Analytical model of compact star in low-mass X-ray binary with de Sitter spacetime

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Abstract In this article, we suggest a de-sitter model in favor of compact stars in low-mass X-ray binary. Here, we have consider the presence of cosmological constant (in small scale) to discuss the stellar structure. We conclude that this doping is very much suitable with the familiar physical mode of the low-mass X-ray binary compact stars. We calculate the probable radii, compactness ($u$) and surface red-shift ($Z_s$) of six compact stars in low-mass X-ray binaries namely Cyg X-2, V395 Carinae/2S 0921-630, XTE J2123-058, X1822-371 (V691 CrA), 4U 1820-30 and GR Mus (XB 1254-690). We also offer possible equation of state (EOS) of the stellar object.

Key words: Mass — compactness — redshift — Equation of state.

1 INTRODUCTION

As compact stars (neutron stars/strange stars) plays a crucial role to relate astrophysics, nuclear physics & particle physics, it motivated many scientists to search for the behaviour of highly dense stars. Commonly neutron stars are built almost fully of neutrons whereas strange stars are supposed to be composed entirely of strange quark matter (SQM) or the conversion (up-down quarks to strange quarks) may be confined to the core of the neutron star (Haensel et al. 1986; Drago et al. 2014). It is well known that neutron stars are bounded by gravitational attraction and on the other hand strange stars are bounded by strong interactions as well as gravitational attractions. Therefore, for lower mass neutron stars gravitational bound becomes much weaker than the strange stars. Hence neutron stars become larger in size in comparison to the strange stars of same mass. All the present EOS of neutron star have zero surface matter density, whereas available EOS of strange star obtained a sharp surface density (Farhi & Jaffe 1984; Haensel et al. 1986; Alcock et al. 1986; Dey et al. 1998). Since after few seconds of birth of a neutron star its temperature decreases to less than Fermi energy, hence for a given equation of state the mass and radius of the star depends solely on central density and also it is very hard to find out mass and radius of a neutron star simultaneously. For a detail study we suggest a review work of Lattimer & Prakash (2007). Theoretical prediction of mass and radii of spherically-symmetric non-rotating compact stars are based on analytical or numerical solutions of Tolman-Oppenheimer-Volkov i.e., TOV equations. From observational point of view, some promising area of measuring mass and radius of compact stars (neutron stars/strange stars) are thermal emission from cooling stars, pulsar timing, surface explosions and gravity wave emissions. Experimental scientists face the recent challenges using giant dipole resonances,
heavy-ion collisions and parity-violating electron scattering techniques to measure the density depend-
ability pressure of nuclear matter. Actually, most challenging task is to determine the proper EOS which
describe the internal structure of neutron stars (Ozel 2006; Özel et al. 2009a; Özel & Psaitis 2009b; Özel et al. 2010; Güver et al. 2010a, 2010b). Though masses of a few dozen compact stars have been
determined very exactly (to some extend) in binaries (Heap & Corcoran 1992; Van et al. 1995; Stickland et al. 1997; Orosz & Kuulkers 1999; Lattimer & Prakash 2005, 2007), no information of radius is avail-
able for these systems. Therefore, theoretical study of the stellar structure is essential to support the
correct direction for the newly observed masses and radii. Here, some of the researcher’s work on com-

Casares et al. (2010) estimated the mass of the compact star in Cyg X-2 by using new high-
resolution spectroscopy and it comes out as $1.71 \pm 0.21 M_\odot$. In another work, Steeghs & Jonker (2007) measured the mass of the compact star in V395 Carinae/2S 0921-630 with the help of MIKE echelle spectrograph on the Magellan-Clay telescope by using high-resolution optical spectroscopy and it comes out as $1.47 \pm 0.10 M_\odot$. On the other hand, Gelino et al. (2003) measured the mass of the compact star in XTE J2123-058 as $1.53^{+0.30}_{-0.47} M_\odot$. Muñoz-Darias et al. (2005) measured the mass of the neutron star in low-mass X-ray binary (LMXB) X1822-371 (V691 CrA) by studying the K-correction for the case of emission lines formed in the X-ray illuminated atmosphere of a Roche lobe filling star and that comes out as $1.61 M_\odot \leq M_{NS} \leq 2.32 M_\odot$. The team of Güver et al. (2010b) measured the mass of the compact star in 4U 1820-30 by using time resolved X-ray spectroscopy of the thermonuclear burst of 4U1820-30 and it comes out as $1.58 \pm 0.06 M_\odot$. Barnes et al. (2007) have also determined the mass of the compact object in GR Mus (XB 1254-690) as $1.20 M_\odot \leq M_{NS} \leq 2.64 M_\odot$.

