Weekly inter-technique combination of SLR, VLBI, GPS and DORIS at the solution level

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Abstract Constructing and maintaining a stable terrestrial reference frame is one of the key objectives of fundamental astronomy and geodesy. The datum realization for all the global terrestrial reference frame versions such as ITRF2014 and its predecessor ITRF2008, etc. is by assuming linear time evolution for transformation parameters and then imposing some conditions on these Helmert transformation parameters allowing defining the combined frame of the long-term solution at a reference epoch and its time evolution. By definition, ITRF adopted only linear motion of the Helmert parameters with uncertainty, while some publications have shown that translation-related uncertainties are underestimated. In this paper, we investigate a new approach, which is based on weekly estimation of station positions and Helmert transformation parameters from a combination of the solutions of four space-geodetic techniques, i.e., SLR, VLBI, GPS and DORIS. For this study, an interval of one week is chosen because the arc length of the SLR solutions is seven days. The major advantage of this weekly estimated reference frame is that both the non-linear station motions and the non-linear origin motion are implicitly taken into account. In order to study the non-linear behavior of station motions and physical parameters, ITRF2008 is used as a reference. As for datum definition of weekly reference frame, on the one hand SLR is the unique technique to realize the origin and determine the scale together with VLBI, on the other hand the orientation is realized via no net rotation w.r.t. ITRF2005 on a subset of core stations. Given the fact that without enough collocations (i.e., at least three co-located stations) an inter-technique combined TRF could not exist, the selection and the relative weight of the local ties surveyed at co-location sites is a critical issue. To get stable results, we first assume that, if there were no events such as equipment changes between the measurement epoch of local tie and that of space-geodetic solution (this time span called the stable period hereafter), the relative position between the two co-located stations should be invariant and this local tie could be used for computing the inter-technique combined reference frame in those weeks during the stable period of this tie. The resulting time series of both station positions and transformation parameters are studied in detail and are compared with ITRF2008. The residual station positions in the weekly combined reference frame are usually in the range of two millimeters without any periodic characteristic, but the residual station positions by deducting the regularized station position in ITRF2008 may reach a magnitude of a few centimeters and seem to have a significant annual signal. The physical parameters series between the weekly reference frame and the ITRF2008 also show the obvious existence of annual signal and reach a magnitude of one centimeter for origin motion and two ppb for the scale.
1 INTRODUCTION

The Earth is a complicated system, and it has been continuously moving since it had been formed. In order to quantify the changes of the Earth system in space and time, it is critically important to define the reference system that can reflect the dynamic variation of the Earth (Collilieux et al. 2010). Therefore, constructing and maintaining a stable terrestrial reference frame is one of the key objectives of fundamental astronomy and geodesy. The International Terrestrial Reference System (abbreviated in ITRS, (Petit et al. 2010), (Dermainis 2015)) realizations are updated at intervals of three to five years through the International Terrestrial Reference Frame (ITRF) consisting in not only a set of regularized station Cartesian coordinates parameterized as a coordinate triple $(x, y, z)$ at a reference epoch $t_0$, a constant velocity per station coordinate component and a full variance-covariance matrix of these previous parameters, but also seven Helmert transformation parameters $H(T_x, T_y, T_z, D, R_1, R_2, R_3)$ at the selected conventional reference epoch $t_k$ of the combination and their time derivatives $\dot{H}(\dot{T}_x, \dot{T}_y, \dot{T}_z, \dot{D}, \dot{R}_1, \dot{R}_2, \dot{R}_3)$. Due to this parameterization, on the one hand linear motions for ground stations are assumed, with some discontinuities and post-seismic deformations enforced at sites affected by major earthquakes or equipment changes, and on the other hand the ITRF origin (Dong et al. 2003) is considered theoretically representative of the long-term center of mass of the total Earth system (CM) and coincides with the long-term averaged CM observed by the Satellite Laser Ranging (SLR). Various publications have shown that the residual nonlinear station motions e.g. (Bloßfeld et al. 2014) can reach a magnitude of a few centimeters due to not considered loading effects and that geo-center variations is within 1 cm level (Dong et al. 1997).

