In this paper, we analyzed 18 repeaters and 12 non-repeating FRBs observed in the frequency bands of 400-800 MHz from CHIME. We investigated the distributions of FRB isotropic-equivalent radio luminosity, considering the K correction. Statistically, the luminosity distribution can be better fitted by Gaussian form than by power-law. Based on the above results, together with the observed FRB event rate, pulse duration, and radio luminosity, FRB origin models are evaluated and constrained such that the gamma-ray bursts (GRBs) may be excluded for the non-repeaters while magnetars or neutron stars (NSs) emitting the supergiant pulses are preferred for the repeaters. We also found the necessity of a small FRB emission beaming solid angle (about 0.1 sr) from magnetars that should be considered, and/or the FRB association with soft gamma-ray repeaters (SGRs) may lie at a low probability of about 10%. Finally, we discussed the uncertainty of FRB luminosity caused by the estimation of the distance that is inferred by the simple relation between the redshift and dispersion measure (DM).

\textbf{Key words:} transients: fast radio burst - methods: statistical - stars: magnetars
1 INTRODUCTION

Fast radio bursts (FRBs) are very strong radio emissions in a couple of milliseconds, which are mostly confirmed to be from cosmic distances. The FRB phenomenon was firstly noticed and reported in 2007 (Lorimer et al. 2007), and the first non-Parkes FRB event, FRB 121102, was observed by the Arecibo telescope in 2014 (Spitler et al. 2014), which is also the first confirmed repeating FRB with the localized host galaxy (Spitler et al. 2016; Chatterjee et al. 2017). With the completion of the advanced radio instruments like Canadian Hydrogen Intensity Mapping Experiment (CHIME) (CHIME/FRB Collaboration et al. 2019a, b; CHIME/FRB Collaboration et al. 2020), Australian Square Kilometre Array Pathfinder (ASKAP) (Shannon et al. 2018; Kumar et al. 2019) and Five-hundred-meter Aperture Spherical radio Telescope (FAST) (Li et al. 2018; Zhu et al. 2020; Luo et al. 2020a; Lin et al. 2020), the number of FRBs has dramatically increased (Petroff et al. 2016). Under the continuous endeavour on FRB searching over a decade (Lorimer 2018), up to now there are 22 repeaters and 107 apparently non-repeaters published with the event rate of $10^{-3} - 10^{-4}$ day$^{-1}$ sky$^{-1}$ (Lorimer et al. 2007; Thornton et al. 2013; Spitler et al. 2014; Kulkarni et al. 2014; Keane & Petroff 2015; Rane et al. 2016; Oppermann, Connor & Pen 2016; Champion et al. 2016; Scholz et al. 2016; Lawrence et al. 2017; Patel et al. 2018; Connor 2019). Recently, FRB-like signals (named as FRB 200428) from a galactic magnetar that is identified as a soft gamma-ray repeater (SGR) 1935+2154 have been detected by STARE2 (Bochenek et al. 2020) and CHIME (CHIME/FRB Collaboration et al. 2020), strongly supporting the idea that the magnetar is the promise candidate for FRB origin. Cosmological FRBs are still remained as the unsettled questions (Kulkarni et al. 2014; Keane 2018; Pen 2018; Cordes & Chatterjee 2015). At present, there are lots of theoretical models proposed to explain the origin and radiation mechanism of FRBs, most of which are mainly centered at the ideas borrowed from the pulsars (Cordes & Wasserman 2016) and gamma-ray bursts (GRBs) (Zhang 2020).

To constrain FRB models, we performed statistical tests on the intrinsic duration and isotropic-equivalent radio luminosity, concluding that the repeaters and apparently non-repeaters should have different origins or physical processes (Cui et al. 2020). This indicates that at least two different models are needed to explain these two classes of FRBs. Since FRB phenomena were observed in the magnetar SGR 1935+2154 (FRB 200428), the FRB-SGR association is confirmed (Bochenek et al. 2020; CHIME/FRB Collaboration et al. 2020; Lin et al. 2020; Li et al. 2020), although its estimated radio emission luminosity is about 4 orders of magnitudes lower than that of the observed cosmological FRBs. Meanwhile, magnetars can act as the central engines of both pulsar-like and GRB-like models (see the review of Zhang (2020)), but the other alternative schemes cannot be completely ruled out. Therefore, it is necessary to investigate more aspects of FRB statistical properties to constrain FRB models.

