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The non-Gaussian distribution of galaxies gravitational fields

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Abstract We perform a theoretical analysis of the observational dependence between angular momentum of the galaxy clusters and their mass (richness), based on the method introduced in our previous paper. For that we obtain the distribution function of astronomical objects (like galaxies and/or smooth halos of different kinds) gravitational fields due to their tidal interaction. Within the statistical method of Chandrasekhar we are able to show that the distribution function is determined by the form of interaction between objects and for multipole (tidal) interaction it is never Gaussian. Our calculation permits to demonstrate how the alignment of galaxies angular momenta depend on the cluster richness. The specific form of the corresponding dependence is due to assumptions made about cluster morphology. Our approach also predicts the time evolution of stellar objects angular momenta within CDM and ΛCDM models. Namely, we have shown that angular momentum of galaxies increases with time.

Key words: Galaxy: General — Galaxy: Formation

1 INTRODUCTION

In the models of galaxies and their structures formation the distribution of gravitational fields of their constituents play the decisive role. Many scenarios of such formation have been around for some time (Peebles (1969), Sunyaew & Zeldovich(1972), Zeldovich (1970), Doroshkevich (1973), Shandarin(1974), Dekel(1985), Efstathiou & Silk(1983)). Under the influence of new observational data, these scenarios are constantly being revised and improved, see (Shandarin et al (2012), Giahi-Saravani & Schäfer (2014)) and references therein for latest discussion. The main controversy here is how galaxies acquire their angular moments, which render subsequently to those of galaxy clusters and larger structures. On the other hand, this angular moment acquisition is intimately related to the above gravitational fields distribution.

The commonly accepted model of the Universe is spatially flat homogeneous and isotropic ΛCDM model. The galaxy clusters in this model are formed as a result of adiabatic and almost scale invariant Gaussian fluctuations (Silk(1968), Peebles & Yu(1970), Sunyaew & Zeldovich(1970)). This assumption is the base of the so-called hierarchical clustering model (Doroshkevich (1970), Dekel(1985), Peebles (1969)), the most popular scenario of galaxies formation. Note, however, the presence of the models with non-Gaussian initial fluctuations, see (Bartolo et al (2004)) and references therein. This non-Gaussianity, however, has been postulated in certain form rather then calculated. At the same time, the non-Gaussian distributions can be obtained from initial Gaussian ones as a result of time evolution in the generalized stochastic models, where probability distribution functions (pdf's) are obtained from the solutions of the differential equations of Fokker-Planck type with so-called fractional derivatives (Garbaczewski & Stephanovich (2009), Garbaczewski, Stephanovich & Kędzierski (2011)).

In other words, the initial Gaussian fluctuations (if any) may become non-Gaussian as a result of primordial, fast time evolution. After it, the slower evolution, dictated by the lambda cold dark matter (ΛCDM) scenario, occurs. Although here we do not present the details of this primordial time evolution, one of the aims of the present paper is to draw attention to the method, which permits to calculate the non-Gaussian distribution function, based solely on the form of interaction between astronomical objects. This distribution function is a terminal function for above initial fast time evolution process.

In hierarchical clustering type of scenarios, the large scale structure forms from bottom to top as a consequence of gravitational interactions between the constituents. This means that the smaller structures like galaxies are formed first with their later merger into larger ones. Consequently, the galaxies spin angular momenta arise as a result of tidal interaction with their neighbours (Schäfer (2009)). In the originally hierarchical clustering scenarios, the completely random distribution of galaxies angular momenta has been obtained. Note however, that it has been shown later that the local tidal shear tensor can generate a local alignment of galaxies angular momenta (Catelan & Theuns (1996), Catelan & Theuns (1996a), Lee & Pen (2002), Navarro et al. (2004)). The mechanisms of this type are known as tidal torque mechanisms, which had first been introduced by Hoyle (1951) and later by White (1984), see also the review paper of Kiessling et al.(2015) that discusses galaxies alignments in the context of gravitational lensing. Note, that in our model the angular momentum is the result of tidal interaction with the entire environment, which occurs via interaction transfer from close to distant galaxies, see below. In this sence our approach is the improvement of those considered by Schäfer & Merkel (2012), Catelan & Theuns (1996), Catelan & Theuns (1996a), Lee & Pen (2002), where the "mean" tidal interaction with the entire environment has been considered.

The above tidal torque mechanism has an opposite idea, constructed on the base of Zeldovich pancake model (Sunyaew & Zeldovich(1972), Doroshkevich (1973), Shandarin(1974)). In this model, the structures in the Universe arise from top to bottom. The crucial role here plays a magneto-hydrodynamic shock wave which makes the large structure to fragment. This shock wave arise as the result of asymmetrical collapse of initial large structure and also imparts galaxies with spin angular momentum. The model predicts a coherent, non-random spatial orientation of galaxy planes with the galaxies rotational axes to be parallel to the main plane of a large structure.

In the model of primordial turbulences, the spin angular momentum is a remnant of the primordial whirl (von Weizsaeker (1951), Gamow (1952), Ozernoy (1978), Efstathiou & Silk(1983)). As result it is obtained that the rotational axes of galaxies are oriented not randomly. The preferred direction of angular momentum of galaxies is perpendicular to the initial large structure's main plane.

It had been pointed out in (Gamow (1946), Goedel (1949)) and later in (Collins, & Hawking (1973)), that if the Universe is rotating, the emerging galaxies angular momentum is a consequence of its conservation in a rotating Universe. At that time, the argument against was that this model predicts the galaxies rotational axes alignment, which had not been confirmed observationally (see Godłowski, 2011 for details). Based on this idea, Li & Li-Xin (1998) introduced a model in which galaxy forms in a rotating Universe.

We emphasize, that simple picture, where each of the above approaches (primordial turbulences, hierarchical clustering and Zeldovich pancakes) predict different ways of galaxies rotational axes ordering is not completely true. The point is that in each of the above models including hierarchical clustering, the phase with shock wave can appear. Latter phase is usually accompanied by the collapse of structures or substructures (Melott & Shandarin (1989), Sahni et al (1995), Paulus et al (1995), Mo et al (2005), Shandarin et al (2012)), which may generate the rotational axes ordering. Apparently, the scale of such orientation is different in different models. For instance, in the Bower's scenario (Bower et al (2005)), we do not have hierarchical clustering for all scales of masses. Instead, we have anti - hierarchical clustering in the small scales as tidal interaction effects yield Zeldovich pancake - like objects emergence (Zeldovich (1970)) rather then spherically collapsing haloes. There is, however, a fundamental difference with above classical pancake scenario. Namely, the anti-hierarchical clustering is local as it occurs in small scale.

The model of hierarchical clustering is the only model explicitly taking into account the dark matter existence. The Li model has originally considered the Universe as dust fluid, however, nothing prevents to introduce the dark matter as a background. As a result, in this model, the dark matter is not interacting with observable matter in any other way than gravitational forces. In the remaining models, namely primordial turbulences and Zeldovich pancake model, the only dust component has been considered so that there are no clear and successful attempts to introduce the dark matter there. Therefore, we exclude both models from the present consideration.

Theoretical models of galaxy formation have problems with explaining the observational dependence between structure angular momentum and its mass. This dependence can be seen only in two classes of models. There are the tidal torque scenario (Heavens & Peacock (1988), Catelan & Theuns (1996), Hwang & Lee (2007), Noh & Lee(2006a), Noh & Lee(2006b)) and Li model (Li & Li-Xin (1998), Godłowski, Szydłowski & Flin (2005)). The remaining models do not anticipate such dependence.

Comparing the two models, we should note that Li model needs a global or at least large scale rotation of the Universe. Li & Li-Xin (1998) studied the dependence between the angular momentum and the mass of spiral galaxies and he estimated the rotation of the Universe to be close to the value obtained by Birch (1982). However, the obtained value is too large compared to observed anisotropy in Cosmic Microwave Background Radiation (CMBR). Hence, in the present paper, we consider the Tidal Torque scenario only.