The combination results of different space geodetic techniques (SLR, VLBI, GPS, DORIS) gives two kinds of products (Belda et al. 2017). On the one hand, the Earth orientation parameters (EOPs) that define the orientation of the rigid TRF in space and, on the other hand, the coordinates of collocation stations by which the ITRF is realized. The Earth Orientation Center of the IERS, located at Paris Observatory, SYRTE, has the task to provide to the scientific community the international reference time series for the EOPs, called "IERS C04" (Combined 04, (Bizouard et al. 2017)), resulting from a combination of intra-technique EOP series, each of them associated with a given space-geodetic technique. The ITRS Center of the IERS, hosted by Institut Géographique National (IGN) in France, is responsible for the realization and maintenance of the official ITRF. Two other ITRF combination centers are also generating combined solutions using ITRF input data: Deutsches Geodätisches Forschungsinstitut (DGFI) of the Technischen Universität München (TUM) in Germany (Seitz et al. 2012) and Jet Propulsion Laboratory (JPL) (Wu et al. 2015).

ITRF2014 (Altamimi et al. 2016), which is available at ftp://itrf.ign.fr/pub/itrf/, is the most up-to-date release of relatively recent generation of combined products based on a two-step combination scheme in use since the release of ITRF2005 (Altamimi et al. 2007): single technique inputs are individually stacked to estimate a long-term solution per technique comprising a set of station position offsets and velocities and daily EOPs which are subsequently combined together along with local tie observations at co-located sites (Sarti et al. 2004). The ITRF2014 origin is defined solely by ILRS SLR data, i.e., satisfying the condition of zero translation and zero translation rate with respect to the SLR cumulative solution. Its scale is specified to have zero scale and zero scale rate between ITRF2014 and the arithmetic mean of intrinsic scales of SLR and VLBI solutions, while its orientation is consistent with the previous ITRF2008 (Altamimi et al. 2011). As each iteration of the ITRF provides improvements in the precision and accuracy of the global reference frame, ITRF2014 introduce two main improvements dealing with nonlinear station motions, i.e., modeling the annual and semi-annual signals for stations with sufficient time span (longer than 2 years) and post-seismic deformation (PSD) for sites affected by major earthquakes.
DTRF2014 ([Seitz et al. 2015](#)) is based on the combination of constraint-free normal equation systems (NEQs) consisting of two main parts since the release of DTRF2008: the time series of NEQs provided by the Technique Centers are accumulated to one NEQs per technique via extending the weekly/session-wise NEQs by station velocities and then the NEQs of the different techniques are combined and solved by adding some necessary pseudo-observations (i.e., local tie vectors and combination of velocities at co-location sites) and the geodetic datum ([Angermann et al. 2004](#)).

Based upon a Kalman filter and smoother algorithm ([Soja et al. 2016](#)), the JPL team adopts a state vector formulation including position-related variables, Helmert transformation parameters from each individual observing space-geodetic network to the final combined frame and daily EOPs, assimilates the pseudo-observations of position derived from the four space-geodetic techniques at a weekly resolution along with daily EOPs and, when available, site tie observations and realizes a sub-secular TRF ([JTRF2014, (Abbondanza et al. 2017)](#)) whose origin is at the quasi-instantaneous CM as detected by SLR, whose scale is the weighted average of the quasi-instantaneous scales materialized by fortnightly/weekly SLR and quasi-daily VLBI sessions and whose orientation is defined via application of weekly no-net-rotation (NNR) conditions with respect to ITRF2008.

The above three secular or sub-secular TRFs are called a multi-year reference frame ([Bloßfeld et al. 2015](#)), since the velocity can be used to calculate a position at any epoch. The advantage of this type of parameterization is the possibility to provide station coordinates of high accuracy over a long time span and realize the reference frame with a high long-term stability ([Ronen & Even-Tzur 2017](#)). However, the calculated station positions are not equal to the instantaneous station positions ([Zhu et al. 2011](#)) and the origin of the multi-year reference frame does not coincide with the CM sensed by SLR ([Riddell et al. 2017](#)), so the difference between each other is a major limiting factor for the accuracy of the secular frames. To overcome these problems, a weekly realization of ITRS is performed by combining SLR, VLBI, GPS and DORIS solutions. Compared to weekly single-technique solutions, the combination ([Tornatore et al. 2016](#)) allows one to utilize the strength of each space geodetic technique. For this study, an interval of one week is chosen because the arc length of the SLR solution is seven days. In this paper, we study this new approach in detail and the resulting time series of weekly inter-technique combination by use of the ITRF2008 input data series. Considering the fact that without enough collocations (i.e., at least three co-located stations) an inter-technique combined TRF could not exist, the selection and the relative weight of the local ties surveyed at co-location sites is a critical issue. To get stable results, we first assume that if there were no events such as equipment changes between the measurement epoch of local ties and that of space-geodetic solution, the relative position between each two co-located stations was invariant and this local tie could be used for computing the inter-technique combined reference frame in the weeks belonging to the stable period of this tie, which is different from the method used in the realization of JTRF ([Wu et al. 2015](#)) and the epoch reference frame at DGFI ([Bloßfeld et al. 2015](#)).