Wilkinson Microwave Anisotropic Probe (WMAP) measurement indicates that nearly 73% of total mass-energy of the Universe is Dark Energy (Perlmutter et al. 1998; Riess et al. 2004) and this dark energy is based on the cosmological constant. To obtain a stable cosmological model, Einstein in 1917, introduced an idea of cosmological constant. Later Zel’dovich (1967, 1968) described this repulsion energy is based on the cosmological constant. To obtain a stable cosmological model, Einstein in 1917, introduced an idea of cosmological constant. Later Zel’dovich (1967, 1968) described this repulsion energy is based on the cosmological constant. However, for viability of the present-day accelerating Universe the earlier cosmological constant $\Lambda$, commonly, assumed to be it time-dependent in the cosmological domain (Perlmutter et al. 1998; Riess et al. 2004). At the same time, space-dependent $\Lambda$ has an desired outcome in the astrophysical point of view as argued by many researchers (Chen & Wu 1990; Narlikar et al. 1991; Ray & Ray 1993) in respect to the behaviour of local massive objects kind of galaxies and elsewhere. In the present motto of compact stars, however, we take cosmological constant, $\Lambda$ as a absolutely constant quantity. This constancy of $\Lambda$ unable to ruled out for the system of very small dimension like compact star systems or elsewhere with various physical needs (MaK 2000; Dymnikova 2002; Dymnikova 2003; Böhmer & Harko 2005). To estimate mass and radii of neutron star Egeland (2007) incorporated the presence of cosmological constant proportionality trust on the density of vacuum. Egeland have done it by applying the Fermi equation of state along with the Tolman-Oppenheimer-Volkov (TOV) equation.

Bottomed by the above knowledge, we organize the presence of cosmological constant in a small scale to study the construction of compact stars in low-mass X-ray binaries namely Cyg X-2, V395 Carinae/2S 0921-630, XTE J2123-058, X1822-371 (V691 CrA), 4U 1820-30 and GR Mus (XB 1254-690) and got a conclusion that incorporation of $\Lambda$ narrates the compact stars in good manners.

2 INTERIOR SPACETIME

We consider stars as static and spherically symmetric body whose interior spacetime as

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

(1)

According to Heintzmann (1969),

$$e^\nu = A^2 (1 + ar^2)^{\alpha}$$
and
\[ e^{-\lambda} = \left[ 1 - \frac{3a r^2}{2} \left( 1 + C \frac{(1 + 4a r^2)}{1 + a r^2} \right)^{-\frac{3}{2}} \right] \]

where A, C and a are metric constants.

We assume that the energy-momentum tensor for the interior of the compact object has the standard form as
\[ T_{ij} = \text{diag}(-\rho, p, p, p) \]
where \( \rho \) and p are energy density and isotropic pressure respectively.

Einstein’s field equations for the metric equation (1) in presence of \( \Lambda \) are then obtained as (taking \( G = 1 \) and \( c = 1 \))
\[ 8\pi \rho + \Lambda = e^{-\lambda} \left( \frac{\lambda'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2} \]
\[ 8\pi p - \Lambda = e^{-\lambda} \left( \frac{\nu'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} \]

Now, from the metric equation (1) and the Einstein’s field equations (2) & (3), we obtain the energy density (\( \rho \)) and the pressure (\( p \)) as
\[ \rho = \frac{3a \left( \sqrt{1 + 4a r^2} (3 + 13a r^2 + 4a^2 r^4) + C (3 + 9a r^2) \right)}{16\pi (1 + a r^2)^{\frac{3}{2}} (1 + 4a r^2)^{\frac{3}{2}}} - \frac{\Lambda}{8\pi} \]
\[ p = -\frac{3a \left( 3\sqrt{1 + 4a r^2} (-1 + a r^2) + C + 7a C r^2 \right)}{16\pi (1 + a r^2)^{\frac{3}{2}} (1 + 4a r^2)^{\frac{3}{2}}} + \frac{\Lambda}{8\pi} \]

From the equation (4) and equation (5) we get the central density (\( \rho_0 \)) and central pressure (\( p_0 \)) of the star gradually:
\[ \rho_0 = \rho(r = 0) = \frac{3a (3 + 3C)}{16\pi} - \frac{\Lambda}{8\pi} \]
\[ p_0 = p(r = 0) = \frac{3a (3 - C)}{16\pi} + \frac{\Lambda}{8\pi} \]

It is known that, \( \Lambda > 0 \) suggests the space is open. In order to explain the accelerating state of the universe, it is supposed that energy in the vacuum is liable for this expansion. In consequence, vacuum energy give some gravitational influence on the stellar structures. It is recommended that cosmological constant is responsible for that energy of the vacuum. The value of cosmological constant, \( \Lambda \) has not been consistent with various scenarios. Though in the cosmological point of view, its order of magnitude is \( 10^{-52} m^{-2} \), in the local scale (for example near the black holes, neutron stars and various massive objects) it is not essential to follow the large scale fine tuning values of \( \Lambda \) (Bordbar et al. 2016).