In the present work we perform the comprehensive theoretical analysis of the influence of tidal interaction between astronomical objects on the larger (then initial constituents) structures formation. The idea of our approach is to use the statistical method originally proposed by Chandrasekhar (1943), where we account also for dark matter haloes. The statistical method of Chandrasekhar (1943) permits to derive the distribution functions of gravitational fields and angular momenta of stellar components. Our main result is that in the stellar systems with multipole (tidal) gravitational interaction, the derived distribution function cannot be Gaussian. Instead we obtain the pdf which rather belongs to the family of so-called "heavy-tailed distributions" (Garbaczewski & Stephanovich (2009), Garbaczewski, Stephanovich & Kędzierski (2011), van Kampen (1981)). As we have mentioned above, the obtained non-Gaussian pdf is a result of fractional time evolution for initial Gaussian fluctuations. This function allows us to calculate the distribution of virtually any observables (like angular momentum) of the astronomical structures (not only galaxy clusters but smooth component like haloes, which mass dominate the total mass of the cluster, see Kravtsov, Borgani (2012)) in any (linear or nonlinear) Eulerian approach.

The paper is organized as follows. To make the paper self-contained, in the section 2 we shortly recollect our method (Stephanovich & Godłowski (2015)) putting more impact to its points, important for present consideration. Some technical details are described in the appendix. In the section 3 we discuss the problem of angular momenta pdf. We show that different (physically reasonable) assumptions about the structure of galaxy clusters generate different relations between their mass M and average angular momentum L. We show that it is possible to derive not only the relation $L \sim M^{4/3}$ (like in Stephanovich & Godłowski (2015)) but also recover well-known empirical relation $L \sim M^{5/3}$. We show that while it is possible to discriminate between the above model assumptions theoretically, the present observational data are not sufficient to come to unambiguous conclusion. We also discuss the possibility of observational testing of our theoretical results related to the time evolution of the distribution function of angular momenta and its mean value L. We conclude our article by the section 4.

2 DISTRIBUTION FUNCTION OF GRAVITATIONAL FIELDS

We consider the tidal interaction in the ensemble of galaxies and their clusters in the Friedmann - Lemaître - Robertson - Walker Universe with Newtonian self-gravitating dust fluid ($p = 0$) containing both luminous and dark matter. The tidal (shape distorting) interaction between the astronomical objects can be derived by the multipole expansion of the Newtonian interaction potential between fluid elements (Poisson (1998)). Limiting ourselves to quadrupolar term, we write the Hamiltonian function of interaction between above elements in the form

$$\mathcal{H} = -G \sum_{ij} Q_i m_j V(\mathbf{r}_{ij}),\ V(\mathbf{r}) = \frac{1}{2}\frac{3\cos^2\theta - 1}{r^3}, \tag{1}$$

where G is the gravitational constant, Q_i and m_i are, respectively, the quadrupole moment and mass of i-th object, $r_{ij} \equiv |\mathbf{r}_{ij}|$, $\mathbf{r}_{ij} = \mathbf{r}_j - \mathbf{r}_i$ is a relative distance between objects while θ is the apex angle. Our Hamiltonian function (1) is obtained for the ensemble of N objects, thus generalizing the result of Poisson (1998) for two particles.

Note, that the Hamiltonian function (1) describes the interaction of quadrupoles, formed both from luminous and dark matter. This is important as in real world the galaxies, formed from luminous matter, reside inside dark matter halos that are much more extended and massive. In other words, the Hamiltonian function (1) (and subsequent results) already contains the information about dark matter haloes. We have discussed this question in our previous work Stephanovich & Godłowski (2015). The main point was that the properties of luminous matter (like galaxies and their clusters) give us information about those of dark matter (sub)structures. This point is corroborated by observations (see, e.g. Paz et al.(2008), Bett et al.(2010), Kim et al.(2011), Varela et al.(2012)) that angular momentum of luminous matter is correlated with that of corresponding dark matter haloes. Below we will calculate the angular momentum of luminous astronomical structures. Our formalism can be generalized to describe not only this situation, but the structures with larger smooth component. Namely, in general, the luminous galaxy matter is not only surrounded by dark matter haloes, but also (along with latter haloes) submerged in the ”mud”, which is hypothetical intergalaxies dark matter. We plan to fulfill this interesting generalization in our subsequent publications.

In the function (1), we split the interaction between many stellar objects (particles) to that in pairs, see Appendix A for details. Such splitting is usual, for instance in the theory of magnetism, where the interacting spins ensemble is represented by the sum of all possible couplings between particle pairs i and j. For instance, the three particle interaction may be decomposed as 123 = 12 + 13 + 23, see, e.g. Mattis (2007).

The Hamiltonian function (1) describes the pairwise, shape-distorting interaction between the structures. Namely, this interaction distorts the shape of a given i-th object, which alters its density field $\rho_i(\mathbf{r})$. As the objects have random shapes, their masses m_i and quadrupole moments Q_i vary randomly likewise the gravity field $\mathbf{E}_{quad}$ from these quadrupoles. One should note that latter field is in fact a gradient of the potential, given by equation (1). It has the form:

$$\mathbf{E}_{quad}(\mathbf{r}) = \mathbf{i}_r E_0 \frac{3\cos^2\theta - 1}{r^4}, \tag{2}$$

where $E_0 = GQ/2$ and $\mathbf{i}_r$ is the unit vector in radial direction.

According to statistical method of Chandrasekhar (1943), the distribution function of random quadrupolar fields is

$$f(\mathbf{E}) = \overline{\delta(\mathbf{E} - \mathbf{E}_i)}, \tag{3}$$

where $\delta(x)$ is Dirac δ - function, while $\mathbf{E}_i \equiv \mathbf{E}_{quad}(\mathbf{r}_i)$ is given by Eq. (2) where the bar means averaging over spatial (and any other possible) disorder. Moreover, if all objects in the ensemble are similar (no randomness), then the distribution function is represented by the simple delta-peak, centered at the field $\mathbf{E}_i$. The disorder broadens this delta-peak, giving rise to ”bell-shaped” continuous probability distribution, see Stephanovich (1997), Semenov & Stephanovich (2002), Semenov & Stephanovich (2003) and references therein.

The explicit averaging in Eq. (3) is performed with the help of the integral representation of Dirac δ - function, see Stephanovich & Godłowski (2015) for details. The idea is that the mass and quadrupole moment of the object in the volume V obey the uniform distribution with probability density equal to $1/V$. In such a case we introduce the number of objects N so that in the limit $N \to \infty$ and $V \to$

∞, their density $n = N/V$ remains constant. Final expression for the distribution function (3) reads (Stephanovich & Godłowski (2015))

$$f(\mathbf{E}) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} e^{i\mathbf{E}\boldsymbol{\rho} - F(\rho)} d^3\rho, \tag{4}$$

$$F(\rho) = n \int_V \left[1 - \frac{\sin \rho E(\mathbf{r})}{\rho E(\mathbf{r})}\right] d^3r. \tag{5}$$

In this case $F(\rho)$ is in fact the characteristic function for random gravitational fields distribution. Note also that characteristic function $F(\rho)$ depends only on modulus ρ and not its angles. This will result (see Eq. (6) below) in the only field modulus dependence of pdf of random gravitational fields. The reason is that we take only zz component of quadrupolar field in Eq.(2). If we need the complete (i.e. including its possible angular dependence) distribution function of vector $\mathbf{E}$, we should account for complete tensor structure of Hamiltonian (1) $\mathcal{H} = -G\sum_{ij\alpha\beta} Q_{i\alpha\beta} m_j V_{\alpha\beta}(\mathbf{r}_{ij})$, $\alpha, \beta = x, y, z$. Such account (Stephanovich (1997), Semenov & Stephanovich (2002), Semenov & Stephanovich (2003)), while not changing our conclusions qualitatively (and in many cases quantitatively, see below), will make the problem to be tractable only numerically. At the same time our present approach permits to gain analytical insights into the problem (for example investigate the implication of non-Gaussian character of distribution function of gravitational fields), which is good starting point for future numerical simulations. One more justification of the radial distribution is the results of numerical simulations in halo model (Schneider, Bridle (2010)), where the axes of galaxies embedded in dark matter halo, were preferentially radially oriented.