2 WEEKLY INTER-TECHNIQUE COMBINATION AT SOLUTION LEVEL

This section describes the used data and the processing scheme mainly including technique-individual pseudo-observation equation construction, the datum realization, the Local Ties selection etc.

2.1 Input data

The weekly inter-technique combination relies not only on space geodesy solutions, but also on local ties (LTs) at co-location sites. In the following, we describe the four sets of data used in the weekly inter-technique combination.

The input space geodesy solutions here are provided on a weekly basis by the IAG International Services of satellite techniques: ILRS, IGS ([Rebischung et al. 2016](#)) and IDS ([Valette et al. 2010](#)) and on a daily (VLBI session-wise) basis by the IVS ([Bachmann et al. 2016](#)). Each per-technique time series has been already a combination of the individual Analysis Center (AC) solutions of that technique.
Table 1: Summary of the input solutions to weekly inter-technique combination

<table>
<thead>
<tr>
<th>Technique type</th>
<th>Solution type</th>
<th>Time span</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLR</td>
<td>Variance-covariance</td>
<td>1994.0-2009.0</td>
<td>Loose</td>
</tr>
<tr>
<td>VLBI</td>
<td>Normal equation</td>
<td></td>
<td>None</td>
</tr>
<tr>
<td>GPS</td>
<td>Variance-covariance</td>
<td></td>
<td>Minimum</td>
</tr>
<tr>
<td>DORIS</td>
<td>Variance-covariance</td>
<td></td>
<td>Minimum</td>
</tr>
</tbody>
</table>

Table 2: Relevant data sources

<table>
<thead>
<tr>
<th>Data</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space geodesy solutions</td>
<td>ftp://dgfi.tum.de/pub/ITRF2008/</td>
</tr>
<tr>
<td>Local ties</td>
<td><a href="http://itrf.ensg.ign.fr/local_surveys.php">http://itrf.ensg.ign.fr/local_surveys.php</a></td>
</tr>
</tbody>
</table>

Table 1 summarizes the input solutions, specifying the technique type, solution type, time span and the constraints applied.

The loose constraint used for SLR means that there is an a priori standard deviation of one meter for station coordinates and the equivalent of at least one meter for EOPs (Pavlis et al. 2009). The concept of minimum constraints approach (Glaser et al. 2015) is based on the minimization of the transformation parameters between an external reference frame with position and velocity vector and the estimated reference frame. For more details regarding the minimum constraints, the reader may refer to (Sillard & Boucher 2001).

The LTs (Sarti et al. 2013) used in the weekly inter-technique combination are provided in SINEX format with known measurement epochs. Most of the local tie SINEX files were provided by the national agencies (Michel et al. 2005) operating co-location sites while the most recent surveys operated at all the DORIS co-location sites were re-adjusted by the IGN survey department in order to generate full SINEX files. Since there is usually only one official local tie file in SINEX format at each co-location site, we first assume that if there were no events such as equipment changes between the measurement epoch of local ties and that of space-geodetic solution, the relative position between each two co-located stations was invariant and this local tie could be used for computing the inter-technique combined reference frame in the weeks included in the stable period for this tie.

In addition, there are two kind of auxiliary data including the ITRF2005-TRF file and four discontinuities files about technique-individual stations. The download links of the data used in this paper are given in Table 2.

2.2 Combination strategy

In the pre-processing, apply minimum constraints over the orientation parameters to all loosely constraint solutions, i.e., SLR solutions. The VLBI daily NEQs are also applied minimum constraints over the origin and orientation parameters, and are subsequently solved. Given the importance of LTs in the inter-technique combination and the limited number of LTs, for the first time, we introduce each LT between two different technique observation stations in those weeks when the LT measurement epoch and the observation epoch of each station related to the LT in the technique-specific SINEX file belonging to the same period supplied in the discontinuity files, i.e., the stable period of the LT.
Table 3 Parameters and their representation in technique-specific pseudo-observation equation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Technique type</th>
<th>Parameterization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Station</td>
<td>SLR</td>
<td>Weekly position</td>
</tr>
<tr>
<td>coordinates</td>
<td>VLBI</td>
<td>Daily position</td>
</tr>
<tr>
<td>(m)</td>
<td>GPS</td>
<td>Weekly position</td>
</tr>
<tr>
<td></td>
<td>DORIS</td>
<td>Weekly position</td>
</tr>
<tr>
<td></td>
<td>SLR</td>
<td>Weekly offset</td>
</tr>
<tr>
<td>Translation</td>
<td>VLBI</td>
<td>Weekly offset</td>
</tr>
<tr>
<td>triple (m)</td>
<td>GPS</td>
<td>Weekly offset</td>
</tr>
<tr>
<td>Scale factor</td>
<td>DORIS</td>
<td>Weekly offset</td>
</tr>
<tr>
<td>((10^{-6}))</td>
<td>SLR</td>
<td>Weekly offset</td>
</tr>
<tr>
<td>Rotation triple</td>
<td>VLBI</td>
<td>Weekly offset</td>
</tr>
<tr>
<td>(as)</td>
<td>GPS</td>
<td>Weekly offset</td>
</tr>
<tr>
<td></td>
<td>DORIS</td>
<td>Weekly offset</td>
</tr>
</tbody>
</table>

