A motion planning algorithm of the feed support system of FAST

Rui Yao1,2, Peng Jiang1,2, Jing-Hai Sun1,2, Dong-Jun Yu1,2 and Chun Sun1,2

1 National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100101, China; pjiang@nao.cas.cn, ryao@nao.cas.cn
2 CAS Key Laboratory of FAST, NAOC, Chinese Academy of Sciences Beijing 100012, China;
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Abstract The paper relates to a motion planning algorithm for the feed support system of the Five-hundred-meter Aperture Spherical radio Telescope (FAST). For enhance the stability of the feed support system, the start/termination planning segments are adopted with an acceleration and deceleration section. The source switching planning adopts a combination of a line segment and focal segment to realize the stable control of the feed support system. Besides, during the observation trajectory, a transition segment which is no used for observation data is planned with a required time. Through the simulation of the example, a smooth change is realized after motion planning algorithm presented in this paper.

Key words: FAST, radio Telescope, motion planning

1 INTRODUCTION

As shown in Figure 1, Five-hundred-meter Aperture Spherical radio Telescope (FAST), which is the largest radio telescope in the word (Nan 2006; Nan et al. 2011), completed with its main structure installed on September 25, 2016, after which it entered the commissioning phase (Jiang et al. 2019; Qian et al. 2019; Lu et al. 2019).

The main structures of the FAST include the active reflector and the feed receiving device. The basic idea of the FAST is to reflect and collect radio waves through the active reflector, which can be deformed from a spherical shape to a paraboloid with a 300m aperture. Then, the feed receivers, which are installed in the feed support system, received all the radio waves at the focal point of the paraboloid reflector. Figure 2 shows the three active control mechanisms of the feed support system (Tang & Yao 2011): a six-cable-driven parallel robot, an A-B rotator and a Stewart parallel manipulator. The Stewart parallel manipulator is divided into upper and lower platforms. All the feed receivers are equipped on the lower platform of the Stewart manipulator. The A-B rotator and the Stewart manipulator are worked in the feed cabin. The six-cable-driven parallel robot can control the feed receivers to the correct position, but the orientation of the feed receivers cannot meet the requirement (Yao et al. 2014; Li & Yao 2014). So, A-B rotator in the feed cabin is used to adjust the orientation of the feed receivers to the direction of the celestial source. Through the simulation analysis of the six-cable driven parallel robot, it is found that the influence of wind disturbance is obvious. Therefore, a Stewart manipulator with six degrees of freedom is adopted to reduce the influence of wind disturbance (Shao et al. 2011, 2012), and further adjust the position and orientation of the feed receivers to receive the radio signal with high accuracy (7).
For meeting the astronomical observation requirements, the position, velocity and acceleration of the feed support system should be planned, so as to ensure the stability and efficiency of the observation. Most of the existing motion planning methods used astronomical coordinate system, which cannot guarantee the motion stability of the mechanisms in Cartesian coordinate system. It may result in a large control error, which exceeds the accuracy requirements of feed positioning, making the observation data unavailable. Therefore, the motion planning algorithm of the feed support system of FAST needs to be improved, especially in the start and end stages, as well as the acceleration and deceleration stages during observation.

This paper introduce a new motion planning algorithm of the feed support system under the Cartesian coordinate system. Section 2 describes the feed support system and its key parameters. In section 3, the motion planning algorithm of the feed support system are presented in detail. Section 4 gives the simulation and discussion of the motion planning algorithm in this paper.

2 DESCRIPTION OF THE FEED SUPPORT SYSTEM

As shown in Figure 3, the feed support system mainly includes the six-cable-driven parallel robot, the A-B rotator, the Stewart manipulator and the multi-feed rotating mechanism.

