Imaging atmospheric turbulence, using entropy to quantify turbulence strength

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Abstract Vertical profile of turbulence strength ($C_n^2$) is one of the most important attributes for evaluating site’s astronomical performances. Although seeing is commonly used for describing turbulence strength, it could only represent the integral of all layers of the turbulence above the observer. Other techniques which could deduct $C_n^2$ such as DIMM-MASS, SCIDAR, etc. all rely on rather complex systems to do the measurements. In this paper, we presented an idea to evaluate the relative strength of turbulence with entropies of short exposure images taken at different conjugated heights of the turbulence. Initial experimental results are also presented in the paper.

Key words: techniques: telescope – site testing

1 INTRODUCTION

As known ever since Newton, turbulence is one of the culprits that deteriorate the image quality of a ground-based telescope. It could blur images taken with long exposure time or create multiple replica of the object in the so-called speckle image with short exposure. To characterize and further alleviate the impact of the atmospheric turbulence, many theoretical/technical efforts have been made. Metrics and measurements methods for quantitizing the strength of turbulence have been proposed in a huge number of papers. Among them, the most popular quantities to represent turbulence strength are refractive index profile, $C_n^2(h)$, where $h$ is the altitude, and the total integral of $C_n^2(h)$ for heights above ground, the seeing $\theta_0$.

The beauty of the these two parameters is that they are measurable with special scientific instruments. Although $C_n^2(h)$ is ideal for learning turbulence strength at different height, to measure such a profile it requires either a balloon equipped with high sensitivity thermometer or using a technique called SCIntillation Detection and Ranging (SCIDAR) which needs a telescope with diameter larger than 50cm. For both cases, it is hard to setup and is difficult to monitor continuously for long term which is important for such a profile that has statistical nature. This leads to the popularity of using seeing as a merit for evaluate a site’s imaging performance. It is a representation of image quality obtained considering only the overall turbulence above the observing height, and thus could be easily combined with designed image quality achieved by telescope to predict how a telescope could performs at a certain location. M. Sarazin (Sarazin et al. 1990) devised a simple instrument called Differential Image Motion Monitor (DIMM) for seeing measurement. The instrument is portable, rugged, and easy to
maintain which is perfect for long term site’s performance monitoring and thus it became a standard instrument for site testing purposes. However, the instrument could not measure turbulence’s structure profile which is getting more and more important ever since the application of adaptive optics.

Due to the fast development of Adaptive Optics (AO) whose performance is strongly correlated with $C_2^2(h)$, study for turbulence structure is necessary. New instrument such as DIMM-MASS, lunar scintillometer, single star SCIDAR, etc. (Masciadri et al. 2009) that could measure turbulence strength at various height have been proposed and utilized with good results. We hereby also proposed a new method for measuring layered structure of turbulence along the line of the sight of a telescope that only requires attaching a fast CMOS to an existing telescope that has an adjustable refocusing stage. By doing fast exposure at different conjugate height and adopting a new metric, the method is fast and simple to acquire knowledge of local turbulence at various height which is beneficial for adaptive optics system’s design and operation.

In section 2, we will introduce the principle of the method. Description of setup and field result will be given in section 3. We will draw a conclusion in section 4.

2 PRINCIPLE

The idea of the method is quite simple as illustrated in Figure 1. A telescope could be considered as a thin lens, $f$ is the focal length of the telescope, while $x'$ is the distance of the image plane from the focal point, and $x$ is the distance of conjugated object plane from the other focal point. When $x' = 0$, as the case for the normal usage of astronomical observation, the telescope is focused to infinity, where $x = \infty$. However, when one moves the imager, either a CCD or a CMOS, away from the focus, the object plane, which is the "plane of interest", would also moves from infinity to a finite closer distance as we known from Newtonian thin lens equation. At the same time, for a powered finite sized aperture focusing at finite distance, the effect of depth of field also begins to play its role. Because the depth of field would "isolate" object within certain distances by smearing other objects outside this range. By combining these two factors, one could image turbulence at a distance within the depth of field by moving the imager to the conjugated image plane. If the imager could move continuously, one could obtain a series of short exposure image of turbulence like the one shown in Figure 2 at different distances from the telescope along the line of sight, and from these obtains information of the vertical strength distribution of turbulence using certain metric.

The metric we propose to use to represent the turbulence strength from these images is the classical "Shannon entropy" from information theory as described in (Gonzalez et al. 2003). We repeat the definition of this entropy in equation 1, where $E$ is the value of the entropy, $p$ is the histogram of the grayscale image, $i$ is number of bins for the histogram, which in our case we chose it to be 256.

$$E = \sum p(i) \cdot \log_2\left(\frac{1}{p(i)}\right)$$ (1)

The reason to use Shannon entropy is that the value of the entropy represent the chaotic nature of random processes, such as the random behaviour of the turbulence image. It is observed that when turbulence is strong, the image of the turbulence would also be more chaotic, which would lead to a higher entropy value than image obtained when turbulence is weaker. Therefore, in principle, it could be used as a metric to reflect the turbulence strength.