mentioned before. As for the introduction of LTs, (Abbondanza et al. 2017) indicated that local tie vectors were applied only once at the epoch of their measurements (as stated in the input SINEX files) for the realization of JTRF2014. For the realization of both ITRF2014 and DTRF2014, the pseudo-observations for the selected LTs were introduced once for the final inter-technique combination. In addition, (Blosfeld et al. 2015) mentioned that the LTs were introduced epoch-wise but did not illustrate how to select LTs epoch-wise.

The general concept of the combination strategy used in this paper is based on the combination of minimum constraint solutions resulting from the observation analysis of space geodetic techniques SLR, VLBI, GPS and DORIS (combination at solution level). In contrast to the multi-year reference frame, no station velocities are introduced as new parameters in the pseudo-observation equations since the station coordinates are assumed to be constant within one week. Table 3 gives an overview of the parameters and their representation as they are included in each pseudo-observation equation. A schematic overview of the combination strategy is shown in Fig. 1.

The related calculation could be distinguished into the following three aspects. Section 2.2.1 deals with the basic theory of the parameterization for the weekly inter-technique combination. Section 2.2.2 presents the methodology to introduce the LTs per week in detail. In Section 2.2.3, the realization of the weekly reference frame datum (origin, scale, orientation) is described.

2.2.1 Construction of weekly inter-technique combination NEQs

The combination of the technique-dependent solutions is done afterwards by setting up a second least squares adjustment where for each individual solution \( s \) and each point \( i \), the position vectors \( X_s \) whose element is \( X_s^i = (x_s^i, y_s^i, z_s^i) \) for each point \( i \), are used as new pseudo-observations with the weighting matrix \( P_s \) \((P_s = \text{the inverse of the covariance matrix of the individual solution})\) and seven individual Helmert transformation parameters \( (\text{i.e., the translation vector } T, \text{ the scale factor } D \text{ and three orientation angle for constructing the orientation matrix } R) \) have to be introduced in order to ensure that the datum of the combined solution can be realized independently from the individual input solutions. A general combination physical model is given by

\[
X_s^i = X_c^i + T_k + D_k X_c^i + R_k X_c^i \quad (1)
\]

Where for each individual-technique reference frame implied in the individual solution \( s \) in one week \( k \), \( T_k, D_k \) and \( R_k \) are the translation vector, the scale factor and the rotation matrix from the weekly combined reference frame \( c \) to each individual-technique reference frame, respectively. The
**Fig. 1** simplified flowchart of the weekly inter-technique combination procedure
pseudo-observation equation for each technique $s$ ($s = L, R, P, D$ representing SLR, VLBI, GPS or DORIS, respectively) can be written in the following form:

$$v_s = A_s \delta x_s - I_s$$  \hspace{1cm} (2)

Herein, $v_s$ is the vector of residuals, $\delta x_s$ is the corrections of parameters with respect to a priori values of these parameters, and $I_s$ is the vector called observed minus computed with a priori values of parameters (O-C):

$$I_s = X_s - A_s x^{apr}_s$$  \hspace{1cm} (3)

Herein, $x^{apr}_s$ are a priori values of these parameters where superscript $apr$ represents a priori value and $A_s$ is the design matrix of partial derivatives constructed upon a priori values of station positions ($\cdots, x^{apr}_{c,i}, \cdots$) in the weekly combined reference frame, wherein $1 < i < n$ ($n$ is the number of stations):

$$A_s = \begin{pmatrix}
\ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & 0 & 0 & x^{apr}_{c,1} & 0 & z^{apr}_{c,1} - y^{apr}_{c,1} \\
0 & 1 & 0 & 0 & y^{apr}_{c,1} - z^{apr}_{c,1} & 0 \\
0 & 0 & 1 & z^{apr}_{c,1} - y^{apr}_{c,1} & x^{apr}_{c,1} & 0 \\
\ddots & \vdots & \vdots & \vdots & \vdots & \vdots 
\end{pmatrix}$$  \hspace{1cm} (4)

