Analysis of the wide area differential correction for BeiDou global satellite navigation system

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Abstract The regional BeiDou Navigation Satellite System, or BDS2, broadcasts a differential correction as Equivalent Satellite Clock Correction to correct both orbit and satellite clock errors. For the global BDS, or BDS3, satellite orbit and clock corrections conforming with RTCA standards will be broadcasted to authorized users. The hybrid constellation and regional monitoring network pose challenge for the high precision separation of orbit and satellite clock corrections. Three correction models as kinematic, dynamic and TWSTFT (Two-way Satellite Time Frequency Transfer)-based dynamic were studied to estimate the satellite orbit and clock corrections. The correction accuracy of the three models is compared and analyzed based on the BDS observation data. Results show that the accuracy (root-mean-squares, RMS) of dual-frequency real-time positioning of the three models are about 1.76 m, 1.78 m and 2.08 m respectively, which is comparable with the performance of WAAS and EGNOS. With dynamic corrections, the precision of Precise Point Positioning (PPP) experiments may reach to about 23 cm after convergence.

Key words: celestial mechanics — methods: data analysis — space vehicles

1 INTRODUCTION

To improve the positioning accuracy of the satellite navigation system, many countries or regions have established the Satellite-Based Augmentation System (SBAS) for GPS and GLONASS, including the American Wide Area Augmentation System (WAAS), European Geostationary Navigation Overlay System (EGNOS), Japanese MTSAT Satellite-Based Augmentation System (MSAS), Indian GPS Aided GEO Augmented Navigation (GAGAN), and so on \cite{Lawrence et al. 2007; Manabe, H. 2008; Seynat et al. 2009}. These SBASs are equipped with signal transponders via GEO satellites, providing real-time orbit correction, clock correction and grid ionospheric correction to authorized users. Next-generation standards, dual-frequency, multi-constellation (DFMC) SBAS, have been proposed in SBAS Interoperability Working Group, to use dual-frequency signals of multi-constellation GNSSs. DFMC SBAS users can eliminate most of ionospheric delay by using dual-frequency signals, there no information on ionospheric delay is broadcasted, and more satellites from multi-constellation GNSS may be used for augmentation purposes. BDS participates in the design for the next-generation DFMC SBAS standards.
Unlike other SBAS, for regional BDS SBAS, orbit and clock corrections are combined as one, as the equivalent satellite clock correction, to make the processing simply. The disadvantage of the method, ignoring the satellite orbit error in different projection directions, leads to the correction accuracy reduction when the satellite orbit error is large. To improve the correction accuracy, the next-generation of BDS SBAS will provide both satellite orbit and satellite clock corrections. Since we focus on the study of orbit and satellite clock correction for DFMC SBAS, and the grid ionospheric correction will not be discussed in the paper.

For the algorithms of orbit and clock correction in SBAS, experts and scholars have conducted some research studies, in which the WAAS uses the snapshot algorithm and kinematic observation equation combined with a Kalman filter to calculate the satellite orbit and clock correction (Tsai, Y. J. 1999). Thales Alenia Space has developed new orbit determination and synchronization modules for EGNOS, the new proposed orbit determination module is based on real time processing using code carrier measurement only, the new synchronization module solves clock errors directly steered to GPS reference time scale, for the stations and satellites, it uses both code carrier and phase carrier measurements as well as the orbits estimated by the orbit determination process (Labé A R et al. 2016). Cao applied the independent two-way time synchronization observation to realize the precision separation of orbit and clock correction (Cao et al. 2014). To meet the requirement of sub-meter level positioning service of BDS SBAS, a new concept of zone correction was proposed by Chen; after applying zone correction, the PPP precision is below 0.15 m horizontally and 0.20 m vertically for a dual-frequency user at a distance of 600 km off the zone center (Chen et al. 2017).

2 DYNAMIC WIDE AREA DIFFERENTIAL CORRECTION MODEL

Satellite orbit error has the characteristics of slow and systematic change. Therefore, it is possible to precisely determine the satellite orbit and then fix the satellite orbit as a known value to determine the satellite clock offset through the pseudorange and carrier-phase observations. In this manner, the residual error of the satellite orbit will be combined into the satellite clock offset, thereby guaranteeing the consistency of the differential correction. This method divides the differential correction into two independent parts, namely, the dynamic orbit correction and the dynamic clock correction, which may also reduce the requirement of the computer memory and improve the operation speed.