Moreover, the spin angular momentum is usually known only for very few galaxies and other structures. For this reason, the spatial orientation of galaxies (see, for example, Flin, Godlowski (1986), Romanowsky, Fall (2012)) is studied instead of their angular momenta. Alternatively, only the distribution of position angles of galaxy planes is analysed in Hawley & Peebles (1975).

In more realistic models of galaxy clustering we can assume that the stellar objects like galaxies density is not a constant but rather depends on their separation $n = n(\mathbf{r})$. The other factor, which may improve the coincidence with observational results is to consider the galaxy clustering within a model of inhomogeneous distribution of masses (and/or quadrupolar moments) in the large scale structure. The idea here is to introduce the distribution function of masses $\tau(m)$, which had been put forward by Chandrasekhar (1943).

It is important that distribution function $f(\mathbf{E})$ (4) in general case could be much more complicated than simple Gaussian. We had shown in Stephanovich & Godłowski (2015) that for multipole interaction between astronomical objects, the function (4) does not admit Gaussian limit. The calculation of $F(\rho)$ (5) generates following explicit form of $f(E)$ (Stephanovich & Godłowski (2015))

$$f(E) = \frac{1}{2\pi^2 E} \int_0^{\infty} \rho\, e^{-\alpha\rho^{3/4}} \sin \rho E \, d\rho, \tag{6}$$
$$\alpha = 2\pi n \cdot 0.41807255 \cdot E_0^{3/4}.$$

The expression (6) is the chief theoretical result of our studies. The distribution function (6) depends parametrically on the objects (i.e. both luminous and dark matter) density n, and on average quadrupole moment Q.

The normalization condition for distribution function (6) reads

$$4\pi \int_0^{\infty} E^2 f(E) dE = 1. \tag{7}$$

As we have shown previously (Stephanovich & Godłowski (2015)), the distribution function of the gravitational fields cannot be Gaussian for multipole interaction between galaxies or any other astronomical objects including elements of dark matter halos. However, all previous theories postulated the distribution function in the Gaussian form rather then calculated it. We mention here that non-Gaussian distribution have also been postulated rather then calculated in Bartolo et al (2004). In our opinion, non-Gausssian, heavy-tailed nature of the above pdf captures the essential physics of the systems with long-range gravitational multipole interaction. Namely, the long-range interaction in such systems makes the objects (galaxies, their clusters and even the dark matter halos) to interact with each other also at very large separations. This, in turn, implies nonzero probabilities of such configurations, contrary to the case of Gaussian distribution, generated by short-range interactions. Below we will see the important implications of this fact.

To plot the function $f(E)$, we define the dimensionless variables $\rho E = x$ and $\beta = E/\alpha^{4/3}$.In these variables the integral (6) assumes the form:

$$f(\beta) = \frac{H(\beta)}{4\pi\beta^2\alpha^4},\ H(\beta) = \frac{2I(\beta)}{\pi\beta}, \tag{8}$$

$$I(\beta) = \int_0^\infty x \sin x \exp\left[-\left(\frac{x}{\beta}\right)^{3/4}\right] dx. \tag{9}$$

The physical interpretation of the function $H(\beta)$ is following. This function gives the effective 1D distribution function of random gravitational fields. This is because the normalization condition for $H(\beta)$ is of effectively one-dimensional form $\int_0^\infty H(\beta)d\beta = 1$, see (7). In this case, the average value $\bar{\beta}$ of dimensionless random field β has the form $\bar{\beta} = \int_0^\infty \beta H(\beta)d\beta$. The mean value $\bar{\beta}$ exists if the integral $H(\beta)$ is convergent. The asymptotic analysis of the function $f(\beta)$, which had been performed in our previous work (Stephanovich & Godłowski (2015)), shows that $f(\beta)$ does not depend on β for small β and decays as $\beta^{-7/4}$ at large β. The character of decay at large β shows that although normalization integral is convergent, already first moment does not exist. Such behavior is a characteristic feature of so-called heavy-tailed distributions (Garbaczewski & Stephanovich (2009), Garbaczewski, Stephanovich & Kędzierski (2011)).

3 DISTRIBUTION FUNCTION OF ANGULAR MOMENTA

Our aim is to derive the distribution function of angular momenta. For that we need to calculate how the angular momentum $\mathbf{L}$ of a stellar object depends on its gravitational field $\mathbf{E}_{quad}(\mathbf{r})$ (2). The expression for angular momentum components L_α ($\alpha = x, y, z$) could be obtained perturbatively in small Lagrangian coordinate $\mathbf{q}$. One should note that the first order terms were obtained in Eq. (11) of Catelan & Theuns (1996), while the second order ones in their next article Catelan & Theuns (1996a) (Eq. (28)). Note that both equations have identical structure i.e. $L_\alpha^{(i)} = f_i(t)\varepsilon_{\alpha\beta\gamma}E_{i\beta\sigma}I_{\sigma\gamma},\ \alpha, \beta, \gamma, \sigma = x, y, z$, where index $i = 1, 2$ defines the order of perturbation theory, $\varepsilon_{\alpha\beta\gamma}$ is Levi-Civita symbol, $E_{\beta\sigma}$ are components of quadrupole (tidal) field (2) while $I_{\sigma\gamma}$ represent the components of inertia tensor.

In order to obtain the distribution function of *modulus* of E (and subsequently L), it is sufficient to take zz component in (2). If we need the complete distribution function of vector $\mathbf{E}$, we should account for complete tensor structure of Hamiltonian (1) $\mathcal{H} = -G\sum_{ij\alpha\beta} Q_{i\alpha\beta}m_jV_{\alpha\beta}(\mathbf{r}_{ij}),\ \alpha, \beta = x, y, z$. Also, as $\mathbf{L}$ is a function of time t by means of the functions $f_i(t)$, the distribution function will be time dependent. With respect to symmetry relations $I_{ab} = I_{ba}$ and $E_{ab} = E_{ba}$ and leaving only E_{zz}, we obtain $L_x = -b(t)E_{zz}I_{yz},\ L_y = b(t)E_{zz}I_{xz},\ L_z = 0,\ L = \sqrt{L_x^2 + L_y^2 + L_z^2} = L_0E,\ L_0 = L_0(t) = f_i(t)\sqrt{I_{xz}^2 + I_{yz}^2}$. Above equations constitute linear relation between angular momentum and tidal field moduli. They are valid both in linear (i =1) and nonlinear (i =2) regimes. Because above relation between gravitational field modulus and angular momentum is linear in both cases, it is easy to see that the shape of distribution function of angular momenta $f(L)$ repeats that of gravitational fields. In the explicit form expression for $f(L)$ can be derived using well known relation from the theory of probability $f(L) = f[E(L)]\left|\frac{dE}{dL}\right|$, which yields

$$f(L) = \frac{1}{2\pi^2 L}\int_0^\infty \rho\, e^{-\alpha\rho^{3/4}} \sin\left(\rho\frac{L}{L_0(t)}\right) d\rho, \tag{10}$$