In this paper, the a priori value of each transformation parameter from the weekly combined reference frame to technique-specific frame equals to zero, while the a-priori values of the station coordinates in combined TRF equal to the solutions in the SINEX file in the first run. During the iterative process, the a-priori values of station coordinates and Helmert transformation parameters should be updated by

$$x^{apr}(j)_s = x^{apr}(j-1)_s + \delta x_s$$  \hspace{1cm} (5)

where $x^{apr}(j)_s, x^{apr}(j-1)_s$ are the a-priori values of technique $s$ parameters in the $(j+1)th$ and $jth$ run, respectively, while $\delta x_s$ is the corrections of parameters with respect to $x^{apr}(j-1)_s$ in the $jth$ run. Since the design matrix $A_s$ is related to the a-priori values of station coordinates, it should be also re-calculated using formula (4).

In terms of the principle of least squares, the technique-individual NEQs is

$$N_s \cdot \delta x_s = U_s$$  \hspace{1cm} (6)

wherein $N_s = A^T_s P_s A_s$ is the normal equation matrix and $U_s = A^T_s P_s l_s$ is the vector of the right hand side of the NEQs.

Since these weekly technique-related reference frames are independent of each other, each technique used its own seven Helmert transformation parameters per week. In other words, the four single technique NEQs are decoupled. As long as these four independent NEQs are combined, it become necessary to distinguish the parameters of each technique and the parameters of the combined which are the common of all the elements of four technique-specific parameters. Assuming that the parameters of SLR, VLBI, GPS and DORIS are denoted by $\delta x_L, \delta x_R, \delta x_P, \delta x_D$, respectively, we denote the parameters of the combined as $\delta x$ with

$$\delta x = \begin{pmatrix}
\delta x_L \\
\delta x_R \\
\delta x_P \\
\delta x_D
\end{pmatrix}$$  \hspace{1cm} (7)

After rows and columns with zero elements have to be inserted in the NEQ matrices $N_s$ and zero elements in corresponding rows of the NEQ right-hand side vectors $U_s$, the four single-technique NEQs
can be combined by adding the elements which corresponding to the same parameters. The combined NEQs can then be written as

\[
\begin{pmatrix}
\lambda_L N_L & 0 & 0 & 0 \\
0 & \lambda_R N_R & 0 & 0 \\
0 & 0 & \lambda_P N_P & 0 \\
0 & 0 & 0 & \lambda_D N_D
\end{pmatrix}
\begin{pmatrix}
\delta x
\end{pmatrix}
= 
\begin{pmatrix}
\lambda_L U_L \\
\lambda_R U_R \\
\lambda_P U_P \\
\lambda_D U_D
\end{pmatrix}
\] (8)

Herein \( N_L, N_R, N_P, N_D \) represent the normal equation matrix of SLR, VLBI, GPS and DORIS, respectively; \( U_L, U_R, U_P, U_D \) are the right-hand side vector of SLR, VLBI, GPS and DORIS NEQs; \( \lambda_L, \lambda_R, \lambda_P, \lambda_D \) are the weighting factors of the SLR, VLBI, GPS and DORIS NEQs. The weighting of the different techniques can be done by, e.g., using equal weights, empirically derived weights or by using an iteratively performed VCE (Koch & Kusche 2002). By considering the number of the parameters for station positions equals to that of the pseudo-observations for each technique in one week, the relative weights are assumed to be constant over time and for each weekly combination they are one for all four single-technique NEQs.

### 2.2.2 Local ties

When combining different technique solutions, it is essential to have sufficient collocation sites (Abbondanza & Sarti 2012) connecting diverse independent technique networks in their respective reference frame together. In view that the local surveys are not always performed each week and different space geodesy instruments at a collocation site are occupying simultaneously or subsequently very close locations and the local ties are very precisely surveyed in three dimensions through classical surveys or the GPS technique, we regard that the motion of two or more collocated stations is the same and the corresponding tie vectors is relatively stable. Thereby, if there are not any abrupt changes such as earthquakes, equipment changes et al. at the collocated stations between the tie measurement epoch and the observation epoch, when we perform the inter-technique combination during this stable period, all the tie vectors at these collocated sites are introduced as pseudo-observations with \( \sigma = 1 \text{mm} \) into the current weekly inter-technique combination.