The calculation processing of the dynamic wide area differential correction model is shown in Figure 1.
Analysis of the Wide Area Differential Correction for BeiDou

Fig. 1 The processing of the dynamic wide area differential correction model.

2.1 Dynamic Orbit Correction

BDS uses the TWSTFT (Two-way Satellite Time Frequency Transfer) measurement to directly measure the satellite clock offset relative to the BDT (BeiDou Time) (Cao et al. 2014). The TWSTFT satellite clock fixed orbit determination strategy is applied to generate the precisely determined satellite orbit as well as the predicted satellite orbit. The details of the orbit determination are described in ref. (Tang et al. 2016; Zhou et al. 2011), and they show that this orbit determination strategy can improve the user equivalent range error of the GEO and IGSO satellites. It is necessary to extrapolate the satellite orbit since the BDS SBAS has provided the real-time service. For the satellite orbit error changes slowly, we process the orbit determination strategy hourly. The orbit prediction of one hour is used to compare with the broadcast ephemeris orbit, and the orbit correction is calculated as equation (1).

\[
(\Delta x, \Delta y, \Delta z) = \text{orbit}_{\text{TWSTFT-based}} - \text{orbit}_{\text{eph}}
\]  

Where, \((\Delta x, \Delta y, \Delta z)\) is the three-dimensions orbit correction, \(\text{orbit}_{\text{TWSTFT-based}}\) is the predicted orbit from the TWSTFT satellite clock fixed orbit determination strategy, and \(\text{orbit}_{\text{eph}}\) is the orbit computed through broadcast ephemeris.

2.2 Dynamic Clock Correction

\[
P C = |X^{sat} - X_{rcv}| + c(\delta t_r - \delta t_s) + \rho_{trop} + \rho_{com} + \varepsilon_P \\
LC = |X^{sat} - X_{rcv}| + c(\delta t_r - \delta t_s) + \rho_{trop} + \rho_{com} + \lambda N + \varepsilon_L
\]

Dual frequency ionosphere-free pseudorange and carrier phase combinations are used to generate the precise satellite clock in the processing of dynamic clock correction. Measurement equation is given as equation (2), where \(PC\) and \(LC\) are ionospheric-free pseudorange and carrier phase combinations, respectively. \(X^{sat}\) is the satellite position vector, \(X_{rcv}\) is the precisely known monitor receiver coordinates, \(\delta t_s\) and \(\delta t_r\) are separately satellite and receiver clock, respectively. \(\rho_{trop}\) is the tropospheric delay, \(\rho_{com}\) indicates the common error, including the satellite transmitting phase center correction, receiver eccentricity correction, tidal correction and periodic relativistic clock correction, which could be corrected following proper models. \(\varepsilon\) is the multipath and observation noise. \(\lambda N\) is carrier phase ambiguity.

Given \(X^{sat}\) as the fixed value to generate the predicted orbit, the processing estimates epoch-by-epoch satellite and receiver clocks using a data arc of 24 hours. Measurement errors such as tropospheric delay that cannot be precisely modeled are treated as unknown parameters, the phase ambiguities estimated as real number rather than integer number.
Considering the fast-change characteristics of satellite clock offset, the strategy of precise satellite clock processing will be operated every ten minutes. The post-processing satellite clock corrections are obtained from the difference between the last ten-minute precise satellite clock offset and the broadcast satellite clock, and it will be predicted with linear model for real-time users.

3 KINEMATIC WIDE AREA DIFFERENTIAL CORRECTION MODEL

BDS-2 broadcast one-dimensional differential correction, as equivalent satellite clock correction to authorized users (BeiDou, ICD. 2013). Separating the satellite clock and orbit corrections, the four-dimensional differential correction processing with kinematic method is proposed in the paper.

Station clock, satellite clock offset and orbit radial errors are highly correlated in kinematic observation equation (3), the normal equation would be seriously ill-conditional if they were solved simultaneously. The solution would have large deflection compared with true value. The processing of kinematic wide area differential correction computation is shown in Figure 2, which is different from the dynamic strategy because it calculates the satellite clock correction first.