In this part we have studied the following features of our model presuming the value of \( \Lambda = 0.00111 km^{-2} \) (nearer to the value of B"Ohmer & Harko 2005a; Bordbar et al. 2016). We have considered this value for the mathematical consistency and stability of the compact star. As a and C specify the central density of the configurations, we calculate it and use it in our model as we know that core properties of the compact star depends on the central density.

Also, we observe (Fig. 1) that, matter density and pressure both are maximum at the centre and decreases monotonically to the boundary. Interestingly, pressure falls to zero at the boundary, though density does not. Therefore, it may be justified to take these compact stars as strange stars where the surface density remains finite rather than the neutron stars for which the surface density vanishes at the boundary (Farhi & Jaffe 1984; Haensel et al. 1986; Alcock et al. 1986; Dey et al. 1998). It is to be mentioned here that, we fixed the values of the constants \( a = 0.0016 km^{-2} \) and \( C = 1.133 \), so that the pressure falls from its maximum value (at centre) to zero at the boundary.
3 EXPLORATION OF PHYSICAL PROPERTIES

In this section we have studied the following property of the compact star in low-mass X-ray binary:

3.1 Energy conditions

In our model we observed that all the energy conditions, namely null energy condition(NEC), weak energy condition(WEC), strong energy condition(SEC) and dominant energy condition(DEC) are satisfied at the centre \( r = 0 \) of the star. From Fig. 1, we observe that all the energy conditions maintain well:

(i) NEC: \( p_0 + \rho_0 \geq 0 \),
(ii) WEC: \( p_0 + \rho_0 \geq 0 \), \( \rho_0 \geq 0 \),
(iii) SEC: \( p_0 + \rho_0 \geq 0 \), \( 3p_0 + \rho_0 \geq 0 \),
(iv) DEC: \( \rho_0 > |p_0| \).

See Table 1 for numerical justification of energy conditions satisfied in our model.

<table>
<thead>
<tr>
<th>( \rho_0 ) (km(^{-2}))</th>
<th>( p_0 ) (km(^{-2}))</th>
<th>( \rho_0 + p_0 ) (km(^{-2}))</th>
<th>( 3\rho_0 + p_0 ) (km(^{-2}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00003669894</td>
<td>0.000078945</td>
<td>0.000123425</td>
<td></td>
</tr>
</tbody>
</table>

3.2 TOV equation

In our stellar model we observe that static equilibrium configurations are present due to the availability of gravitational \( F_g \) and hydrostatic \( F_h \) forces.

\[
F_h + F_g = 0
\]

where,

\[
F_g = \frac{1}{2} \nu' (\rho + p)
\]

\[
F_h = \frac{d}{dr} (p - \frac{\Lambda}{8\pi})
\]

Fig. 2 shows that equilibrium state of the compact object under gravitational and hydrostatic forces in our stellar model.
3.3 Stability

Now, we examine the stability of the model. For a stable stellar model it is always required that the speed of sound should be less than speed of light \( (c = 1) \) everywhere within the stellar object i.e. \( 0 \leq v^2 = \left( \frac{dp}{d\rho} \right) \leq 1 \) (Herrera 1992; Abreu et al. 2007). For these purpose we plot the sound speed in Fig. 3(left) and observed that it satisfies well the inequalities \( 0 \leq v^2 \leq 1 \). Therefore our stellar model is well stabled.

Our stellar model is also dynamical stable in present of thermal radiation. The dynamical stability examined by adiabatic index \((\gamma)\). The adiabatic index \((\gamma)\) is express as

\[
\gamma = \frac{\rho + p \frac{dp}{d\rho}}{\frac{\rho}{p}}
\]

If the value of adiabatic index \(\gamma > \frac{4}{3}\) throughout the interior of the stellar body then the stellar model will be stable. From Fig. 3(right) we observe that our stellar model is stable in present of thermal radiation. This type of stability executed by several author namely Chandrasekhar (1964), Bardeen et al. (1966), Knutsen (1988), Mak & Harko (2013) gradually in their work.