In Figure 3, related coordinates are defined as: The origin point of an inertial frame $\mathcal{R}: O - XYZ$ is located at the spherical center of the active reflector. $X -$ axis points to east direction, and $Y -$ axis points to north direction. Three cable-cabin anchoring points shape the moving plane of the six-cable driven parallel manipulator. The origin point of moving frame $\mathcal{R}_C: O_C - X_C Y_C Z_C$ is located at the center of the moving plane. Initially, the moving plane of the six-cable driven parallel manipulator places horizontally. In this condition, $X -$ axis points to east direction and $Y -$ axis points to north direction. The origin point of frame $\mathcal{R}_{AB}: O_{AB} - X_{AB} Y_{AB} Z_{AB}$ is located at the center of A-B rotator. Initially, the A-B rotator places horizontally. In this condition-axis points to east direction and $Y -$ axis points to north direction. The origin point of frame $\mathcal{R}_S: O_S - X_S Y_S Z_S$ is located at the center of the
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Fig. 2 Feed support system. (a) Design of the feed support system of FAST; (b) Feed cabin;(c) Stewart manipulator.

moving platform of the Stewart manipulator. In this condition, $X$ – axis points to east direction and $Y$ – axis points to north direction. All Coordinates comply with the right hand theorem.

The Coordinates of the feeds under the inertial frame $\mathcal{R} : O – XYZ$ can be described as:
Fig. 3 Main mechanisms of the feed support system.

\[ P_{\text{feed}} = P_{AB} + R_C \cdot (r_{AB} - s - r_{AB-feed}) \]  \hspace{1cm} (1)

Where \( P_{\text{feed}} \) is the position vector of the \( R_{AB} \) under the frame \( \mathbb{R} \). \( R_C \) is the coordinate rotation matrix of \( \mathbb{R}_C \) relative to \( \mathbb{R} \). \( P_{AB} \) is the position vector of the feed center under the frame \( \mathbb{R}_{AB} \). In this paper, Tilt-and-Torsion angle (Bonev 2002) is used to describe the coordinate-axis rotation matrix \( R_C \), which can directly discriminate the azimuth and tilt angles. Three angles are defined: azimuth \( \phi \) tilt \( \theta \) and torsion \( \omega \). These angles are easier to interpret geometrically and allow simple computation and representation of the 3D orientation workspace.

The rotation matrix is derived as follows:

\[
R_C = R_a(\theta) R_z(\omega) = R_z(\phi) R_y(\theta) R_z(-\phi) R_z(\omega) = \begin{bmatrix}
\cos \theta \cos \omega - \sin \theta \sin \omega & -\cos \theta \sin \omega + \sin \theta \cos \omega & \cos \theta \sin \phi \\
\sin \theta \cos \omega + \cos \theta \sin \omega & \cos \theta \cos \omega - \sin \theta \sin \omega & -\cos \theta \sin \phi \\
-\sin \phi \cos \omega & \sin \phi \sin \omega & \cos \phi
\end{bmatrix}
\]  \hspace{1cm} (2)

Where \( c(*) \) and \( s(*) \) correspond to the \( \cos(*) \) and \( \sin(*) \), respectively.

For the six-cable-driven parallel robot, the torsion \( \omega \) is designed as zero, so,

\[
R_C = \begin{bmatrix}
\cos \phi \cos \theta - \sin \phi \sin \theta & -\cos \phi \sin \theta + \sin \phi \cos \theta & \cos \phi \\
\sin \phi \cos \theta + \cos \phi \sin \theta & \cos \phi \cos \theta - \sin \phi \sin \theta & -\cos \phi \sin \theta \\
-\sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta
\end{bmatrix}
\]  \hspace{1cm} (3)

The \( P_{AB-feed} \) can be described as:

\[ P_{AB-feed} = R_{AB} \cdot (r_{AB} - s - r_{s-feed}) \]  \hspace{1cm} (4)
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Where $R_{AB}$ is the coordinate rotation matrix of $\mathbb{R}_{AB}$ relative to $\mathbb{R}_C$, $r_{AB-\text{feed}}$ is the position vector of the $\mathbb{R}_S$ under the frame $\mathbb{R}_{AB}$, $r_{\text{feed}}$ is the position vector of the feeds under the frame $\mathbb{R}_S$.

In Equation (4), $R_{AB}$ should be decomposed into $\theta_A$ and $\theta_B$ in two rotational axis. According to the required azimuth angle $\phi_{AB}$ and tilt angle $\theta_{AB}$ of the A-B rotator. The relationship of the between them satisfies:

$$R_{AB} = R_z(\alpha) R_y(\theta_A) R_x(\theta_B) R_z(\alpha)$$

(5)

$$R_{AB} = R_z(\phi_{AB}) R_y(\theta_{AB}) R_x(\theta_{AB}) R_z(\omega_{AB})$$

(6)

where $\alpha$ is the angle between axis $A$ and axis $Y$.