Since the local tie SINEX file \( I \) provides station coordinates with the full covariance matrix \( \text{cov} \), the pseudo-observations, i.e., the tie vectors \( \Delta_I (\Delta_I = (\ldots, \Delta_{i,j}^i, \ldots)^T, \Delta_{i,j}^i = (\Delta x_{i,j}^i, \Delta y_{i,j}^i, \Delta z_{i,j}^i)) \) can be achieved by the following equation

\[
\begin{pmatrix}
\Delta x_{i,j}^i \\
\Delta y_{i,j}^i \\
\Delta z_{i,j}^i \\
\vdots
\end{pmatrix}
= 
\begin{pmatrix}
0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\
\vdots & & & & & & & & \\
\vdots & & & & & & & & \\
\end{pmatrix}
\begin{pmatrix}
x_i^I \\
y_i^I \\
z_i^I \\
\vdots
\end{pmatrix}
\] (9)

where \((\Delta x_{i,j}^i, \Delta y_{i,j}^i, \Delta z_{i,j}^i)\) are the geocentric components of the tie vector \(\Delta_{i,j}^i\) linking two collocated stations \(i\) and \(j\) in a given local tie SINEX file \(I\). According to equation (9), the matrix in the right side of the equation is called coefficient matrix \(A_I\) and then the weight matrix \(P_I\) of the pseudo-observations \(\Delta_I\) are computed by

\[
P_I = (A_I \text{cov}(A_I)^T)^{-1}
\] (10)
In this study, the origin of the weekly combined reference frame (WCRF) is realized by setting \( T_{SLR} = 0 \) for all input weekly SLR solutions. As for the scale, we choose to use a simple mathematical mean of the two scales implied by weekly SLR and session-wise VLBI solutions. Given the fact that the orientation cannot be observed by any space geodetic technique, it is conventionally defined using NNR condition over a well-behaved core network so that each new realization has no net rotation with respect to the previous one at a reference epoch. In this paper, we define the orientation by applying the NNR condition over a subset of stable stations w.r.t. ITRF2005. Each core station coordinate \( x^{05} \) at the epoch of 2000.0 and its linear velocity \( v^{05} \) in ITRF2005 can be used to calculate its coordinate \( x^{05}(t_k) \) at any epoch \( t_k \) by the following formulation

\[
x^{05}(t_k) = x^{05} + v^{05} \cdot (t_k - 2000.0)
\]

3 RESULTS AND COMPARISONS

In this paper, the weekly inter-technique combination is performed only in those weeks when there are four space geodesy solutions in SINEX format and the number of the local ties is enough to connect different techniques. In addition to using ITRF2008 as a reference, we utilize the same input data for the realization of ITRF2008 to estimate a time series of weekly reference frames during the period of 1994.0 to 2009.0.

3.1 Input data analysis

Fig. 2 shows the amount of individual technique SINEX files per week during the time span of GPS week 730 (1994.0) to 1511(2009.0). The number of weekly SINEX files are 722, 782 and 777 for SLR, GPS and DORIS, respectively. There is one week when no session-wise solution file for VLBI is
In total, there are 710 weeks when four space geodesy solutions are provided. As shown in Fig. 3, the increase or decrease of the total amount of stations in the weekly combination is almost totally explained by the change of the number of GPS stations.

In addition that no local ties could be available in two weeks, there are 72 weeks during which not sufficient local ties could be used. Due to lack of local ties, the result is the NEQ with the rank deficiency cannot be solved. Thereby, we only compute 636 weekly reference frames via the weekly inter-technique combination with enough local ties. The number of introduced LTs per week between GPS and other technique stations is shown in Fig. 4. The number of LTs between GPS and DORIS stations increases to be more than ten from the middle of the year 1999. With the decrease of GPS stations, the number of LTs between GPS and SLR or VLBI stations remains relatively more stable, compared with that between GPS and DORIS stations.

As for the change of tie vectors with time, we utilize the space geodesy solutions to investigate their changing characteristics. In this study, after the weekly baseline vectors between every two collocated stations are calculated through their coordinates in their input SINEX file and the corresponding seven Helmert parameters linking their weekly single-technique reference frame and ITRF2008, the discrepancies between the baseline estimated in ITRF2008 and the tie derived from the local tie SINEX file at the measurement epoch are computed weekly. By this, we could preliminarily detect the change of the relation between collocated stations in each pair. For example, the change of the baseline series, with respect to the local tie between the collocated GPS station WTZZ and SLR station 7224, which was measured at the epoch of 2002: 266 (i.e., year: day of year), as shown in Fig. 5, is almost random without apparent trend and most of the tie discrepancies are smaller than 1 cm, which suggests that this local tie linking GPS station WTZZ and SLR station 7224 could be introduced as the pseudo-observation with its covariance information per week when both of these two co-located stations have space geodesy es-

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**Fig. 3** Number of stations per week in the weekly inter-technique combination

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**Fig. 4** Number of LTs per week between GPS and other technique stations.
timates. In other words, the relative position change of these two co-located stations implied by space geodesy estimate series is at 1 cm level in each direction while the change of their distance with time is only in the range of 5 mm.