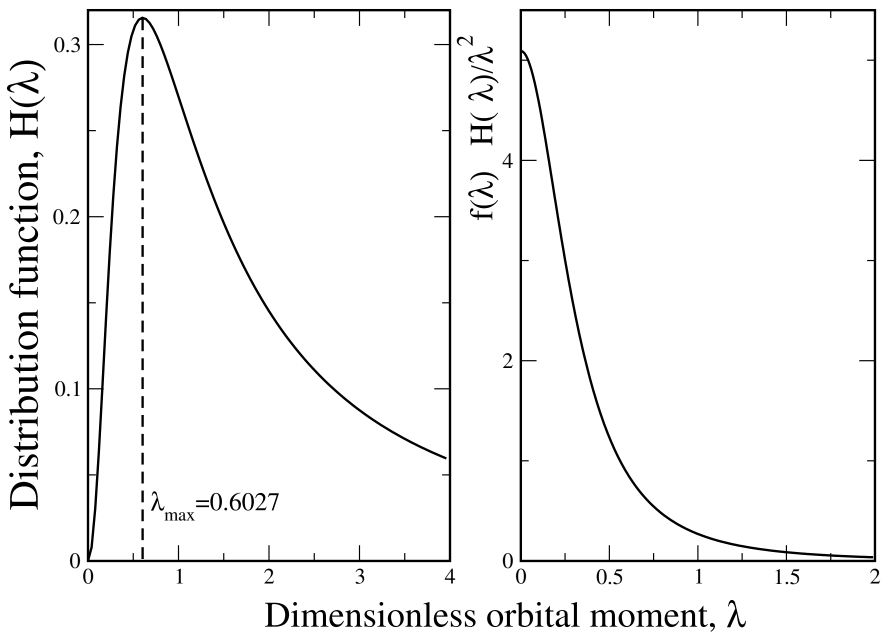

Fig. 1 Left panel shows the effective 1D distribution function $H(\lambda)$ (11). The shape of the function is the same as the distribution function (9). Dashed line represents the value of argument λ_{max}, related to maximum of $H(\lambda)$. Right panel shows 3D distribution function $4\pi\alpha^4 L_0 f(\lambda) = H(\lambda)/\lambda^2$.

where $L_0(t)$ is defined above. Dimensionless variables $\rho(L/L_0) = x$, $\lambda = L/(L_0\alpha^{4/3})$ generate the pair of functions which are similar to those obtained for the gravitational fields distribution. They read:

$$f(\lambda) = \frac{H(\lambda)}{4\pi\lambda^2\alpha^4 L_0},\ H(\lambda) = \frac{2I(\lambda)}{\pi\lambda}, \tag{11}$$

where $I(\lambda)$ is defined by the expression (9) and is usually referred to as spin parameter.

The effective 1D distribution function for gravitational fields or angular momenta is presented in left panel of Fig. 1. It is seen that while initial 3D function $f(\lambda)$ decays monotonically (right panel), this function is strongly asymmetric and has characteristic bell shape. Note, that as the initial equation (1) allows for the interaction between all astronomical objects in an ensemble, it considers naturally the interaction with surrounding structures and dark matter haloes also. This fact renders the distribution functions of gravitational fields (8) and angular momenta (11) to account not only for the isolated cluster regions, but for long-range interactions with surrounding structures as well. To be specific, the narrow peak of distribution function in left panel of Fig.1 stems from the closely situated cluster region, while its long tails stem from the long-range (quadrupole) interaction with surrounding structures. In other words, the interaction with surrounding structures is essential (and our distribution functions takes this fact into account) as the interaction between objects in stellar ensembles have long-range multipole character.

As we have shown in the previous article (Stephanovich & Godłowski (2015)), the integral for the first moment of angular momenta pdf diverges. It is well - known (see, for example, van Kampen (1981)) that for the distribution functions, which decay slowly at infinities, the corresponding mean value can be approximately estimated as the maximum of such function. In this spirit we calculate λ_{max}, corresponding to the maximum of distribution function $H(\lambda)$, as presented on the Fig. 1. The analysis of λ_{max} in dimensional units makes possible to obtain some useful relations, which earlier had been guessed only empirically. To consider the characteristics of galaxies, i.e. luminous matter, here we use the ideas of halo model (Schneider, Bridle (2010)), which states that galaxies (i.e. "pieces" of luminous matter) are embedded in the dark matter haloes so that their observable characteristics like angular momentum emerge from the mass and hence gravitational field of dark matter. Also, as the galaxies and their clusters reside in the larger structures like voids and filaments, the gravitational field of latter large objects also influence galaxies, see, e.g. Joachimi et al. (2015). As our distribution function (11) takes these effects into account by virtue of model (1), our subsequent calculations of mean angular momentum of the galaxies take above effects into account.

Let us first consider the simplest possible CDM model in the first order of perturbation theory. In such model the evolution of scale factor is given by the equation $a(t) = D(t) = (t/t_0)^{2/3}$ (Doroshkevich (1970)) so that $L_0 = \frac{2I}{3t_0}\tau$, $\tau = t/t_0$, $I = \sqrt{I_{xz}^2 + I_{yz}^2}$. The equation $dH/d\lambda = 0$ has solution $\lambda_{max} = 0.602730263$, which give in dimensional units

$$L_{max} = 0.7281884 n^{4/3} \frac{t}{t_0^2} GIQ \approx \approx \kappa n^{4/3} \frac{t}{t_0^2} GR^4 m^2, \tag{12}$$

where $n = N/V$, $\kappa \sim 1$ is a constant. To derive the equation (12), we estimate galaxy quadrupole moment Q and its mean inertia moment I as being proportional to mR^2, where m is mass of galaxy while R is its mean radius. In our approach we represent volume V as $V = R^3$, then R cancels down in Eq. (12) so that $L_{max} \sim (t/t_0^2) m^2 N^{4/3}$. Then, we introduce the mass of a galaxy cluster $M = mN$ and obtain

$$L_{max} \sim \frac{t}{t_0^2} M^{5/3} \left(\frac{m}{N}\right)^{1/3} \equiv \frac{t}{t_0^2} M^{5/3} \frac{\rho^{1/3}}{n^{1/3}}, \tag{13}$$

where $\rho = m/V$ is a mass density and $n = N/V$ is galaxies density. Following Catelan & Theuns (1996), we assume that mass density ρ is a function of time, defined by Friedmann equation in CDM model $\dot{a}/a = H_0 = \sqrt{8\pi G\rho/3}$, where H_0 is the Hubble constant. This generates the dependence $\rho \propto t^{-2}$, which, being substituted to Eq. (13), yields

$$L_{max} \sim \frac{t^{1/3}}{t_0^2} \frac{M^{5/3}}{n^{1/3}} \sim t^{1/3} M^{5/3}. \tag{14}$$

To derive Eq. (14), we assume that $n =$ const. We see that equations (13) and (14) recover the expression (27) of Catelan & Theuns (1996), giving the theoretical derivation of well-known empirical relation between mean value angular momentum of galaxies ensemble (galaxy clusters) and their mass $L_{max} \sim M^{5/3}$ (see Catelan & Theuns (1996) and references therein). Note, that within tidal torque model the $M^{5/3}$ - law has been first obtained by Heavens & Peacock (1988) while reasonable values for lambda in Eq. (11) within the tidal torque approach has been derived by Schäfer & Merkel (2012), who followed Heavens & Peacock (1988).

There is also other approach to interpret the dependence of L_{max} on stellar parameters. Namely, suppose that volume $V = R_A^3$, where R_A is a mean cluster radius, proportional to the autocorrelation radius (see Longair (2008) and references therein). In such approach (see Stephanovich & Godłowski (2015) for details) n is still a constant for any particular cluster, but now it varies from cluster to cluster with increasing richness N. In this case we may rewrite $N = M/m$ to obtain the alternative (to Eq. (14)) form of expression for L_{max}

$$L_{max} \sim \frac{t}{t_0^2} \left(\frac{R}{R_A}\right)^4 m^{2/3} M^{4/3}, \tag{15}$$

which does not contain ρ.

It is instructive to comment on time dependence $L_{max}(t)$ in Eq. (15). On the first sight, it follows from (15) that $L_{max} \sim t$, but the problem complicates a lot by the intricate time dependence of the quantities R and R_A (Longair (2008)). We plan to study this question in future works.

It is clear from the equation (12) that mean orbital moment of a galaxy increases with the number of galaxies N and it is proportional to $N^{4/3}$. Moreover, even in the model with constant galaxies density n, number (richness) N varies from cluster to cluster so that the dependence $L_{max}(N) = \kappa_2 N^{4/3}$ holds and shows that angular momenta increase with number of galaxies N in analysed structure.

The sample of 247 Abell cluster has been analysed by Godłowski et al.(2010). Namely, the orientation of galaxies in particular clusters has been studied. The idea was to test hypotheses that the galaxies angular momenta increase with the cluster richness. If galaxy cluster do not rotate (see Regos & Geller (1989), Hwang & Lee (2007)), then increasing alignment of galaxies in clusters mean the increase of the angular momentum of whole cluster. In the paper of Godłowski et al.(2010) the orientation of galaxies was quantified by distribution of the angles. Specifically, the position angle of galaxy plane p and two angles δ_d, giving spatial orientation of the normal to galaxy plane, have been considered. The authors have also studied two additional angles. One is the angle between the normal to the galaxy plane and the main plane of the coordinate system. The second is the angle η between the projection of this normal onto the main plane and the direction toward the zero initial meridian (Flin, Godlowski (1986)).