FRB luminosity distribution has been studied by several researchers (Kumar, Lu & Bhattacharya 2017; Li et al. 2017; Luo et al. 2018; Hashimoto et al. 2020; Luo et al. 2020a), the power-law type with various indices is usually preferred (Zhang 2020). In the work from Li et al. (2017), they analyzed observed fluence of 16 non-repeaters, deriving a power-law type for intensity distribution function. Meanwhile Luo et al. (2020a) assumed the Schechter function as a likelihood function in Bayesian method, while Hashimoto et al. (2020) applied a simple $V_{\text{max}}$ method without any presupposition functional shape in their analysis. As expected, the different fitting functions for FRB luminosity distributions should be rooted in the particular physical origins or radiation processes of FRBs, by which the FRB models could be constrained.

In this paper, we divide the isotropic-equivalent radio luminosity data from CHIME with K correction into two sample sets according to the repeatability (Petroff, Hessels & Lorimer 2019; Cui et al. 2020). Then, we compare the two different types of fitting functions, the Gaussian type and power-law type, respectively, based on the goodness of fittings. Furthermore, combining the statistical properties of FRB duration and luminosity, as well as the event rate of FRBs, we obtain that models associated with GRBs for the non-repeater are ruled out, while the repeating FRBs favor magnetars or supergiant pulses as their origins. In addition, if we consider the low probability of SGRs to exhibit FRBs (e.g., ~10%),
and the small beaming angle of FRB emission (e.g., 0.1 sr), then the tension between the birth rate of magnetars and the event rate of FRBs can be settled down.

The structure of our paper is as follows. In Section 2, we describe the data selection of two samples. In Section 3, we fit the FRB isotropic-equivalent radio luminosity distribution by both power-law and Gaussian functions. In Section 4, we discuss the constraints on the different FRB models and evaluate the possibility of producing repeating FRBs by magnetars. Finally, in Section 5, we summarize our results.

2 SELECTION OF OBSERVATION DATA

To limit the uncertainties of the FRB flux density resulted from the different types of radio telescopes, we select the FRB data only from CHIME (12 non-repeaters and 18 repeaters), detected at frequency band of 400-800 MHz (CHIME/FRB Collaboration et al. 2019a,b), since the thresholds and beam patterns of FRBs are related to the features of the radio telescopes (Caleb et al. 2016).

In previous investigation about the FRB luminosity function, a Schechter type is selected as described in the following Eq. (1), which is usually considered as a likelihood function in Bayesian method (Luo et al. 2018, 2020a),

$$\phi(\log L)d\log L = \rho \left( \frac{L}{L_{\max}} \right)^{\alpha+1} e^{-\frac{L}{L_{\max}}} d\log L,$$

where $\rho$ is a normalization coefficient, $L$ is the radio luminosity, $L_{\max}$ is the upper limit of FRB luminosity (hereafter we take it as the maximum value of FRBs) and $\alpha$ is the power-law index. The form of the likelihood function has effect on the final luminosity function. Inappropriate selection of likelihood function may cause errors and misunderstandings when we attempt to explain these observations. Hence, it is necessary to check the selected type of the likelihood function. The Schechter type and power-law are different in terms of the function type, but Schechter function is still a special case of power-law type, containing the exponential cutoff at the high end. Here we employ the Schechter function to represent the power-law type. Without considering the scattering (Lorimer et al. 2013), the isotropic-equivalent radio luminosity (hereafter referred to as radio luminosity) of an FRB is defined by Eq. (2),

$$L_{\text{iso}} \sim S_{\nu_c} D_L^2,$$

where $S_{\nu_c}$ is the flux density at central frequency $\nu_c$ and $D_L$ is the FRB luminosity distance.