3.2 WCRF datum parameters

The weekly SLR solution is used to define the WCRF origin, by fixing to zero the three translation parameters of the corresponding weekly solution. Moreover, the weekly SLR solution and the session-wise VLBI solution are used together to define the WCRF scale via a simple mathematical mean method. Fig. 6 illustrates the temporal behavior of the weekly SLR origin components w.r.t. WCRF and that of the weekly SLR and VLBI scale factor w.r.t. WCRF, showing most of weekly translation offsets between SLR solution and WCRF are smaller than 5 mm and the scale factors between SLR solution and WCRF are in two ppb and opposite to that between VLBI solution and WCRF. As the translation and scale parameter series of SLR are re-derived from the 7-parameters Helmert transformation between station coordinates in SLR-specific reference frame and those in WCRF whose origin is already fixed to SLR solution via setting SLR translation parameters $T_L = 0$, the discrepancy of the translation parameters might be caused by the residual errors in these station coordinates in WCRF, and the SLR residual translation parameters are almost random without apparent seasonal characteristic. The scale factor of SLR and VLBI have a clear characteristic of opposite sign due to the scale realization means of WCRF, i.e., setting SLR scale factor $D_L + \text{VLBI scale factor} D_R = 0$. The sum of the scale factor of both is almost equal to zero while the discrepancy might be also on account of the residual errors in these station coordinates in WCRF.

For validation, the station coordinates which have been used for the datum realization are transformed weekly with a seven-parameter similarity transformation to ITRF2008. The obtained time series
of transformation parameters are shown in Fig. 7. Sometimes, the number of LTs is enough to transfer the datum information between different technique networks but the spatial information is poor. Therefore, the estimated WCRF would not have a stable datum and an outlier in the time series might occur. As shown in Fig. 7, the difference of the frame parameters between the WCRF and ITRF2008 is almost at the level of one centimeter. Possible reasons for this difference may lie as following. Firstly, the distribution of the weekly stations used for WCRF realization is much sparser and less homogeneous than that for the ITRF2008. Secondly, the realization of datum definition of WCRF and ITRF2008 are different. To be more specific, the origin of the former coincides with the weekly coordinate origin of the SLR solutions, which equivalently is the quasi-instantaneous CM, while that of the latter is realized through a long-term average of the instantaneous CM as tracked by SLR data. The scale of WCRF is the arithmetic mean of the quasi-instantaneous weekly scales contained in SLR and VLBI solutions, while the scale of ITRF2008 is realized by averaged linear solutions of VLBI and SLR. The orientation of WCRF is realized by NNR over a subnet of stations at every week, while that of ITRF2008 is conventionally defined with NNR over a well-behaved core network at the reference epoch (only once) such that ITRF2008 has no rotational offset and rate from ITRF2005 at the reference epoch J2005.0. In other words, the datum realizations of each WCRF are independent which might allowed non-linear variations over time while the datum realization of ITRF2008 only takes linear variation of each datum parameter into account. Thirdly, in addition that the number of available LTs in the WCRF computation, as shown in Fig. 4, is significantly lower than that in the ITRF2008 realization and the space distribution of the colocation sites used for WCRF establishment, the discrepancies between the weekly space geodesy estimates and LTs introduced per week are up to 30mm as shown in Fig. 5, while for ITRF2008 computation the tie discrepancies at their measurement epochs are less than 10mm. However, it can be

Fig. 5 Weekly time series of the differences of the baseline vector between the collocated station WTTZ (GPS) and 7224 (SLR) in Germany w.r.t. the tie vector values calculated from the local tie SINEX file (14201_BKG_2002-266-ZA.SNX), in millimeter along the X, Y and Z-axes: upper left, upper right, left bottom, and for distance differences: right bottom.
seen that the change of the geo-center sensed by SLR per week w.r.t the origin of ITRF2008 is almost at one centimeter, and shows a certain periodicity which can be further investigated and modeled using other geophysical data. As for the scale factor, it changes with time at the level smaller than 2 ppb. It is expected to find that three rotational parameters of WCRF w.r.t ITRF2008 before the year of 2000 are larger but most of them after 2000 change in five millimeters at the Earth’s surface (Earth radius, 6378137m).