3.1 Kinematic Clock Correction

Dual frequency ionosphere-free pseudorange residual is computed by removing geometric range, satellite clock offset, and tropospheric delay from the CNMC (Code Noise and Multipath Correction) smoothed pseudorange (Wu et al. 2012; Cao et al. 2012) and can be simplified as:

\[ \Delta \rho^i_j = c \delta t_i - \epsilon_{orb} - \epsilon_{satclk} + \epsilon^i_j \]  

Where \( \Delta \rho^i_j \) is the pseudorange residual, \( c \delta t_i \) is the station clock offset; \( \epsilon_{orb} \) and \( \epsilon_{satclk} \) are orbit and satellite clock offset in broadcast ephemeris, respectively.
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According to the ref. (Cao et al. 2012), the equivalent satellite clock correction \( ESclkcor \) is used to correct satellite clock errors and orbit radial errors as well as average projection of orbit tangential and normal errors in combination, given as equation (4).

\[
ESclkcor = \varepsilon_{orb} + \varepsilon_{satclk}
\] (4)

The station clock and equivalent satellite clock correction will be unknown parameters and solved with least-square estimation.

3.2 Kinematic Orbit Correction

As equivalent satellite clock correction does not take into account the effect of orbit error projection difference, the residual orbit errors is estimated with the pseudorange residual corrected the equivalent satellite clock correction and station clock solved.

\[
\delta\varepsilon_{orb} = \Delta\rho_{ji} - c\delta t_i + ESatcor = a_i \cdot x + b_i \cdot y + c_i \cdot z
\] (5)

Where \( \delta\varepsilon_{orb} \) is the orbit projection error, \( a_i, b_i, c_i \) are the orbit error projection coefficients.

If no cycle slip is detected, the epoch-differenced carrier phase residual is also used to compute the epoch-wise variation of the satellite orbit errors.

\[
\Delta L^i_j = c(\delta t_i) - \delta\varepsilon_{orb} - ESatcor_i + \lambda N + \varepsilon^j_i
\] (6)

\[
d\Delta L^i_j = c(d\delta t_i) - (\delta\varepsilon_{orb} - \delta\varepsilon_{orb-1}) - (ESatcor_i - ESatcor_{i-1}) + \Delta\varepsilon^j_i
\] (7)

Where \( \Delta L^i_j \) is the dual-frequency ionosphere-free carrier phase residual, \( d\Delta L^i_j \) is the epoch-differenced carrier phase residual, \( d\delta t \) is the epoch-differenced station clock, which can be achieved using multiple common satellites (Cao et al. 2012; Chao Y C. 1999).

Ignoring the variation of epoch orbit error projection coefficients, the epoch-wise variation of the satellite orbit errors could be estimated as equation (8).

\[
d\tilde{\Delta} L^i_j = d\Delta L^i_j - c(d\delta t_i) - cESatcor
\]

\[
= a_i(x_i - x_{i-1}) + b_i(y_i - y_{i-1}) + c_i(z_i - z_{i-1})
\]

\[
= a_i dx_{\phi,i} + b_i dy_{\phi,i} + c_i dz_{\phi,i}
\] (8)

The MV (Minimum-Variance) estimation approach is adopted for orbit error estimation, which is less sensitive to the observing geometry and the measurement noise (Tsai, Y. J. 1999).

The pseudorange-based satellite orbit \( x_{c,i} \) is taken as the virtual observation of the actual parameters as follows:

\[
v_{c,i} = x_i - x_{c,i}
\] (9)

Where \( x_i \) is the true value of the satellite orbit at epoch \( i \), and \( v_{c,i} \) is the residual.

The phase-based epoch-wise variation of satellite orbit \( dx_{\phi,i} \) is also taken as the virtual observation of the actual parameters.

\[
v_{\Delta \phi,i} = (x_i - x_{i-1}) - dx_{\phi,i}
\] (10)

Where \( (x_i - x_{i-1}) \) is the true value of the epoch-wise variation of the satellite orbit at epoch \( i \), and \( v_{\Delta \phi,i} \) is the residual.

By combining equation (9) and equation (10), we can obtain the smoothed orbit error correction (Chen et al. 2017).

4 DYNAMIC WIDE AREA DIFFERENTIAL CORRECTION MODEL BASED ON TWSTFT

For TWSTFT-based dynamic wide area differential correction model, the processing of the satellite orbit correction is the same as chapter 2.1. The satellite clock correction is calculated based on comparisons between the real-time TWSTFT clock offset and the broadcast satellite clock offset.

The detail processing of the real-time TWSTFT clock offset shall refer to (Liu et al. 2009), which indicate that the precision of the satellite clock estimation from the TWSTFT measurements is better than 1 ns.
5 EXPERIMENTS AND DISCUSSION

The correction accuracy of the three wide area differential correction models compared with BDS observation data from China regional monitoring network, in the form of User Differential Range Error (UDRE), real-time positioning accuracy and precise positioning accuracy. The performance of BDS SBAS is also compared with other SBAS such as WAAS and EGNOS.