Whether the repeaters and apparently non-repeaters originated from the same population is still unclear, there are several works that have discussed this issue. In the views of Fonseca et al. (2020) and Cui et al. (2020), they believe that two types of FRBs follow different origins. However, the simulation from Gardenier et al. (2020) show that the observed FRBs in the sky can be accounted by a single population with varying repetition rate. The above works share similar views that the repeaters and apparently non-repeaters may come from different physical processes or traits. So in the following analysis, based on the occurrence property (repeaters and apparently non-repeaters), we divide the CHIME FRB data into two sample groups. Group one is the published repeating FRB data (CHIME/FRB Collaboration et al. 2019a,b, Fonseca et al. 2020), and group two is the announced non-repeating FRB sample, corresponding to the database from CHIME and FRB Catalogue (FRBCAT) \footnote{http://www.frbcat.org/} (Petroff et al. 2016). Some repeating FRBs were previously thought to be the non-repeater ones (Kumar et al. 2019), however it is difficult to foresee whether all non-repeaters are bound to burst again in the future. So, based on the current situation, we presume the "apparently" non-repeating sources as the real non-repeaters. Considering that the repeaters contain multiple bursts, we take their mean value as a statistical variable for each repeater.
Up to now, the distances of 13 FRBs are measured directly by the redshift of their host galaxy, while the distances of the other FRBs are all estimated by their dispersion measures (DMs). In the FRBCAT, Petroff et al. (2016) obtained the maximum redshift by DM and inferred the FRB luminosity distance. The ΛCDM cosmological parameters that they employed are listed below: $H_0 = 69.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_M = 0.286$ and $\Omega_{\text{vac}} = 0.714$, where $H_0$ is the Hubble constant, and $\Omega_M$ and $\Omega_{\text{vac}}$ are the matter and dark energy fraction in the universe, respectively. In our analysis, the FRB luminosity distance is directly taken from the FRBCAT, since the estimated distances by Luo et al. (2018) share the similar results to those of FRBCAT (see Appendix B). As noted, the distances of FRBs in the above two samples are corresponding to the redshift ($z$) from 0.05 to 2.1, so the FRB luminosity needs a K correction, which is described in Eq.(3) as below (Hogg et al. 2002; Xiao, Wang, & Dai 2021).

$$L_k \sim S_{\nu} \frac{D_L^2}{(1 + z)}.$$ (3)

3 LUMINOSITY DISTRIBUTION OF FRBS

Here, for the two groups of FRB luminosity data as mentioned above, we fit their distributions by the two types of functions, a power-law type and Gaussian type, respectively. Then we compare the goodness of fitting results to test which form of function is more appropriate. The histograms of repeaters and fitted curves of the different types are shown in Figure 1. The goodness of power-law and Gaussian form are 0.013 and 0.927, respectively, for the repeater samples. Figure 2 shows the histogram and fitted curves of non-repeaters, and the goodness of power-law and Gaussian form are 0.006 and 0.734, respectively.

What we have to clarify here is that although the goodness of Gaussian form is much better than power-law type, we still can not draw a conclusion that the Gaussian is the best form for luminosity distribution. But at least, the fittings of power-law type is not suitable enough for the luminosity distribution from CHIME data, and Gaussian type is better than power-law type.

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2 [http://frbhosts.org/](http://frbhosts.org/)

**Fig. 1** Histogram and different fitted curves of repeaters at 600MHz. The solid line is the curve of Gaussian form and the dashed line is the curve of power-law form. The square dots are the data points.
Fig. 2 Histogram and different fitted curves of non-repeaters at 600MHz. The solid line is the curve of Gaussian form and the dashed line is the curve of power-law type. The square dots are the data points.

4 RESULTS AND DISCUSSIONS

For the radio luminosity distributions of FRBs in two sample groups, they are more likely to favor the Gaussian type rather than power-law type (Schechter function), which can restrict the FRB models. The burst mechanism or radiation process of FRBs prefer the sources that emit the radio flashes to follow the Gaussian distribution of luminosity. This fact should rule out or favor some procedures of FRB origins, as described below.