From the Equations (5)–(6), $\theta_A$ and $\theta_B$ can be derived as:

$$\sin \theta_B = \sin \alpha \cdot \cos \phi_{AB} \cdot \sin \theta_{AB} - \cos \alpha \cdot \sin \phi_{AB} \cdot \sin \theta_{AB}$$

(7)

$$\sin \theta_A = \frac{\cos \alpha \cdot \cos \phi_{AB} \cdot \sin \theta_{AB} - \sin \alpha \cdot \sin \phi_{AB} \cdot \sin \theta_{AB}}{\cos \theta_B}$$

(8)

From Equations (1)–(8), the position vector $P_{AB}$, which is the control point of the cable-driven parallel robot, can be derived as:

$$P_{AB} = P_{\text{feed}} - R_c \cdot P_{AB-\text{feed}}$$

(9)

3 A NEW MOTION PLANNING ALGORITHM OF THE FEED SUPPORT SYSTEM

As shown in Figure 4, the motion planning process of the feed support system includes as follows:

a) According to the types of astronomical trajectories to be observed, the time parameters needed for motion planning are calculated and sent to the astronomical trajectory planning unit.

b) According to the required observational position and time of the feeds, the motion planning of the feed support system completes the start/termination planning, the observation trajectory planning, and the source switching planning.

c) Based on the Equations (1)–(9), the related planning parameters of the six-cable-driven parallel robot and the A-B rotator can be calculated.

Therefore, a complete planning trajectories includes four parts which are shown in Figure 5: The source switching planning trajectory $P_0P_1$, the start planning trajectory $P_1P_s$, the observatory planning trajectory $P_sP_t$, and the termination planning trajectory $P_tP_2$.

As shown in Figure 6, all the acceleration curve adopted in this paper are curves of the first degree, so that the velocity planning curves are quadratic.

3.1 Start and Termination Planning algorithm of the Feed Support System

In order to reduce the system disturbance of acceleration and deceleration of the mechanisms, a start and termination segments are added to buffer the disturbance and enhance the efficiency of the observatory trajectory.

According to the velocity and acceleration at the beginning of the observatory trajectory $P_1P_2$, the planning time and distance of the start planning trajectory can be calculated based on the given maximum velocity limit and the acceleration limit of the feed support system. The coordinates $P_1$ can be derived by the observatory trajectory on the focal surface. So, time and coordinate value planning can be calculated for the six-cable-driven parallel robot, the A-B rotator, the multi-beam rotation mechanism separately. Then, based on the Equations (1)–(9), the related planning parameters of the six-cable-driven parallel robot and the A-B rotator can be calculated.

Similarly, the termination planning of the feed support system can be completed on the same way.
3.2 Source switching planning algorithm of the Feed Support System

Source switching means the feed needs to move from the current position \( P_0 \) to the start position \( P_1 \) of the next observation task. The Source switching planning can be decomposed into four steps:

a) Finding out \( P_h \): As shown in Figure 7, based on the geometric parameters of the FAST, the closest point on the focal surface to the start position \( P_0 \) can be found, which is denoted as \( P_h \). From the point \( P_0 \) to the point \( P_h \), the straight line segment is used for the source switching. From the point \( P_h \) to the point \( P_1 \), the focal segment is used for the source switching.

b) Calculating the source switching time: determine the source switching time of each stage of the feed support system, including the source switching time of the straight line segment (\( T_{time} \)) and the focal segment (\( T_{focal} \)).

\( T_{focal} \): The coordinate values of the inflection point \( P_h \) is given after Step a). Thus the vector \( P_h P_0 \) is obtained. By setting up a maximum velocity and acceleration of the source switching trajectory and using the acceleration planning curve in Figure 6, the \( T_{focal} \) can be calculated.
Fig. 6 acceleration and velocity planning curves.

Fig. 7 A sketch map of the source switching trajectory.

$T_{time}$: The $T_{time}$ is chosen as the maximum time of the source switching time of the three mechanisms and the feed position of the feed support system on the straight line segment. The three mechanisms include six-cable-driven parallel robot, the A-B rotator and the multi-feed rotational mechanism.
\[ T_{\text{line}} = \max (T_{p-\text{line}}, T_{\text{cable-\text{line}}}, T_{AB-\text{line}}, T_{\text{multifeed-\text{line}}}) \]  

The four kinds of the source switching time are calculated as follows:

- \( T_{p-\text{line}} \) is the source switching time of the feed position on the straight line segment, which is calculated by the coordinate values of the start point \( P_0 \) and the inflection point \( P_h \) based on a given maximum velocity limit and the acceleration limit of the feed support system.