The entire range of all investigated angles was arranged into n bins. As we would like to detect non-random efect in the galaxies orientation, we first check whether the orientation is isotropic. To be specific, we check if the disribution of analyzed angles in the clusters under investigation is isotropic. The distribution of the above angles has been investigated using the statistical tests. They were χ^2 and the Fourier Test. However, in the present paper we extend the analysis for first auto-correlation and Kolmogorov-Smirnov tests (K-S test) (Hawley & Peebles (1975), Flin, Godlowski (1986), Godłowski et al.(2010), Godłowski (2012)).

The statistics χ^2 is:

$$\chi^2 = \sum_{k=1}^{n} \frac{(N_k - N\,p_k)^2}{N\,p_k} = \sum_{k=1}^{n} \frac{(N_k - N_{0,k})^2}{N_{0,k}}, \tag{16}$$

where p_k are probabilities that chosen galaxy falls into k-th bin, N is the total number of galaxies in a sample (in our case in a cluster), N_k is the number of galaxies within the k-th angular bin and $N_{0,k}$ is the expected number of galaxies in the k-th bin.

The first auto-correlation test quantifies the correlations between galaxy numbers in neighboring angle bins. The statistics C reads

$$C = \sum_{k=1}^{n} \frac{(N_k - N_{0,k})(N_{k+1} - N_{0,k+1})}{[N_{0,k} N_{0,k+1}]^{1/2}}, \tag{17}$$

where $N_{n+1} = N_1$.

If the deviation from isotropy is a slowly varying function of the analyzed angle θ, one can use the Fourier test:

$$N_k = N_{0,k}(1 + \Delta_{11}\cos 2\theta_k + \Delta_{21}\sin 2\theta_k + \Delta_{12}\cos 4\theta_k + \Delta_{22}\sin 4\theta_k + ...). \tag{18}$$

In this test, the crucial statistical quantities are amplitudes

$$\Delta_1 = \left(\Delta_{11}^2 + \Delta_{21}^2\right)^{1/2}, \tag{19}$$

(only the first Fourier mode is taken into account) or

$$\Delta = \left(\Delta_{11}^2 + \Delta_{21}^2 + \Delta_{12}^2 + \Delta_{22}^2\right)^{1/2}, \tag{20}$$

where the first and second Fourier modes are analysed together. During our investigations we analyzed statistics $\Delta_1/\sigma(\Delta_1)$ and $\Delta/\sigma(\Delta)$ (see Godłowski et al.(2010) for details).

In the case of K-S test we investigate statistics λ:

$$\lambda = \sqrt{N}\, D_n \tag{21}$$

which is given by limiting Kolmogorov distribution, where

$$D_n = \sup |F(x) - S(x)| \tag{22}$$

and $F(x)$ and $S(x)$ are theoretical and observational distributions of the investigated angle respectively.

The aim of the paper of Godłowski et al.(2010) was to test the hypotheses that alignment of galaxies increases with cluster richness (Godłowski, Szydłowski & Flin (2005)). The main result of Godłowski et al.(2010) was that the values of investigated statistics increase with increasing number of cluster galaxy members. This permits to conclude that there exist a relation between anisotropy and the number of galaxies in a cluster. Note, that above testing has been performed assuming linear model $y = aN + b$ where y is a value of investigated statistics, N is the cluster members number while a and b are linear regression coefficients. In the paper of Godłowski et al.(2010), the null hypothesis H_0 (that the investigated statistics is random one, i.e. neither increases nor decreases so that parameter $t = a/\sigma(a) = 0$) has been confronted against H_1 hypothesis that statistics increases with the cluster richness i.e. $t > 0$. In our previous paper (Stephanovich & Godłowski (2015)), as well as in the present paper, we show that dependence between the alignment of galaxies in clusters and number of members galaxies is not necessarily linear but could be, according to above assumptions as either $N^{4/3}$ or $N^{5/3}$.

For this reason, in the figure 2, we present statistics (χ^2, Fourier and First autocorrelation tests Hawley & Peebles (1975), Flin, Godlowski (1986), Godłowski et al.(2010)) for the case of the position angles obtained for the sample of 247 rich Abell clusters, analysed by Godłowski et al.(2010). We present linear dependence $\sim N$ as well as the cases when analysed statistics increases as $N^{4/3}$ and $N^{5/3}$. The error bars presented in the figure 2, suggest that the data points seem to be not sufficient do discriminate between models. For this reason we analyze the dependence between the number of galaxies in a cluster and the value of analyzed statistics in more details. We performed the investigation of the linear regression given by $y = aN + b$ counted for various parameters. Namely, we have studied the linear regression between different statistics χ^2, $\Delta_1/\sigma(\Delta_1)$, $\Delta/\sigma(\Delta)$, C or λ and the number of analyzed galaxies in each particular cluster. This has been done for the case of linear dependence $\sim N$ or power laws $\sim N^{4/3}$ and $\sim N^{5/3}$ in the case of remaining models.

Now we assume that the theoretical, uniform, random distribution contains the same number of clusters as the observed one. To be specific, we consider null hypothesis H_0 that the distribution is a random one and neither increases nor decreases. This means that expected value of statistics $t = a/\sigma(a) = 0$, while t statistics has Student's distribution with $u - 2$ degrees of freedom, where u is the number of analyzed clusters. In other words, we test H_0 hypothesis that $t = 0$ against H_1 hypothesis that $t > 0$. Of course, in order to reject the H_0 hypothesis, the value of the observed statistics t should be greater than t_{cr} which we could obtain from staistical tables. Note that for our sample containing only 247 clusters, the critical value at the significance level $\alpha = 0.05$ is equal to $t_{cr} = 1.651$.

The result of our statistical analysis is presented in the Table 1. We analysed two samples of data. In the first sample A all galaxies lying in the area regarded as a cluster, were taken into account. In the second sample B, to avoid the "contamination" by the background objects, we restrict ourselves by consideration of the galaxies brighter than $m_3 + 3$ only.

Note, that the cases of linear dependence for statistics of χ^2, $\Delta_1/\sigma(\Delta_1)$ and $\Delta/\sigma(\Delta)$ have usually been analysed in the paper of Godłowski et al.(2010) (Table 1). Note, however, that our present results are somewhat different from those obtained by Godłowski et al.(2010). For example in the case of χ^2 instead of $t = 0.025/0.015 = 1.67$ we obtain $t = 1.87$. The reason is that in the paper of Godłowski et al.(2010) the error bars of the data points (i.e. statistics for individual clusters) has been estimated from the sample, while now it is taken from exact theoretical analysis (Godłowski (2012), Wang et al. (2003)).