5.1 Comparison of Three Wide Area Differential Correction Models

Based on the analysis of three wide area differential correction algorithms, Table 1 summarizes the differences among the three differential correction models.

As seen from Table 1, the kinematic correction model uses pseudorange and epoch-difference carrier phase observation to calculate the satellite orbit and clock correction epoch by epoch. The dynamic model calculates the satellite orbit and clock correction in batch processing with 24-hour data.

Table 1 The differences of three wide area differential correction models.

<table>
<thead>
<tr>
<th></th>
<th>Kinematic</th>
<th>Dynamic</th>
<th>TWSTFT-based</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clock correction</td>
<td>Based on the pseudorange observation, epoch-by-epoch processing</td>
<td>Based on pseudorange and carrier phase observation, batch processing with 24 hours data</td>
<td>Based on real-time TWSTFT clock offset, epoch-by-epoch processing</td>
</tr>
<tr>
<td>Orbit correction</td>
<td>Based on the pseudorange and epoch-difference carrier phase observation, epoch-by-epoch processing</td>
<td>Based on pseudorange and carrier phase observation, batch processing with 24 hours data</td>
<td>Based on pseudorange and carrier phase observation, batch processing with 24 hours data</td>
</tr>
<tr>
<td>Observation data</td>
<td>Phase-smoothed pseudorange and epoch-difference carrier phase observations</td>
<td>Raw pseudorange and carrier phase observations</td>
<td>Real-time TWSTFT clock offset, raw pseudorange and carrier phase observations</td>
</tr>
<tr>
<td>Ambiguity processing method</td>
<td>Eliminated with epoch differenced method</td>
<td>Estimate</td>
<td>Estimate</td>
</tr>
<tr>
<td>Tropospheric parameter</td>
<td>The Saastamoinen model</td>
<td>Estimated every three hours</td>
<td>Estimated every three hours</td>
</tr>
</tbody>
</table>

5.2 User Differential Ranging Error (UDRE)

The User Equivalent Range Error (UERE) under the open service and the UDRE corrected by the three models of differential corrections were calculated and compared with BDS observation data on July 27, 2016. To compare the performance among the different wide area difference models, all three wide area differential models are used with BeiDou observation data from China’s regional monitoring network, and the stations are distributed in Beijing, Sanya, Chengdu, Kashi, Harbin and Wulumuqi.

The formula of the UERE and UDRE calculation are as follows:

\[
\text{UERE} = PC - \left| X^{sat} - X^{rcv} \right| - c(\delta t_r - \delta t_s) - \rho_{trop} - \rho_{cor}
\]

\[
\text{UDRE} = PC - \left| X^{sat} - X^{rcv} \right| - c(\delta t_r - \delta t_s) - \rho_{trop} - \rho_{cor} - \Delta \rho_{cor}
\]

\[
\rho_{cor} \text{ is the systematic errors in length, which can be modeled precisely. According to the system design, } \Delta \rho_{cor} \text{ is the differential correction and is provided to authorized users only to correct their pseudorange observations, for the meaning of other variables, please refer to equation (2).}
\]

Figure 3 shows the statistic of UERE and UDRE of all the visible satellites from six monitoring stations in China, where \( UERE_{os} \) is the open service UERE, \( UDRE_{kin} \) is the UDRE with kinematic differential correction, \( UDRE_{dyn} \) is the UDRE with dynamic differential correction, and \( UDRE_{dynTS} \) is the UDRE with TWSTFT-based dynamic differential correction.
It shows that the three models of differential corrections all effectively improve the user range accuracy compared with UERE under open service. Table 2 gives the detail of statistic of UERE and UDRE resulting from the six monitoring stations.

Table 2 The RMS of UDREs.

<table>
<thead>
<tr>
<th></th>
<th>UEREos/m</th>
<th>UDREkin/m</th>
<th>UDREdyn/m</th>
<th>UDREdynTS/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEIJING</td>
<td>0.91</td>
<td>0.53</td>
<td>0.48</td>
<td>0.52</td>
</tr>
<tr>
<td>SANYA</td>
<td>1.07</td>
<td>0.78</td>
<td>0.59</td>
<td>0.70</td>
</tr>
<tr>
<td>KASHI</td>
<td>0.91</td>
<td>0.66</td>
<td>0.66</td>
<td>0.73</td>
</tr>
<tr>
<td>CHENGDU</td>
<td>0.87</td>
<td>0.57</td>
<td>0.54</td>
<td>0.63</td>
</tr>
<tr>
<td>HAERBIN</td>
<td>0.85</td>
<td>0.50</td>
<td>0.50</td>
<td>0.53</td>
</tr>
<tr>
<td>WULUMUQI</td>
<td>0.90</td>
<td>0.58</td>
<td>0.54</td>
<td>0.58</td>
</tr>
<tr>
<td>AVERAGE</td>
<td>0.92</td>
<td>0.60</td>
<td>0.55</td>
<td>0.62</td>
</tr>
</tbody>
</table>