3.3 Station coordinates

In the following, the station coordinate estimates of the single-technique reference frame and the WCRF are compared. The remaining individual station coordinates here are regarded as the differences between the coordinate parameter estimates in WCRF and the corresponding station coordinate triple calculated by use of the corresponding station coordinates from the technique-related SINEX files and the seven Helmert transformation parameters between single-technique reference frame and the WCRF. Fig. 8 depicts the individual differences between the single-technique reference frame and the WCRF for the GPS station IRKT in Russia between 1995.7 and 2009.0 and ALBH in Canada from 1994.0 to 2009.0. For IRKT and ALBH, the residuals in the horizontal station components and the height component are all relatively larger and more disperse before 1997. In total, the residuals are at the level of 1 mm.

In addition, the station coordinate estimates of the WCRF and the ITRF2008 are also compared here. The remaining individual station motions in the WCRF w.r.t. the ITRF2008 are analyzed in the following. For both IRKT station and ALBH station, it is clearly visible from Fig. 9 that the dominant signal in the time series has a seasonal period and the residuals are at the level of several centimeters. In
Table 4 The estimated annual and semiannual signals (Am=amplitude; Ph=phase) in each component of IRKT and ALBH station position differences between WCRF and ITRF 2008 over the time period 1997.0-2009.0. The estimated phases are referred to epoch 1997.0.

<table>
<thead>
<tr>
<th>station</th>
<th>IRKT</th>
<th>ALBH</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>north</td>
<td>east</td>
<td>up</td>
<td>north</td>
<td>east</td>
<td>up</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Am(mm) of the annual signals</td>
<td>8.94</td>
<td>3.74</td>
<td>2.28</td>
<td>6.10</td>
<td>1.91</td>
<td>6.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ph(°) of the annual signals</td>
<td>152.8</td>
<td>141.9</td>
<td>198.6</td>
<td>256.8</td>
<td>99.5</td>
<td>156.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Am(mm) of the semiannual signals</td>
<td>2.77</td>
<td>0.71</td>
<td>1.34</td>
<td>2.33</td>
<td>0.72</td>
<td>2.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ph(°) of the semiannual signals</td>
<td>31.9</td>
<td>252.9</td>
<td>332.1</td>
<td>60.0</td>
<td>1.9</td>
<td>316.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

addition, for the seasonal adjustments of phases, we use the cosine convention, i.e., \( \cos(\omega t - \phi) \) where \( \omega = 2\pi \) for annual signal, \( \omega = 4\pi \) for semiannual signal, and \( \phi \) is the corresponding phase. Then, the estimated amplitudes and phases of the annual and semiannual signals are also given in Table 4 in the north, east and up components of IRKT and ALBH station position differences between WCRF and ITRF2008. It can be clearly seen that the estimated magnitudes of the annual signals were generally larger than those of the semiannual signals in each components of IRKT and ALBH station position differences between WCRF and ITRF2008.

4 CONCLUSIONS

Weekly inter-technique combination are valuable not only to take advantage of the strengths of each technique but also to provide some direct result for the study about the nonlinear motion of both station and weekly reference frame w.r.t. secular reference frame like ITRF, DTRF, JTRF. In this paper, we
Fig. 8 Individual station position differences (in mm) between the coordinates calculated by use of the transformation parameters w.r.t. WCRF and the coordinates in SINEX files and the station coordinate parameter estimates in WCRF of the GPS stations. IRKT (blue) and ALBH (red)

make an assumption of the stability of the relationship between each pair of the collated stations during the time span when no abrupt changes happened at these collocated sites, so that the tie vectors can be reused in many weeks, which make the weekly inter-technique combination to be possible. Although the weekly network geometry is not as well as that for the realization of the secular reference frame, it doesn’t require the linearization of the change of the weekly single-technique reference frame w.r.t. the secular reference frame, such that the nonlinear motion of the weekly frame parameter like origin and scale can be studied further.

Because the number of GPS stations is smaller and their distribution is not perfect, the results of weekly inter-technique combination before 1999 are not good enough. However, the accuracy of the weekly inter-technique combination results is improved significantly afterwards. Since the network geometry is one of the crucial aspects for the realization of the terrestrial reference frame, it is also necessary to set up more technique-related observation stations. In addition, in order to achieve more precise weekly inter-technique combined reference frame, it is essential to make more accurate and denser local surveys at co-location sites in both space and time.

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Fig. 9 Individual station position differences (in mm) between WCRF and ITRF2008 of the GPS stations IRKT (blue plots) and ALBH (red plots)

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