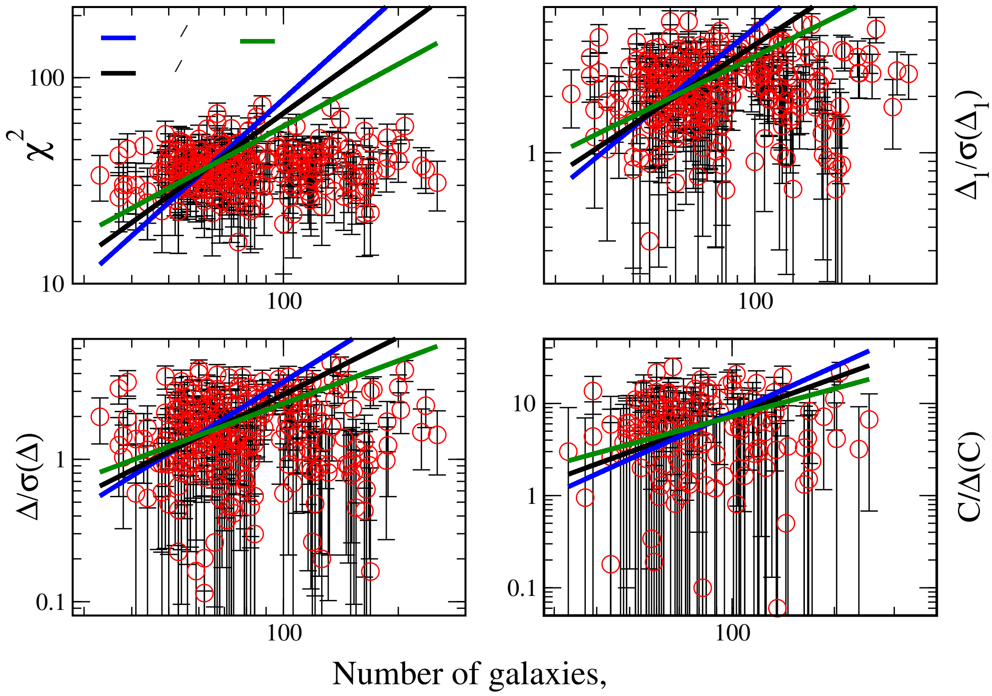

Fig. 2 The dependence between the number of galaxies in the cluster N and the value of analyzed statistics (χ^2 - left upper panel, $\Delta_1/\sigma(\Delta_1)$ - right upper panel, $\Delta/\sigma(\Delta)$ - left lower panel, $C/\sigma(C)$ - right lower panel) for the position angles p. Mind double log scale, chosen to make the dependences N^α (α =1, 4/3, 5/3) straight lines.

In majority of cases, exept for the first autocorrelation test, the values of the obatined statistics are greater than critical one $t_{cr} = 1.651$. One could observe that for all three analyzed models (i.e. linear dependence $\sim N$ and the increased statistics like $\sim N^{4/3}$ or $\sim N^{5/3}$) we can eliminate H_0 hypothesis (that statistics $t = a/\sigma(a) = 0$) in favour of hypothesis H_1 that $t > 0$. The effect increases if we analyse Sample B which mean that we restrict the cluster membership to galaxies brighter than $m_3 + 3$. The significance of the effect decreases with increasing powers m in the models like N^m, but in majority of cases the effect is significant. The above results allow us to conclude that the presented data is not sufficient to discriminate between above three models so that we need future investigations based on the larger cluster samples.

In our investigations, we have also studied the time dependence of galaxies gravitational fields pdf (Stephanovich & Godłowski (2015)). The distribution function (11) evolves in time. It relies on explicit dependences $f_1(t)$ and $f_2(t)$. The functions $f_1(t) = a^2(t)\dot{D}(t)$ while $f_2(t) = \dot{E}(t)$ (we use standard notations where dot means time derivative) could be obtained from the differential equations set, derived in i - th order of perturbation theory by Bouchet et al (1992):

$$t_0^2\ddot{D}(t) + a(t)D(t) = 0, \tag{23}$$

$$t_0^2\ddot{E}(t) + a(t)E(t) = -a(t)D(t)^2, \tag{24}$$

where $0 \leq t < \infty$ is dimensional physical time. The dimensionless function (scale factor) $a(t)$ is determined from the first Friedmann equation. In our investigations we consider the ΛCDM model, however we compare its predictions with those obtained in classical CDM model.

Table 1 The statistics $t = a/\sigma(a)$ for 247 rich Abell clusters. Sample A - all galaxies, Sample B - galaxies brighter than $m_3 + 3$

Test	N	$N^{4/3}$	$N^{5/3}$
SampleA			
χ^2	1.872	1.766	1.667
$\Delta_1/\sigma(\Delta_1)$	1.613	1.588	1.580
$\Delta/\sigma(\Delta)$	1.964	1.941	1.821
C	1.352	1.381	1.417
λ	2.366	2.500	2.400
SampleB			
χ^2	1,979	1.801	1.625
$\Delta_1/\sigma(\Delta_1)$	2.182	1.962	1.702
$\Delta/\sigma(\Delta)$	2.104	1.885	1.596
C	1.225	1.170	1.125
λ	2.421	2.000	1.765

To obtain the dependence $L_0(t)$, we use substitution $\lambda \to \lambda/f_i(\tau)$, ($\tau = t/t_0$) which yields from (11)

$$H(\lambda,\tau) = \frac{2I(\lambda/f_i(\tau))}{\pi\lambda},\ i = 1,2. \tag{25}$$

To derive $f_{1,2}(\tau)$ in particular model (ΛCDM model in our case), it is necessary to calculate $a(t)$ from the first Friedmann equation, see Stephanovich & Godłowski (2015) for details:

$$\frac{da}{dt} = H_0\sqrt{\Omega_\Lambda a^2 + \frac{1-\Omega_\Lambda}{a}}. \tag{26}$$

The solution of the equation (26) has the form

$$a(t) = \alpha \sinh^{2/3}(t/t_0),\ \alpha = \left(\frac{1-\Omega_\Lambda}{\Omega_\Lambda}\right)^{1/3}, \tag{27}$$
$$t_0 = \frac{2}{3H_0\sqrt{\Omega_\Lambda}},$$

where $\Omega_\Lambda = \Lambda/(3H_0^2)$ is cosmological constant or so-called vacuum density, Λ is cosmological constant and H_0 is Hubble constant.

Having the function $a(t)$, we can solve equation (23) numerically for $D(\tau)$ and then determine the function $f_1(\tau) = a^2(\tau)D'(\tau)$ ($D' = dD/d\tau$). Accordingly, in the nonlinear regime, the function $f_2(\tau) = E'(\tau)$ could be calculated numerically from the equation (24).

One should note that functions $f_2(\tau)$, which are related to the second perturbative corrections, are negative. For instance, in Einstein - de Sitter model $f_1(\tau) = (2/3)\tau$ and $f_2(\tau) = (-4/7)\tau^{1/3} < 0$ (Doroshkevich (1970), Catelan & Theuns (1996a)). The same result ($f_2(\tau) < 0$) can be obtained numerically for ΛCDM model.

The dependences $H(\lambda,\tau)$ (25) for CDM (with above analytical expressions for $f_i(\tau)$) and ΛCDM models are shown in the Fig. 3. It is easy to observe that as time increases, the distribution function decreases, while its peak grows to infinity at $t \to 0$. As time grows, the whole distribution function "blurs" as its maximum shifts towards large t. It is also easy to notice that "blurring" of distribution function at large times is much faster for ΛCDM model. Also, both in linear and nonlinear regimes $H(\lambda,\tau)$ increases with time. We emphasize once more that in ΛCDM model this growth is much faster than in the CDM model. It is the consequence of the fact that functions $f_i(\tau)$ enter the exponent in the integrand (25). The comparison of upper and lower panels of Fig. 3 show that the behaviour of $H(\lambda,\tau)$ is qualitatively similar in linear and nonlinear regimes of fluctuation growth. This leads to conclusion that even linear regime give qualitatively correct approximation to the function $H(\lambda,\tau)$.