The statistic results show that the average UDRE is improved by 35%, 40% and 34% for the kinematic correction model; dynamic correction model and TWSTFT-based dynamic correction model respectively, comparing with the average UERE under open service.

5.3 Dual-frequency Real-time Positioning Accuracy with BDS SBAS Corrections

Dual frequency ionosphere-free pseudorange is used to compute the real-time positioning. The observation equations under open service and SBAS service are respectively as follows:

\[
P_{C_{\text{open}}} = |X_{\text{sat}} - X_{\text{rcv}}| + c(\delta t_r - \delta t_s) + \rho_{\text{trop}} + \rho_{\text{com}} + \epsilon_{\rho}
\]

\[
P_{C_{\text{SBAS}}} = |X_{\text{sat}} - X_{\text{rcv}}| + c(\delta t_r - \delta t_s) + \rho_{\text{trop}} + \rho_{\text{com}} + \epsilon_{\rho} - \epsilon_{\text{orb}} - \epsilon_{\text{satclk}}
\]

Where \(P_{C_{\text{open}}}\) is the ionospheric-free pseudorange measurement under open service; \(P_{C_{\text{SBAS}}}\) is the ionospheric-free pseudorange measurement under SBAS service; \(\epsilon_{\text{orb}}\) and \(\epsilon_{\text{satclk}}\) are orbit and satellite clock differential correction, for the meaning of other variables, please refer to equation (2).
With precise-known coordinate monitoring station data, the real-time position coordinates are estimated and compared with the precise coordinates. The position errors are analyzed under open service and SBAS service with three models of differential corrections. The dual-frequency real-time positioning results for 24 hours are shown in Figure 4.

![Figure 4](image)

**Fig. 4** The dual-frequency real-time positioning RMS (the blue line indicates the no-differential model; the aqua line indicates the kinematic correction model; the yellow line indicates the dynamic correction model; and the red line indicates the TWSTFT-based dynamic correction model).

Table 3 shows that the positioning accuracy is improved by 31%, 32% and 24% for the kinematic correction model, dynamic correction model and TWSTFT-based dynamic correction model respectively, corresponding to open service.

In order to verify the reliability of the three differential correction models, we select 12 stations in China to estimate the real-time positioning accuracy. The distribution of the stations is shown in Figure 5.

<table>
<thead>
<tr>
<th>Station</th>
<th>Vertical RMS</th>
<th>3D RMS</th>
<th>Vertical RMS</th>
<th>3D RMS</th>
<th>Vertical RMS</th>
<th>3D RMS</th>
<th>Vertical RMS</th>
<th>3D RMS</th>
<th>Vertical RMS</th>
<th>3D RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beijing</td>
<td>2.28</td>
<td>2.82</td>
<td>1.75</td>
<td>1.96</td>
<td>1.48</td>
<td>1.67</td>
<td>1.84</td>
<td>2.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sanya</td>
<td>1.55</td>
<td>1.94</td>
<td>1.73</td>
<td>1.87</td>
<td>1.09</td>
<td>1.42</td>
<td>1.41</td>
<td>1.82</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kashi</td>
<td>2.32</td>
<td>3.04</td>
<td>1.72</td>
<td>2.05</td>
<td>1.72</td>
<td>2.02</td>
<td>2.08</td>
<td>2.46</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chengdu</td>
<td>1.62</td>
<td>2.25</td>
<td>0.97</td>
<td>1.27</td>
<td>1.27</td>
<td>1.53</td>
<td>1.62</td>
<td>2.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Haerbin</td>
<td>1.55</td>
<td>2.4</td>
<td>1.39</td>
<td>1.65</td>
<td>1.33</td>
<td>1.87</td>
<td>1.53</td>
<td>2.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wulumuqi</td>
<td>2.66</td>
<td>2.98</td>
<td>1.65</td>
<td>1.86</td>
<td>2.13</td>
<td>2.32</td>
<td>2.11</td>
<td>2.37</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>2</td>
<td>2.57</td>
<td>1.54</td>
<td>1.78</td>
<td>1.46</td>
<td>1.76</td>
<td>1.71</td>
<td>2.08</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The day average positioning accuracy of 12 stations is listed in Table 4 from 6 May 2017 to 11 May 2017.