- \( T_{\text{cable-\text{line}}} \) is the source switching time of the angle of the cable-driven parallel robot on the straight line segment, which can be calculated by the given maximum angular acceleration and the angular velocity limit. That is to say, considering the influence of the angle of the cable-drive parallel robot on the cable force, the stability of the feed support system mechanism can be further ensured.

- \( T_{AB-\text{line}} \) is the source switching time of the A-B rotator on the straight line segment. According to the angle switching value of the A-B rotator and the maximum angular acceleration and the angular velocity limit, the whole switching time of the A-B rotator is obtained as \( T_{AB} \). Thus, \( T_{AB-\text{line}} = T_{AB} - T_{\text{focal}} \).

One of the feed of the FAST is the multi-feed receiver (19-beam 1.05-1.45GHz receiver). A rotational mechanism is installed with the multi-feed receiver to realize the rotation motion according to the astronomical observation requirement. Similarly, the \( T_{\text{multifeed-\text{line}}} \) which is the source switching time of the multi-feed rotational mechanism on the straight line segment can be calculated as the \( T_{AB-\text{line}} \).

c) Completing the source switching planning: According to the relationship between time and velocity (angular velocity) and acceleration (angular acceleration), the time and coordinate value planning can be performed for the six-cable-driven parallel robot, the A-B rotator, the multi-beam rotation mechanism separately.

d) Based on the Equations (1)–(9), the related planning parameters of the six-cable-driven parallel robot and the A-B rotator can be calculated.

### 3.3 The Observatory Planning algorithm of the Feed Support System

The observation mode of FAST includes but is not limited to the tracking scan, drift scan, basketweave scan and motion scan. During the observation trajectory, adding acceleration and deceleration planning at inflection point in the motion planning algorithm can ensure the motion stability and accuracy of the feed support system, which will enhance the efficiency of the observation. This paper describes the motion planning algorithm of the motion scan along the right ascension direction and the motion scan along the declination direction.

Figure 8 shows a sample observation trajectory of the motion scan along the right ascension direction in astronomical coordinate system and in Cartesian coordinate system, which includes observation segment along the declination direction (vertical line) and transition segment along the right ascension direction (horizontal line). The inflection is formed at the junction of observation section and transition section. The data of the transition segment has no use, thus the transition segment can be planned. For reducing the mechanism disturbance at the inflection, an acceleration section and a deceleration section can be added on the transition segment. Figure 9 gives the motion planning process. The planned time of the transition segment is defined as \( t_2 \), which is the most important parameter of the observatory planning algorithm.

Figure 10 shows the related parameters of the \( t_2 \) under the Cartesian coordinate system. The dotted line part is the actual route in the time \( t_2 \). The \( V_m \) is the velocity at the end of the observation segment (also at beginning of the transition segment), \( p_m \) is the slope of observation curve, and the \( p_c \) is the slope of the transition segment. Based on the analysis of the observation curve, the \( V_m, p_m, V_c \) and \( t_2 \) can be described as:

\[ V_m = 0.008V_{\text{dec}}^2 + 0.292V_{\text{dec}} + 10.19 \]  

\[ p_m = 0.070V_{\text{dec}} - 0.0133 \]  


Fig. 8 A sample observation trajectory along the right ascension direction.

![Diagram showing motion planning process]

**Fig. 9** Motion planning process of the motion scan along the right ascension direction.

- Scanning velocity along the declination direction
- Calculating the time $t_2$ used for transition segment
- Maximum acceleration and velocity of the feeds during observation
- Distance for acceleration and deceleration on the inflection
- Endpoint of deceleration/acceleration at inflection point
- Completing the observation planning
Fig. 10 Related parameters of the \( t_2 \) under the Cartesian coordinate system.

\[
V_c = -0.71 \frac{60}{t_2} + 10.68 \quad (13)
\]

\[
l_2 = 0.5t_2V_c \quad (14)
\]

Where the unit of the \( V_m \) is mm/s; \( V_{\text{dec}} \) is the scanning speed at declination direction under the astronomical coordinate system and its unit is \(^\circ/h\); \( V_c \) is the scanning speed at right ascension direction under the Cartesian coordinate system.