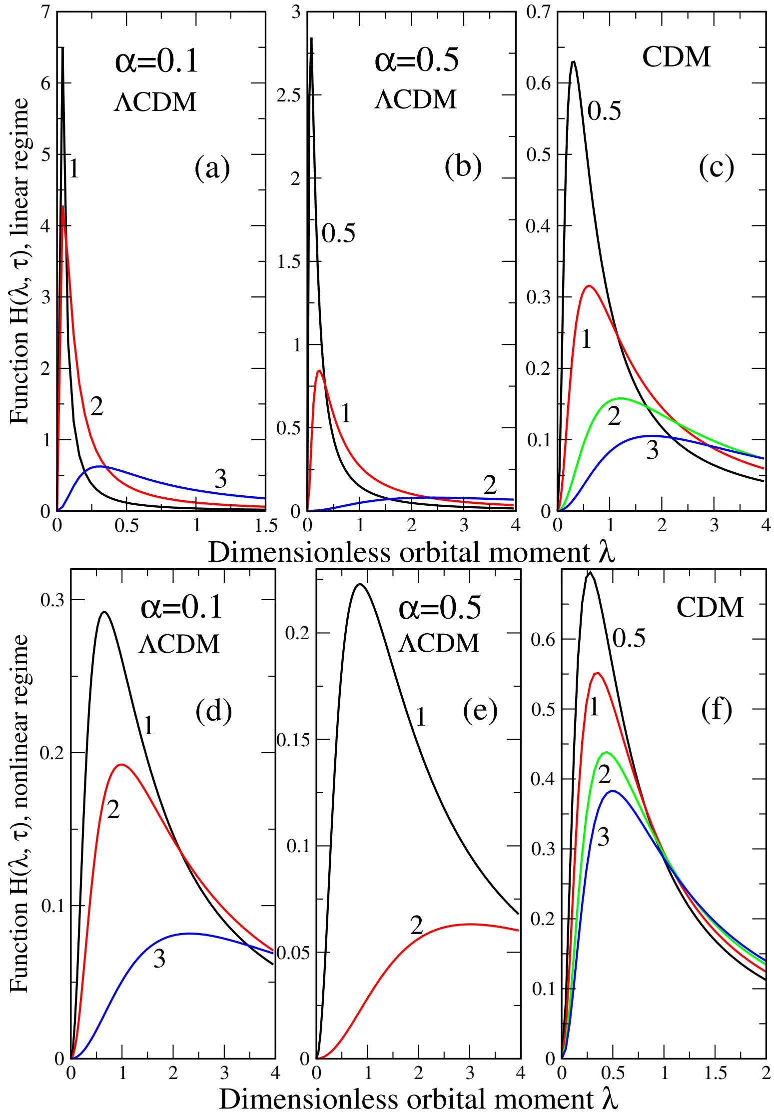

Fig. 3 One dimensional effective distribution function $H(\lambda, \tau)$. The figure reports time evolution of above function in both ΛCDM and CDM (panels (c) and (f)) models, see legends. We present also differences between linear (upper panels) and nonlinear (lower panels) regimes. Figures near curves correspond to dimensionless time $\tau = t/t_0$.

The above results lead to conclusion that angular momentum of galaxy clusters should increase in time. This hypothesis could be tested theoretically. This is because limited speed of light causes that the age of the astronomical objects with different redshifts z is different. So, assuming that galaxy clusters form in the same time instant, we expect that clusters with higher redshift z are younger. This means that galaxies alignment should decrease with z. Our preliminary analysis of the sample of 247 Abell cluster shows that in the case of χ^2 and Kolmogorov - Smirnov tests (Godłowski (2012), Aryal et al (2013)), the analysed statistics decreases with z. Unfortunately this effect is not significant since parameter $t = a/\sigma(a)$ is less then 1.

One should note however, that basic catalog of galaxies is complete up to magnitude $m = 18.^m3$, which means that the red shift of the most distant cluster $z < 0.12$. As result it is very difficult to detect such subtle effect for small cluster sample. Moreover, although the vast majority of clusters do not rotate (Regos & Geller (1989), Hwang & Lee (2007)), this is not completely true for all clusters (Hwang & Lee (2007)). Hwang & Lee (2007) study the dispersions and velocity gradients in 899 Abell clusters. They have found possible evidence for rotation in only six of them i.e. less then 1%. Latter sample of rotating clusters has been studied by Aryal et al (2013). The random orientation of galaxies angular momenta vectors in the analysed clusters was found. Similarly, Narayan et al (2017) found no preferred alignments of angular momenta vectors of galaxies in a sample of six dynamically unstable clusters. Presence of such cluster types, even relatively small, could give additional difficulties in the observational investigation of the time evolution of the clusters angular momenta. So, larger sample of the cluster stretched for higher z is necessary to make unambiguous conclusions regarding above effect.

4 RELATION TO OBSERVATIONAL RESULTS

Our calculations demonstrate that although the gravitational interaction between stellar components (including dark matter halos) is of long-range multipole character, the observations (see below) give some confirmations that there is additional short-range (like $\sim \exp(-r/r_c)$ with range r_c) interaction. As a result, if the distance r between two objects (say galaxies) is smaller then r_c, they are correlated which means that their orbital moments are aligned. This assumption works for the dense (rich) galaxy clusters, which, by this virtue, have high degree of orbital moments alignment. For the sparse (poor) clusters the situation is opposite. For such type of clusters the intergalaxy distance $r > r_c$, the long-range multipole interaction prevails so that there is no alignment of the orbital moments. The above statistical method accounts for this situation if we add the (empirical) short-range interaction term to the initial potential (2). In the analysed case we obtain that the distribution function of random fields would depend on the average angular momentum $L_{max} \equiv L_{av}$ (see Stephanovich (1997), Semenov & Stephanovich (2003)) and as result we obtain the self-consistent equation for L_{av}

$$L_{av} = \int L(E) f(E, L_{av}) d^3E. \tag{28}$$

where $f(E, L_{av})$ is the distribution function of gravitational field E, depending on L_{av} as parameter. This function substitutes the expression (6) in the case of inclusion of the possible short-range interaction term. One should note that in the case of finite r_c, the distribution function decays at $E \to \infty$ faster then (6) so that the integral (28) converges. As total interaction potential contain both luminous and dark matter components, the equation (28) allows us to ask the question about alignment of sub-dominant galaxies, even though the majority of cluster angular momentum is related to the smooth dark matter halo component. For instance, in the halo model (Schneider, Bridle (2010)), when the luminous matter of galaxies is embedded in dark matter halo, this halo by virtue of its mass may mediate the intergalaxy interaction, adding possible short-range terms to it. The self-consistent equation (28) permits also to include the temperature into consideration (Semenov & Stephanovich (2002), Semenov & Stephanovich (2003)) and study the galaxies and their clusters (with respect to dark matter haloes) time evolution within ΛCDM model. Also, the combination of stochastic models (Garbaczewski & Stephanovich (2009), Garbaczewski, Stephanovich & Kędzierski (2011)) of primordial dynamics along with those of ΛCDM, most probably, would permit to answer (at least theoretically) the question if the galaxies are initially aligned at the time of their formation, or such alignment is generated in some merger events, and how dark matter haloes influence (mediate) this alignment.

Here we also show that there are different possible relations between angular momentum and the mass (richness) of the cluster. Note that $M^{5/3}$ - law for such dependence as well as reasonable values of parameter λ in Eq. (11) had been obtained by Heavens & Peacock (1988) followed by Schäfer & Merkel (2012). Figure 2 reports our preliminary results of the dependence between analysed statistics obtained for the sample of 247 rich Abell clusters (Godłowski et al.(2010)). We conclude here that our comparison of the cases when the statistics grows as N, $N^{4/3}$ and $N^{5/3}$ does not permit to establish unambiguous correspondence of different dependences between angular momentum and richness of the structure. However, such unambiguous discrimination would be possible if larger statistical sample of galaxy clusters is available. Moreover, we show that angular momentum of galaxies should increase with time. Latter fact follows from equations (12) - (14) for CDM model and from Fig. 3 for ΛCDM model. The physical mechanism of that has been discussed in details in our previous paper (Stephanovich & Godłowski (2015)). It is related to the growing time evolution of scale factor $a(t)$ both in CDM (Doroshkevich (1970)) and ΛCDM models, see Eq. (27) for details. This means that above theoretically predicted effect could be tested by observations as galaxies angular momentum should decrease with redshift z. Once more, the enlarged sample containing clusters with much higher z is necessary for such studies.

5 CONCLUSIONS

To summarise, in the present paper we analyze theoretically the observational dependences of the galaxies and their clusters angular momenta on their mass (richness). To do so, we use the method, introduced in our previous paper (Stephanovich & Godłowski (2015)). Observational data are in agreement with our theoretical results and mainly Eqs. (15) and (13) where we have shown that under reasonable assumptions about cluster morphology the angular momentum of galaxy structures increases with their richness. The solution of equation (28) will permit to establish a relation between the characteristics of possible short-range intergalaxy interaction and character of their spins alignment.