4.1 Ruling out the GRB origin for FRBs

Usually there are two distinct classes of GRBs, long GRBs (LGRBs) and short GRBs (SGRBs), divided by their duration of \(\sim 1\text{-}2\) seconds, which are assumed to be produced by the collapses of massive stars and mergers of compact stars, respectively (Berger 2014; Abbott et al. 2017; Jespersen et al. 2020). In the early years of FRB discovery, the origins of FRBs associated with GRBs have been proposed (Egorov & Postnov 2009; Falcke & Rezzolla 2014; Zhang 2014; Romero, del Valle, & Vieroy 2016; Metzger, Berger, & Margalit 2017; Margalit, Berger, & Metzger 2019). Based on the statistical properties of FRBs, together with the comparisons of event rates between FRBs and GRBs, we find that non-repeating FRBs do not favor the origin models associated with both LGRBs and SGRBs, and the reasons are listed below and in Table 1.

I. LGRBs happen in the final collapses of massive stars, whose luminosity distribution is usually described in Schechter function form (Dwek & Krennrich 2013; Trenti, Perna, & Tacchella 2013; McGuire et al. 2016). While the luminosity distribution of FRBs prefers the Gaussian type.

II. The birth rate of LGRBs is not consistent with that of FRBs, which are approximately \(7 \times 10^2 Gpc^{-3} yr^{-1}\) (Chapman et al. 2007; Sun, Zhang, & Li 2015; Luo et al. 2020a) and \(2 \times 10^4 Gpc^{-3} yr^{-1}\) (Kulkarni et al. 2014), respectively. III. The durations of LGRBs are usually much longer than 2 seconds, which are far from those of FRBs (milliseconds timescale).

SGRBs are considered as the mergers of compact objects in binary systems, e.g., double neutron star (DNS) (Berger 2014; Abbott et al. 2017) and black hole (BH)-NS system (D’Avanzo 2015). Although the luminosity distribution of DNS mergers can be taken as a Gaussian type, the duration and event rate
The reasons are that the merger rate of WD-WD (Bade & Maoz 2012), WD-NS (Ginzburg & Maoz 2012) are not preferably taken as the origins of FRBs, we cannot rule out other merger and collision models, such as the ones by the double white dwarf (WD) (Kashiyama, Ioka, & Meszaros 2013), BH-WD (Abbott et al. 2016), and BH-NS (Gu et al. 2016) and BH-NS (Li et al. 2018). The reasons are that the merger rate of WD-WD (Bade & Maoz 2012), WD-NS (Ginzburg & Maoz 2012) are not preferably taken as the origins of FRBs, we cannot rule out other merger and collision models, such as the ones by the double white dwarf (WD) (Kashiyama, Ioka, & Meszaros 2013), BH-WD (Abbott et al. 2016), and BH-NS (Gu et al. 2016) and BH-NS (Li et al. 2018). The reasons are that the merger rate of WD-WD (Bade & Maoz 2012), WD-NS (Ginzburg & Maoz 2012) are not preferably taken as the origins of FRBs, we cannot rule out other merger and collision models, such as the ones by the double white dwarf (WD) (Kashiyama, Ioka, & Meszaros 2013), BH-WD (Abbott et al. 2016), and BH-NS (Gu et al. 2016) and BH-NS (Li et al. 2018).
4.2 Supporting the magnetar origin for FRBs

The magnetar origin models for the repeating FRBs seem to be promising. I. Magnetars can release soft gamma-ray bursts repeatedly, and FRB phenomena on the magnetar SGR 1935+2154 (Bochenek et al. 2020, CHIME/FRB Collaboration et al. 2020) have been observed recently. Meanwhile, if the non-repeaters are partly the repeaters with a long repeating time, they may come from the highly energetic SGRs of magnetars with a very long recurrence time. II. Some FRB repeaters are observed to have polarization characteristics (Luo et al. 2020b), which are related to the strong magnetic activities.