3.4 Matching Conditions

Here, we match the interior metric of the star with the exterior Schwarzschild de Sitter metric at the boundary \((r = b)\)

\[
ds^2 = -\left(1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}\right)dt^2 + \left(1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)
\]

From the continuity of the metric function across the boundary, we get the compactification factor as

\[
\frac{M}{b} = \frac{1}{2} \frac{3ab^2 \left(1 + C(1 + 4ab^2)^{-\frac{1}{2}} \right)}{2 \left(1 + ab^2\right)} - \frac{\Lambda b^2}{3}
\]
3.5 Mass-Radius relation and Surface redshift

For a static spherically symmetric perfect fluid sphere maximum allowable mass-radius ratio should be \[ \frac{M_{\text{Mass}}}{{R_{\text{Radius}}}} < \frac{2}{3} \] (Buchdahl 1959). In our stellar model we have calculated the gravitational mass (\( M \)) in presence of cosmological constant as

\[
M = 4\pi \int_0^b \rho r^2 dr = \frac{3ab^3 \left( 1 + C(1 + 4ab^2)^{-\frac{1}{2}} \right)}{4 \left( 1 + ab^2 \right)} - \frac{\Lambda b^3}{6} \quad \text{(8)}
\]

where the radius of the star is taken as \( b \).

Hence, the compactness (\( u \)) of the star be able to written as

\[
u = \frac{M}{b} = \frac{1}{2} \left[ \frac{3ab^2 \left( 1 + C(1 + 4ab^2)^{-\frac{1}{2}} \right)}{2 \left( 1 + ab^2 \right)} - \frac{\Lambda b^2}{3} \right] \quad \text{(9)}
\]

The behaviour of Mass function and Compactness of the star in our model are shown in Fig. 4 and Fig. 5(left).

The surface redshift (\( Z_s \)) analogous to the above compactness (\( u \)) will be

\[
Z_s = \left[ 1 - 2u \right]^{-\frac{1}{2}} - 1 \quad \text{(10)}
\]

Therefore the maximum surface redshift, from Fig. 5(right) for the different compact stars can be easily find out. The radii, compactness and surface redshift of the different compact stars are obtained from Fig. 6, equation (9) and equation (10) and a comparative analysis has been done in Table 2.

<table>
<thead>
<tr>
<th>Star</th>
<th>Observed Mass (( M_\odot ))</th>
<th>Radius from Model (in km)</th>
<th>Redshift from Model</th>
<th>Compactness from Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cyg X-3</td>
<td>1.71 ± 0.21</td>
<td>11.55 ± 0.65</td>
<td>0.331 ± 0.0395</td>
<td>0.2169 ± 0.017</td>
</tr>
<tr>
<td>2S 0921-630</td>
<td>1.44 ± 0.10</td>
<td>10.75 ± 0.35</td>
<td>0.2834 ± 0.0194</td>
<td>0.1962 ± 0.0092</td>
</tr>
<tr>
<td>XTE J2123-058</td>
<td>1.53 ± 0.30</td>
<td>10.8 ± 1.1</td>
<td>0.2897 ± 0.0614</td>
<td>0.1973 ± 0.0287</td>
</tr>
<tr>
<td>X1822-371 (V691 CrA)</td>
<td>1.61 &lt; M &lt; 2.32</td>
<td>11.2 &lt; R &lt; 13.2</td>
<td>0.3740 ± 0.0655</td>
<td>0.2334 ± 0.0254</td>
</tr>
<tr>
<td>4U 1820-30</td>
<td>1.58 ± 0.06</td>
<td>11.2 ± 0.2</td>
<td>0.3087 ± 0.0117</td>
<td>0.208 ± 0.0052</td>
</tr>
<tr>
<td>GR Mus (XB 1254-690)</td>
<td>1.92 ± 0.72</td>
<td>12 ± 2.1</td>
<td>0.3734 ± 0.1352</td>
<td>0.2285 ± 0.0547</td>
</tr>
</tbody>
</table>
4 DISCUSSION AND CONCLUDING REMARKS

It is to be noted here that the model described by Heint Ilia (1969) is useful to study both neutron and strange stars depending upon the choice of the metric parameter $a$, $C$ (Kalam et al. 2016; Kalam et al. 2017). In this article, we have investigated that whether the same Heint Ilia metric is capable to explain the compact stars within low-mass X-ray binaries or not. For which, we have exhibited the physical behavior of the six compact stars within the low-mass X-ray binary (LMXB) namely Cyg X-2, V395 Carinae/2S 0921-630, XTE J2123-058, X1822-371 (V691 CrA), 4U 1820-30 and GR Mus (XB 1254-690) by considering isotropic pressure in nature. Here we have also merged the previous cosmological constant $\Lambda$ in the Einstein’s field equation in favour of study the stellar construction. Effectively, we obtained an analytical solution for the fluid sphere which are really interesting in respect to many physical property, which are as follows:

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**Fig. 4** Variation of the mass function $M(r)$ of our star model (taking $a=0.0016 \ km^{-2}$, $C=1.133$).