Table 4 The dual-frequency real-time positioning accuracy of 12 stations.

<table>
<thead>
<tr>
<th>Data</th>
<th>Open service/m</th>
<th>Kinematic model/m</th>
<th>Dynamic model/m</th>
<th>TWSTFT-based dynamic model/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.6</td>
<td>2.48</td>
<td>1.72</td>
<td>1.87</td>
<td>1.98</td>
</tr>
<tr>
<td>5.7</td>
<td>2.37</td>
<td>1.71</td>
<td>1.91</td>
<td>2.04</td>
</tr>
<tr>
<td>5.8</td>
<td>2.66</td>
<td>1.91</td>
<td>1.87</td>
<td>1.87</td>
</tr>
<tr>
<td>5.9</td>
<td>2.51</td>
<td>1.78</td>
<td>1.88</td>
<td>2.05</td>
</tr>
<tr>
<td>5.10</td>
<td>2.48</td>
<td>1.71</td>
<td>1.88</td>
<td>1.88</td>
</tr>
<tr>
<td>5.11</td>
<td>2.26</td>
<td>1.69</td>
<td>1.94</td>
<td>1.99</td>
</tr>
<tr>
<td>Average</td>
<td>2.46</td>
<td>1.75</td>
<td>1.89</td>
<td>1.97</td>
</tr>
</tbody>
</table>

It indicates that the real-time positioning accuracy of the three differential correction models is reliable. After applying the differential correction, the average 3D real-time positioning RMS of all three models are below 2 meters.

5.4 The Performance of WAAS and EGNOS

To compare BDS SBAS orbit and satellite clock correction accuracy with other SBAS, the performance of WAAS and EGNOS are established, just considering the orbit and clock corrections. The EGNOS differential corrections can be downloading via FTP at ftp://ems.estec.esa.int/, and the WAAS differential corrections can be downloading via FTP at ftp://nstb.tc.faa.gov/. The GPS users with WAAS/EGNOS capable receivers will be able to perform differential positioning across the United States and Europe.

The observation data from IGS stations within the coverage of EGNOS and WAAS were selected on July 27, 2016. With WAAS orbit and clock differential corrections, dual-frequency real-time positioning accuracy of stations of amc2, whc1, wide, gode, gol2 and holp distributed in United States were
estimated. And with EGNOS orbit and clock differential corrections, the dual-frequency real-time positioning accuracy of stations of hueg, sofi, gras, bzrg, glsv, and hert distributed in Europe were estimated.

Table 5 presents the statistics of positioning errors from Figure 6 and Figure 7 and Section 5.3 is also listed to compare with WAAS and EGNOS. The results show that the real-time positioning accuracy of BDS SBAS is comparable with WAAS and EGNOS, and WAAS has slightly higher precision.

Figure 6 gives the comparison of real-time positioning accuracy of GPS and WAAS, and Figure 7 shows the real-time positioning accuracy of GPS and EGNOS. Table 5 presents the statistics of the positioning errors from Figure 6 and Figure 7. In addition, BDS real-time positioning accuracy resulting from Section 5.3 is also listed to compare with WAAS and EGNOS.

The results show that the real-time positioning accuracy of BDS SBAS is comparable with WAAS and EGNOS, and WAAS has slightly higher precision.