In the following section we will show how we did our tests and the results we have obtained so far.
3 TEST SETUP AND RESULTS

We used an 80cm telescope at Xing Long Observatory\textsuperscript{1} for the test. The focal distance of the telescope is 8 meter. An Andor DU-888E back illuminated EMCCD camera was mounted on the electrical focusing stage behind the telescope’s Cassegrain focus. The camera could travel along the electrical focusing stage continuously, and the maximum offset distance for the camera from the focal point is 23mm. There are 1024 × 1024 pixels on the camera. The size of each pixel is $13 \times 13 \mu m^2$ which corresponds to a pixel scale of 0.33 arcsec/pixel. Considering the typical size of the "blob" in the image is approximately the seeing at the conjugate height, and the typical integral seeing for our test site is 2 arcsecs, the pixel scale of our setup should be enough to sense turbulence with strength that could degrade image size larger than 0.33 arcsecs and verify our thoughts. For our observation, since the source we chose was very bright, we set the exposure time of the CCD to be 5 milliseconds, and the frame rate was set to be the maximum frame rate of the camera which is 26 frames per second. No binning was applied since it will change the detecting threshold of the turbulence strength. A Johnson V band filter was installed in front of the camera.

With this setup, we pointed the telescope at a bright source close to the zenith. After locating the focus point where the size of the source on the image was smallest, we move the camera away from the telescope. By doing this, the conjugated object plane was getting closer to the primary of the telescope. In another word, the camera would begin to image atmosphere at a finite distance rather than stars at infinity when the camera was moving in such a fashion.

The source we chose was a 5 magV star at a zenith angle around 10°. It was bright enough for identifying the chaotic structure in images (as the one shown in figure 2) with 5 milliseconds exposure for conjugated heights from 6.4 kilometer to 2.8 kilometer. Out of this conjugated height range, either the "doughnut" shape was too big that light was spread out so that the

\textsuperscript{1} http://www.xinglong-naoc.org/html/en/
An image obtained with an 80cm telescope focused at finite distance. One could see the shapes of the telescope’s primary, secondary and spider. The most important are the “boiling blob structure” shown in the image which is caused by the atmospheric turbulence. These boiling blobs’ structure becomes more chaotic when turbulence is stronger.

Structure was too weak to identify, or the doughnut shape was too small to observe any structure within it with our setup.

We then set our camera at 6 different conjugated heights consecutively. At every conjugated height, we took 50 frames with 5ms exposure time which took about 1 minute in total, then continued on to the next conjugated height by moving the camera along electrical focusing stage which would take about another 2 to 3 minutes each time. We repeated this observation sequence three times. The zenith angle of the star for each of these three exposure sequences were 82°, 80.7° and 79.3° respectively.

From these images, we extracted the value within the doughnut shape in the image, and calculated the entropy without any binning. The calculated results are shown in figure 3. From top to bottom, the three panels shows entropy variation for each of the observation sequence when the star at different elevation angles. Lines in different colors within one panel represent entropy obtained at different conjugated heights in one observation sequence. X axis represents the number of frame that was used calculate the entropy value. Therefore, it could also be considered as time past since the first exposure at a certain conjugate height. Y axis is the entropy value, and each curve represent the continuous evolution of entropy observed at a certain height.
There are three interesting behaviors of entropy that could be noticed from figure 3. Firstly, entropys of different heights tend to be separated from each other, in a “layering” fashion. Secondly, entropys for each of the heights evolves with time. These three behaviors actually corresponds to what we have known about atmospheric turbulence strength perfectly. The atmospheric turbulence profile is layer-like. The strength of turbulence is stronger in lower altitudes than higher altitudes. Turbulence strength evolves with time. Therefore, qualitatively as the first step, that using entropy as a metric for representing turbulence strength at different observed height of the atmosphere is reasonable and feasible.

To further verify the relationship between entropy and $C_N^2(h)$, we calculated entropy of similar images of figure 2 generated from a numerical Monte Carlo simulation (1) given specific $r_0$ (coherence length) value. In this simulation, only one layer of turbulence was considered, and because of this, $r_0$ is directly related to the single layer’s $C_N^2$ value. We generated evolving phase screens with constant $r_0$ value, used these phase screens to generate images to calculate entropy, after calculated the mean value of entropy of this group of images at this $r_0$ then we changed $r_0$ value and repeat this process. In figure 4 we plotted the mean values of entropy vs. the $r_0$ values in dot shape, and a polynomial function fitting these dots. We can see that as the $r_0$ is approaching larger value which indicate a milder turbulence, the entropy value is also dropping down as we have already discussed. This simulation clearly indicated a relationship between the entropy and the strength of turbulence represented by $r_0$ here. However, further theoretical study is still needed to understand this relationship quantitatively that could help guiding instrument parameters selection of this method.

4 DISCUSSION

In this paper, we proposed a new metric, Shannon entropy, to be used for representing turbulence strength. We also demonstrate a practicable method to obtain knowledge of turbulence distribution at different height by adjusting camera’s object plane at different conjugated height and calculate Shannon entropy. The results of our tests shows that the new metric has correlation with atmospheric turbulence strength qualitatively. Initial simulation shows a clear relationship between entropy obtained with method and turbulence strength. For the next step, we are planning to incorporate more detailed simulation to study the relationship between entropy and the commonly used metric $C_N^2(h)$ and theoretical works to fully understand this relationship.

References

Fig. 3  Entropy calculated for three different observation sequences when the source was at elevation angles of 82°, 80.6° and 79.3°. X axis is the number of frame during one continuous exposure sequence for a certain height. Y axis the calculated Shannon entropy.
Fig. 4  Entropy calculated with simulated images vs $r_0$ used for generating these images. Dots are the simulation results, the blue continuous line is a fitting curve of these dots.