**Fig. 5** Variation of the compactness ($u$) and surface red-shift ($Z_s$) of our star model (taking $a=0.0016 \ km^{-2}$, $C=1.133$).
Fig. 6 Probable radii of Cyg X-2, 2S 0921-630, XTE J2123-058, X1822-371 (V691 CrA), 4U 1820-30 and GR Mus (XB 1254-690).

(i) In our model at the interior of the compact stars density and pressure are well function (positive definite at the centre) (Fig. 1). It is to be mentioned here that pressure and density are both maximum at the origin and interestingly pressure falls to zero (monotonically decreasing) towards the boundary while density does not. Therefore it is justified to designated these compact stars as strange stars where the surface density does not vanishes in place of the neutron stars dissimilar the surface density vanishes at the boundary. Here, we assume the values of constants ($a, C$) in the metric and $\Lambda$...
so that pressure reduces to zero at the boundary. By assuming of the constant’s values \( a, C \) and \( \Lambda \), we calculate the central density, \( \rho_0 \) as \( 5.67 \times 10^{-6} \text{km}^{-2} (7.651 \times 10^{14} \text{gm/cm}^3) \) and central pressure, \( p_0 \) as \( 2.245 \times 10^{-7} \text{km}^{-2} (5.557 \times 10^{15} \text{dyne/cm}^2) \) (Table 1). It satisfies energy conditions, TOV equation and Herrera’s stability condition. It is also stable regarding infinitesimal radial perturbations. From mass function (equation 8), all desired inside properties of a compact star be possible to evaluated which satisfies Buchdahl mass-radius relation \( \frac{2M}{R} < \frac{8}{9} \) \( (\text{Figs. 4, 5(left))} \) which is optimal \( (Z_s \leq 0.85) \) \( (\text{Haensel et al. 2000}) \). We estimated the EOS and that would be like \( p = \alpha e^{(-p/\beta)} + \eta e^{(-p/\delta)} + \xi \) \( \text{whereinto } \alpha, \beta, \eta, \delta, \xi \text{ are constants and their units are } \text{km}^{-2} \) \( (\text{Fig. 7}) \) \( \text{indicates that a stiff equation of state (Özel (2006); Lai & Xu (2009) and Guo et al. (2014)) rather be a soft equation of state.} \)

(ii) From our mass function graph Fig. 6, equation (9) and equation (10), we obtain the radii, compactness and surface red-shift of six compact stars within the low-mass X-ray binary (LMXB) as like Cyg X-2, V395 Carinae/2S 0921-630, XTE J2123-058, X1822-37 1 (V691 CrA), 4U 1820-30 and GR Mus (XB 1254-690). The detail comparison chart are shown in Table 2.

It is to be mentioned here that we actually considering Heinl IIa metric with de-Sitter spacetime to describe the compact stars within low-mass X-ray binaries where inlaid metric parameters \( a, C \) are assess by computing all modes of necessary situations. When metric parameters values are known, the EOS additionally the central density are settled. In general, the mass-radius curve are considered under a conferred equation of state for different values of central density; with a definite value of the central density, the mass and radius of a compact star are settled. But our model is diverse and theoretically attractive. According to our model, six compact stars within the low-mass X-ray binary (LMXB) namely Cyg X-2, V395 Carinae/2S 0921-630, XTE J2123-058, X1822-371 (V691 CrA), 4U 1820-30 and GR Mus (XB 1254-690) derive the identical values of \( a, C \) and therefore the identical central density and the identical equation of state. Further interesting part of our stellar model is that, if we begin out of the center by a particular central density, the construction of a compact star can be determined by preventing at any radius whereinto pressure arrive to zero.

Therefore, our conclusion is that we may find useful relativistic model in the sake of compact stars within low-mass X-ray binaries by suitable choice of the values of the metric parameters \( a, C \) in the metric given by Heinl IIa (1969).
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