**Table 5** The RMS of dual-frequency real-time positioning with differential correction (unit/m).

<table>
<thead>
<tr>
<th>WAAS station</th>
<th>wide</th>
<th>whc1</th>
<th>amc2</th>
<th>gode</th>
<th>gol2</th>
<th>holp</th>
<th>AVERAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS</td>
<td>2.38</td>
<td>1.9</td>
<td>1.63</td>
<td>2.04</td>
<td>2.13</td>
<td>1.88</td>
<td>1.99</td>
</tr>
<tr>
<td>GPS + WAAS</td>
<td>1.82</td>
<td>1.52</td>
<td>1.27</td>
<td>1.15</td>
<td>1.73</td>
<td>1.51</td>
<td>1.50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>EGNOS station</th>
<th>hueg</th>
<th>sofi</th>
<th>gras</th>
<th>bzrg</th>
<th>glsv</th>
<th>hert</th>
<th>AVERAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS</td>
<td>2.43</td>
<td>2.48</td>
<td>1.93</td>
<td>2.44</td>
<td>2.26</td>
<td>2.17</td>
<td>2.29</td>
</tr>
<tr>
<td>GPS + EGNOS</td>
<td>2.16</td>
<td>1.81</td>
<td>1.3</td>
<td>1.68</td>
<td>1.64</td>
<td>1.52</td>
<td>1.69</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BDS station</th>
<th>beijing</th>
<th>sanya</th>
<th>kashi</th>
<th>chengdu</th>
<th>haerbin</th>
<th>wulumuqi</th>
<th>AVERAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BDS</td>
<td>2.82</td>
<td>1.94</td>
<td>3.04</td>
<td>2.25</td>
<td>2.4</td>
<td>2.98</td>
<td>2.57</td>
</tr>
<tr>
<td>BDS + BDS SBAS</td>
<td>1.67</td>
<td>1.42</td>
<td>2.02</td>
<td>1.27</td>
<td>1.87</td>
<td>2.32</td>
<td>1.76</td>
</tr>
</tbody>
</table>
5.5 Real-Time PPP Using BDS SBAS Corrections

Rho and Langley investigated the use of WAAS corrections for PPP. They successfully applied WAAS orbit and clock corrections to correct carrier phase observation in PPP processing and obtained centimeter-level accurate in horizontal positioning for a 24-hour statistic (Rho et al. 2007; Heßelbarth et al. 2013).

To promote the development of the BDS real-time precise applications, the National BDS Augmentation Service System (NBASS) was established in 2014. With the real-time corrections estimated with 150 nationwide reference stations in China, BDS dual-frequency PPP can achieve the horizontal and vertical accuracy as high as 0.2 m and 0.3 m respectively, at the 95% confidence level (Shi et al. 2017).

In our study, we applied BDS SBAS orbit and clock corrections estimated with only six monitoring stations for PPP processing, and the stations are distributed in Beijing, Sanya, Chengdu, Kashi, Harbin and Wulumuqi. The PPP results with IGS post-precise satellite ephemeris and clock products were also analyzed as a reference.

The observation functions can be simplified as:

\[
P_C = |X_{\text{sat}} - X_{\text{rcv}}| + c(\delta t_r - \delta t_s) + m \cdot ZTD + \varepsilon_p \\
L_C = |X_{\text{sat}} - X_{\text{rcv}}| + c(\delta t_r - \delta t_s) + m \cdot ZTD + \lambda N + \varepsilon_L
\]  \tag{14}

Where \(m\) and \(ZTD\) are mapping function and zenith tropospheric delay, for the meaning of other variables, please refer to equation (2). Errors resulting from the offset and variation of the antenna phase center, tidal displacements, relativity and phase wind-up at the satellite antenna are assumed to have been corrected by models.
Satellite orbit and clock offset which are used to compute geometrical distance in equation (13), are achieved from BDS broadcast ephemeris and BDS SBAS differential corrections, or IGS post-precise satellite ephemeris and clock products respectively.

Table 6 Summarizes the strategy of PPP in the paper.

<table>
<thead>
<tr>
<th>Modeled observable</th>
<th>Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ionosphere-free combination: LC/PC</td>
<td></td>
</tr>
<tr>
<td>30 s</td>
<td></td>
</tr>
<tr>
<td>5°</td>
<td></td>
</tr>
<tr>
<td>LC: 0.01 m; PC: 1.00 m</td>
<td></td>
</tr>
<tr>
<td>The Kalman filter</td>
<td></td>
</tr>
</tbody>
</table>

PPP performance with three models of differential corrections as well as IGS post-precise products were assessed with BDS data from stations as JFNG, GUA1, SHA1 and KUN1 on July 27, 2016. We also use BeiDou observation data from China’s regional monitoring network, and the stations are distributed in Chengdu, Harbin, Wulumuqi and Shantou. All these stations are located in China and the coordinates are precisely known.

The time series of the PPP processing errors were shown in Figure 8, where the blue lines were the results related to kinematic correction model, the green lines were the results from dynamic correction model, and the yellow lines were the results with TWSTFT-based dynamic correction model. The red lines were the results with IGS post-precise products, which may be shown as refer. The top subfigures give the positioning errors in North-south direction, the middle subfigures show the positioning errors in East-west direction, and the bottom subfigures give the positioning errors in Height direction.