A serious problem of the FRB-SGR association is that the birth rate of magnetars is much higher than the event rate of FRBs. As proposed by many researchers, the birth rate of magnetars is about $\sim 10^{-2} - 10^{-3} \text{yr}^{-1} \text{galaxy}^{-1}$ (Duncan & Thompson 1992, Kouveliotou et al. 1998, Gill & Heyl 2007, Ferrario & Wickramasinghe 2008, Gullón et al. 2015, Mereghetti, Pons, & Melatos 2015, Beniamini et al. 2019), or equivalently expressed as $\sim 10^6 - 10^7 \text{Gpc}^{-3} \text{yr}^{-1}$, which is about two orders of magnitude higher than the event rate of FRBs (see Table 1). However, if we consider a factor of beaming effect and not all hard X-ray bursts generate radio bursts (Lin et al. 2020), the event rate of magnetars to release FRBs may drop down at least two orders of magnitude, which can reconcile the difficulty in the event rates of FRBs and SGRs. Here, we assume that the beaming solid angle of the FRB is about 0.1 sr (about one hundredth of the whole sphere). Therefore, the rate of magnetar SGR for FRBs can be reduced to $\sim 10^{-5} \text{yr}^{-1} \text{galaxy}^{-1}$, which is satisfied with the observational requirement of FRB event rate. As remarked, in calculating the FRB luminosity, Eq. (2) and Eq. (3) are employed to present the luminosity in the approximated unit solid radian. For the FRB repeaters, if the beaming solid angle is 0.1 sr, the FRB radio luminosity needs to be reduced by one order of magnitude. Furthermore, there are 16 SGRs detected so far (12 confirmed, 4 candidates, Olausen & Kaspi 2014), and only one case of FRB associated with SGR has been observed (Bochenek et al. 2020, CHIME/FRB Collaboration et al. 2020), implying a low rate of FRB-SGR event, e.g., expressed by a rarity factor of FRB-SGR association of $< 10\%$. Thus, the reasonable event rate of FRBs can be obtained by constraining the beaming angle and rarity factor of emitting the radio bursts.

Similar to the FRB-SGR association in magnetar, FRBs originated from the supergiant radio pulses of the NSs could be also possible (Cordes & Wasserman 2016, Lyutikov, Burzawa, & Popov 2016, Lorimer 2018), although their actual occurrence rate is uncertain. Giant pulses with the radio flux density over ten times the normal pulses were noticed long before Crab pulsar (Staelin & Reifenstein 1968, Heiles & Campbell 1971, Staelin 1970). Then, lots of giant pulses, with intensity even as high as thousand times that of the average of normal pulses, have been observed from several young pulsars (Romani & Johnston 2001, McLaughlin & Cordes 2003, Cordes et al. 2004, Mickaliger et al. 2012). In addition, the similarity between the slope of the fluence distribution for Crab giant pulses and the repeating FRB 121102 was noticed (Bera & Chengalur 2019), which should have interesting implications on the nature of the FRB phenomena and giant pulses. Thus, it is meaningful to consider the association between the FRB and supergiant pulse, although the relations between magnetars and NSs of emitting supergiant pulses are not clear yet. Furthermore, asteroid or planetary system dominated by NS can also generate repeating FRBs in some models (Dai et al. 2016, Kuerban et al. 2021), which are also instructive for us to understand the physical nature of FRBs.

Finally, we discuss the uncertainty in deriving the values of FRB luminosity, which comes from the observational errors of three quantities, i.e., the flux density, the distance inferred by the redshift and emission solid angle of FRB. The latter two quantities depend on some assumptions, which act as the dominant effects on the luminosity. For example, the uncertainty of the emission beaming solid angle may cause the uncertainty of about one order of magnitude in determining the FRB luminosity if we

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[3] https://solomon.as.utexas.edu/magnetar.html
ascribe the FRB as a magnetar origin, as discussed above. In contrast, the luminosity errors caused by the measurement flux density will not act as a main factor. Therefore, our statistical analysis based on the FRB luminosity distribution implies the following idealized assumptions. To begin with, the one-one correlation between the redshift and DM (Macquart et al. 2020). Then, each FRB has a similar beaming solid angle of emission. Moreover, it is assumed that all FRB measurements are accurately measured and are not affected by different telescope beams. Apparently, more precise determinations of FRB parameters are needed for the derivation of robust FRB luminosity properties.