We emphasize however, that the above observational results about lack of alignment of galaxies for poor clusters, as well as evidence for such alignment in the rich galaxy clusters (Godłowski, Szydłowski & Flin (2005), Aryal et al (2007), see also Godłowski, 2011 for later improved analysis) clearly shows that angular momentum of galaxy groups and clusters increases with their richness. The problem of clusters angular momenta in context of their mutual interactions as well as those with dark matter haloes has been discussed by Hahn et al. (2007) based on the results of computer simulations. The presence of threshold value of the cluster mass (that is to say richness) has been noticed in these simulations. This threshold value is related to mutual alignment of clusters and dark matter haloes axes. As we have shown above, this fact can be explained by our model.

We finally note that the direct computer simulations of stellar ensembles are still quite computationally expensive to simulate realistic (i.e. sufficiently large) parts of the Universe. Hence it seems to be a good idea to put some effort into developing new theoretical models for galaxy alignment with respect to dark matter haloes and (possible) merger into larger structures like superclusters. Since galaxy morphology plays important role in this behavior, our approach, linking the galaxy shapes with their characteristics distribution (especially in view that it permits to calculate the non-Gaussian pdfs), will improve the overall understanding, which can additionally be tested against observed galaxy shape distributions and alignments.

Appendix A:

Here we present some more details of our model, based on Hamiltonian (1). In this Hamiltonian, the explicit expression for i - th galaxy quadrupolar moment Q_i has the form (Poisson (1998))

$$Q_i = \int_{V_i} \rho_i(\mathbf{x})|\mathbf{x}|^2 P_2(\mathbf{s}\cdot\mathbf{x})d^3x, \tag{A.1}$$

where $P_2(x) = (3x^2 - 1)/2$ is corresponding Legendre polynomial (Abramowitz & Stegun (1964)), V_i is a volume of i-th galaxy, $\rho_i(\mathbf{x})$ is a density of its mass.

The geometry of the problem under consideration is shown in Fig.A.1. It is seen first, that the origin is not related to any specific galaxy or other astronomical object. Rather, it is situated in the arbitrary point in the Universe. Although $\mathbf{r}_{ij}$ is directed from one galaxy (in our case j) towards another (in our case i) it is by no means bounded to these galaxies. It is simply means the difference in their radius - vectors, which connect the coordinates origin and position of each galaxy.

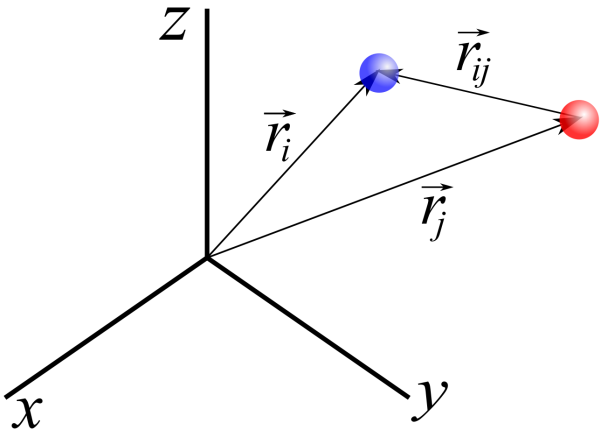

Fig. A.1 The reference frame of the problem under consideration. Radius - vectors of galaxy (or dark matter halo element) i (blue ball) and j (red ball) ($\mathbf{r}_i$ and $\mathbf{r}_j$ respectively) as well as their difference $\mathbf{r}_{ij}$ are shown.

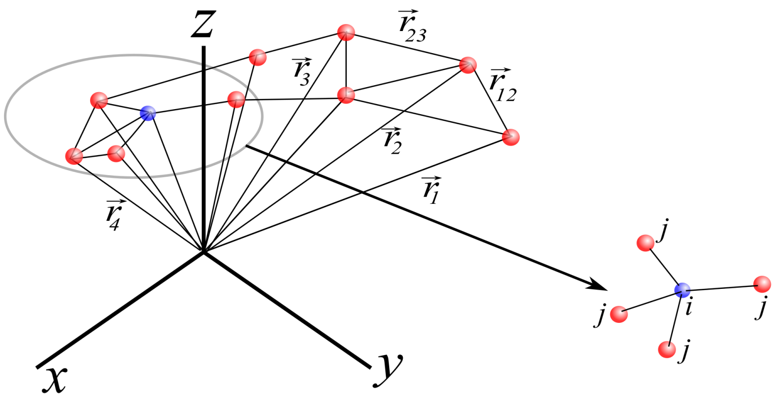

Fig. A.2 Geometry of the problem with many galaxies (or other astronomical objects marked by red and blue balls) situated randomly in the Universe. Radius - vectors of those elements (like $\mathbf{r}_1$, $\mathbf{r}_2$ etc) as well as their separations (like $\mathbf{r}_{23}$) are shown selectively. Blue ball (in the ellipse on the main panel and in the inset) shows the example of i-th object with the rest being j-th objects. Division on i and j objects is arbitrary and made to calculate the gravitational field, exerted on i-th object from the rest of the ensemble. In other words, any galaxy can be either of i or j type. Inset shows this situation (from the ellipse on the main panel): the gravitational field on the (arbitrary chosen) blue ball i is a sum of the fields from its neighboring objects j. The dimensions of the ellipse on the main panel visualize the range of interaction (A.3); this range is very long (decays as r^{-4} so that much more galaxies will be in the range of interaction, but the distant j-th galaxies make almost zero contribution to the gravity field on i-th one), it does not have clear boundary but the ellipse gives some guide for eyes. As the number of galaxies is actually infinite and their separations become progressively smaller, the galaxies connecting polyline (i.e. line consisting of all $\mathbf{r}_{ij}$) tends to continuous curve (not shown). In this case all sums are converted to integrals, as described in the text.

The Hamiltonian (1) can be identically rewritten through the interaction energy

$$\mathcal{H} = -GM^2 \sum_i p_i m_i W_i, \tag{A.2}$$
$$W_i = W(\mathbf{r}_i) = \sum_j m_j V(\mathbf{r}_{ij}) \equiv \sum_j m_j V(\mathbf{r}_j - \mathbf{r}_i).$$

The interaction energy W_i is the energy exerted by the rest of the galaxy ensemble (due to intergalaxy interaction) to the galaxy in the point i. We can see that after summation (actually integration, see below) over $\mathbf{r}_j$ the relative intergalaxy distance $\mathbf{r}_{ij}$ has actually disappeared.

The gradient of the energy (A.2) is indeed the gravity field, which acts on i-th galaxy (or other astronomical object) from the rest j of these objects ensemble

$$\mathbf{E}_{quad}(\mathbf{r}_i) \equiv \mathbf{E}_{quad,i} = \sum_j m_j \nabla V(\mathbf{r}_j - \mathbf{r}_i) = \mathbf{i}_r E_0 \sum_j m_j \frac{3\cos^2\theta_{ij} - 1}{r_{ij}^4}, \tag{A.3}$$

which is the expression (2) from the text, rewritten explicitly in terms of vectors $\mathbf{r}_i$ and $\mathbf{r}_j$.

Having the expression (A.3), we can write explicitly the distribution function of random quadrupolar fields, Eq. (3) from the text

$$f(\mathbf{E}) = \overline{\delta(\mathbf{E} - \mathbf{E}_i)} \equiv \overline{\delta(\mathbf{E} - \mathbf{E}_{quad}(\mathbf{r}_i))} = \overline{= \delta\left(\mathbf{E} - \mathbf{i}_r E_0 \sum_j m_j \frac{3\cos^2\theta_{ij} - 1}{r_{ij}^4}\right)}, \tag{A.4}$$

where bar means the averaging over random spatial configurations of galaxies and other astronomical objects.

In performing the actual averagings in the expression (A.4) (see Fig.A.2), with respect to the fact that number of galaxies is infinite and their "elementary separations" $\mathbf{r}_{ij}$ become very small, we can change summations in (A.3) and (A.4) to integrations using the expression for gravity field $\mathbf{E}_i$ in the form (2). Further averagings in (A.4) are prescribed in the text, see also Stephanovich & Godłowski (2015).

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