According to the IGS post-precise products, the dynamic differential correction model shows the best converge characteristic. The convergence time of the dynamic differential correction model needs about 80 min to achieve the accuracy better than 0.3 m. However, the convergence of kinematic differential correction model needs more than 120 min to obtain the accuracy better than 0.5 m, and the convergence of TWSTFT-based dynamic wide area differential correction model needs more than 300 min to obtain the accuracy better than 0.7 m.

The statistic bias achieved after a 24-hour static PPP are listed in Table 7.

It shows that centimeter-level positioning results can be achieved with IGS post-precise orbit and clock products, which also proved that the PPP processing strategy is correct. After 24-hour static PPP, the positioning results can all reach the sub-meter level, with the dynamic wide area differential correction model being obviously better than the other two models. Because the dynamic wide area differential correction model uses phase observations directly, the kinematic wide area differential correction model only uses epoch differenced phase observations and still has residual tropospheric error. For the TWSTFT-based dynamic wide area differential correction model, there are systematic errors caused by the equipment time delay in satellite clock offset; as a result, the positioning result is worse than the dynamic wide area differential correction model.
Fig. 8 Precise point positioning results with IGS products (red), kinematic wide area differential corrections (blue), dynamic wide area differential corrections (green) and TWSTFT-based dynamic wide area differential corrections (yellow) at station JFNG and SHA1 on July 27, 2016.

Table 7 A 24-hour dual-frequency static PPP results with Open Service (OS), kinematic wide area differential corrections (KIN), dynamic wide area differential corrections (DYN), TWSTFT-based dynamic wide area differential corrections (TSDYN), and IGS post precise satellite ephemeris and clock products (IGS).

<table>
<thead>
<tr>
<th>Station</th>
<th>OS/m</th>
<th>KIN/m</th>
<th>DYN/m</th>
<th>TSDYN/m</th>
<th>IGS/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>JFNG</td>
<td>1.22</td>
<td>0.45</td>
<td>0.15</td>
<td>0.54</td>
<td>0.02</td>
</tr>
<tr>
<td>SHA1</td>
<td>1.61</td>
<td>0.59</td>
<td>0.22</td>
<td>0.62</td>
<td>0.01</td>
</tr>
<tr>
<td>KUN1</td>
<td>1.18</td>
<td>0.46</td>
<td>0.17</td>
<td>0.55</td>
<td>0.01</td>
</tr>
<tr>
<td>GUA1</td>
<td>1.35</td>
<td>0.43</td>
<td>0.27</td>
<td>0.87</td>
<td>0.02</td>
</tr>
<tr>
<td>CHENGDU</td>
<td>0.91</td>
<td>0.48</td>
<td>0.20</td>
<td>0.79</td>
<td>0.01</td>
</tr>
<tr>
<td>HAERBIN</td>
<td>1.54</td>
<td>0.34</td>
<td>0.34</td>
<td>0.63</td>
<td>0.01</td>
</tr>
<tr>
<td>WULUMUQI</td>
<td>1.77</td>
<td>0.67</td>
<td>0.27</td>
<td>0.74</td>
<td>0.01</td>
</tr>
<tr>
<td>SHANTOU</td>
<td>1.44</td>
<td>0.52</td>
<td>0.19</td>
<td>0.60</td>
<td>0.01</td>
</tr>
<tr>
<td>AVERAGE</td>
<td>1.38</td>
<td>0.49</td>
<td>0.23</td>
<td>0.67</td>
<td>0.01</td>
</tr>
</tbody>
</table>

6 CONCLUDING REMARKS

In this paper, we studied the orbit and clock differential correction methods for BDS SBAS, using kinematic, dynamic and TWSTFT-based dynamic models respectively. The accuracy of UDRE, dual-frequency real-time positioning and PPP with the three differential correction models were compared. It
shows that dual-frequency real-time positioning accuracy with the three differential correction models are all below 2 meters, and is improved by 31%, 32% and 24% for the kinematic correction model, dynamic correction model and TWSTFT-based dynamic correction model respectively, corresponding to open service. The dual-frequency real-time positioning accuracy of BDS SBAS is comparable with WAAS and EGNOS.

The PPP precision with dynamic correction model is about 23 cm, PPP precision with kinematic differential corrections and TWSTFT-based dynamic differential corrections may reach to sub-meter level. It may be due to the reason that for the kinematic and TWSTFT-based dynamic correction models, the pseudorange data are still the main data resource when calculating the clock differential corrections. The results of the paper may give certain reference for developing the new generation of BDS SBAS.

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