The maximum tolerance of the \( |p_m - p_c| \) and the maximum acceleration \( A_{\text{max}} \) can be used as the limit condition to calculate the \( t_2 \) by using the acceleration curve which is shown in Figure 6.

In Figure 11, a sample observation trajectory of the motion scan along the declination direction in astronomical coordinate system and Cartesian coordinate system includes observation segment along the right ascension direction (horizontal line) and transition segment along the declination direction (vertical line).

As shown in Figure 12, motion planning process of the motion scan along the declination direction is proposed, which adopts two acceleration sections and two deceleration sections on the transition
Fig. 12 Motion planning process of the motion scan along the declination direction.

The planned time of the transition segment is defined as $t_2$, which is the most important parameter of the observatory planning algorithm.

Figure 13 shows the related parameters of the $t_2$ under the Cartesian coordinate system. The dotted line part is the actual route in the time $t_2$. The $V_m$ is the velocity at the end of the observation segment (forward direction along the right ascension), and $V_{m-inv}$ is the velocity at the beginning of the next observation segment (reverse direction along the right ascension). Based on the analysis of the observation curve, they can be described as:

$$V_m = -0.751V_{asc} + 11.3 \quad (15)$$

$$V_{m-inv} = 0.751V_{asc} + 11.3 \quad (16)$$

Where the unit of the $V_m$ and $V_{m-inv}$ are mm/s; $V_{asc}$ is the scanning speed at declination direction under the astronomical coordinate system and its unit is $h/sec$.

So,

$$t_3 = t_0 + t_1 = 2V_m/A_{max} + 2V_{m-inv}/A_{max} \quad (17)$$
Then, the scanning speed at declination direction under the astronomical coordinate system and the Cartesian coordinate system can be described as:

\[ V_{\text{dec}} = \frac{60}{t_3} \] \hspace{1cm} (18)

\[ V_D = 0.008V_{\text{dec}}^2 + 0.292V_{\text{dec}} + 10.19 \] \hspace{1cm} (19)

Therefore,

\[ t_2 = \frac{(2A_{\text{max}}t_3 + 8V_D) + \sqrt{(2A_{\text{max}}t_3 + 8V_D)^2 - 4(A_{\text{max}}t_3)^2}}{2A_{\text{max}}} \] \hspace{1cm} (20)

### 4 SIMULATION AND DISCUSSION

The main parameters of the feed support system are shown and listed in the Figure 14 and Table 1.

For an example, a motion scan trajectories along the right ascension direction is planned as Figure 15.

Then, the velocity and acceleration result of this trajectory with a start and termination planning is shown in Figure 16.
Fig. 14 A sketch map of the feed support system.

Fig. 15 A motion scan trajectories along the right ascension direction.
Fig. 16  A motion planning result of a motion scan trajectory along the right ascension direction with a start and termination planning. (a) The velocity of the trajectory; (b) The acceleration of the trajectory.

According to the Figure 16, the motion planning algorithm ensure the moving velocity of the feed support system is consistent with the observation speed on the observation segment. Meanwhile, the lower change rate of the velocity and the acceleration of the feed support system reduce the disturbance of the mechanisms, leading to a stability feed support system and a high observation efficiency. Then, the source switching planning can be completed based on the $P_0$ and $P_1$, and the Figure 17 shows the one cable planning result of the six-cable driven parallel manipulator, which is stable.

5 CONCLUSIONS

This paper presents a motion planning algorithm for the feed support system. Firstly, a start/termination planning is adopted for reducing the mechanism disturbance at the beginning/termination of the observation. Secondly, in the process of source switching, a combination of the line segment and focal segment is adopted to reduce the force change of the six-cable-driven parallel robot, which can control stability of the feed support system. Thirdly, a transition segment with planned time is used for reducing the control error on the inflection point of the observation trajectory, which can enhance the observation data efficiency. Finally, a simulation shows the smooth change by the motion planning algorithm used in this paper.

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Fig. 17 A motion planning result of the cable driven parallel robot. (a) The velocity of the trajectory; (b) The acceleration of the trajectory.

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