5 CONCLUSIONS

To study the model constraints for FRBs, we investigate the statistical properties of FRB luminosity and FRB event rates, and find that magnetars or supergiant pulse of NSs are favored as the origins for the repeating FRBs and GRBs should be excluded for the non-repeaters. In the data selection, we only analyze the CHIME data for FRB luminosity distribution, which avoids the uncertainty of the flux density from different radio telescopes. If we assume that the current FRB distance inferred from FRBCAT based on the redshift and DM relation is reliable, then the FRB luminosity distribution should be more consistent with the Gaussian type in logarithmic scale, which will present a tight constraint on the FRB origins. Considering the FRB duration, radio burst luminosity and event rate, we discuss the non-repeater and repeater models, as listed in Table 1. For the non-repeater FRB models, we exclude the LGRB and SGRB origins, meanwhile some models based on the mergers and collisions of compact objects with the non-degenerate stars cannot be easily excluded at present. For the repeater FRB models, we favor the magnetar or NS supergiant pulse models. The big difference of the birth rate of magnetar or NS from the FRB event rate can be solved by introducing the small beaming solid angle (0.1sr) for FRB emissions and/or the rarity factor (∼0.1) of FRB-SGR associations.

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DATA AVAILABILITY

The data underlying this article are available in the references below: (1) Repeating FRBs are from (CHIME/FRB Collaboration et al. 2019a,b); (2) non-repeaters are from the database of FRB Catalogue (FRBCAT), available at http://www.frbcat.org/; (3) information of SGRs are from McGill Online Magnetar Catalog, available at http://www.physics.mcgill.ca/~pulsar/magnetar/main.html.

References

APPENDIX A: DIFFERENT EXPRESSIONS FOR THE FRB EVENT RATE

The event rate of FRBs given by observations is often expressed in different units, e.g., $day^{-1}sky^{-1}$, $Gpc^{-3}yr^{-1}$ and $yr^{-1}galaxy^{-1}$ (Lorimer et al. 2007; Kulkarni et al. 2014; Cordes & Chatterjee 2019). To reconcile these equivalent expressions, in this appendix, we make the conversion of the observation rate of $10^4 day^{-1}sky^{-1}$ into the units of $Gpc^{-3}yr^{-1}$ and $yr^{-1}galaxy^{-1}$. First, we convert it into the unit of $Gpc^{-3}yr^{-1}$,

$$10^4 day^{-1}sky^{-1} = \frac{10^4 \times 365}{4\pi D_{H_0}} Gpc^{-3}yr^{-1}$$

(4)

where $D_{H_0}$ is Hubble distance around $4.3 Gpc$ with Hubble constant $H_0 \sim 70 km s^{-1} Mpc^{-1}$. Second, we convert the FRB event rate into the unit of $yr^{-1}galaxy^{-1}$,

$$10^4 day^{-1}sky^{-1} = \frac{10^4 \times 365}{N_G} yr^{-1}galaxy^{-1}$$

(5)

where $N_G$ is the galaxy number in the observable universe, which ranges from $2 \times 10^{11}$ (Gott et al. 2005) to $2 \times 10^{12}$ (Conselice et al. 2016), hence we take the conservative value of $N_G \sim 2 \times 10^{11}$ for calculation. Meanwhile, what we need to clarify here is that the above is just a rough estimation. This conversion is without considering the different detection limits of various radio telescopes, the curvature of the space, the luminosity scattering, etc.

APPENDIX B: THE EVALUATION OF THE UPPER LIMIT OF REDSHIFT FOR FRBCAT

To evaluate the FRB distances estimated by the redshift of the different models, we make the comparison for the upper limits of redshift of the two models, given by FRBCAT ($z_{CAT}$) and those by Luo et al. (2018) ($z_{Luo}$), respectively, as listed in Table 2. The upper limit of redshift hints the assumptions that
the contribution of the host galaxy or surrounding material to DM is ignored. Interestingly, we find that the values of \( z_{Luo} \) are systematically higher than those of \( z_{CAT} \) with one extraordinary case. Especially for the well known source FRB121102, \( z_{Luo} (0.32) \) and \( z_{CAT} (0.31) \) are similar, and both are deviated from the real value of \( z_{host} (0.19) \) by about 30 \%. Therefore, it is reasonable for us to conclude that the values of \( z_{Luo} \) may not be necessarily better than those of \( z_{CAT} \). So, it is acceptable to directly quote the inferred values of redshift and distance listed in FRBCAT.

### Table 2: Comparison of the upper limit of redshift for FRBCAT

<table>
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<tr>
<th>FRB</th>
<th>( z_{CAT} )</th>
<th>( z_{Luo} )</th>
<th>( (z_{Luo} - z_{CAT})/z_{Luo} )</th>
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