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Strongly screening electron capture rates of chromium isotopes in presupernova

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Abstract Taking into account the effect of electron screening on the electron energy and electron capture threshold energy, by using the method of Shell-Model Monte Carlo and Random Phase Approximation theory, we investigate the strong electron screening capture rates of chromium isotopes according to the linear response theory screening model. The strong screening rates can decrease by about 40.43% (e.g., for ^{60}Cr at $T_9 = 3.44, Y_e = 0.43$). Our conclusions may be helpful to the researches of supernova explosion and numerical simulation.

Key words: nuclear reactions, electron capture, supernovae

1 INTRODUCTION

At the presupernova stage, beta decay and electron capture on some neutron-rich nuclei may play an important roles in determining the hydrostatic core structure of massive presupernova stars, thereby affect the subsequent evolution during the gravitational collapse and supernova explosion phases (e.g., Dean et al. 1998; Caurier et al. 1999; Juodagalvis et al. 2010; Liu 2013a; 2014; 2016a; 2016b; 2017). For example, the beta decay (electron capture) strongly influences the time rate of change of the lepton fraction (e.g., the time rate of change of electron fraction $\dot{Y}_e$) by increasing (decreasing) the number of electrons. Some isotopes of iron, chromium, and copper can also make a substantial contribution to the overall changes in the lepton fraction (e.g., $\dot{Y}_e$), electron degenerate pressure, and entropy of the stellar core during its very late stage of evolution. Many of these nuclei could be appropriately tracked in the reaction network in the stellar evolution calculations. The lepton fraction (e.g., Y_e) is bound to lead to an unstoppable process of gravitational collapse and supernova explosion.

Some research shows that the EC of iron group nuclei (e.g., iron and chromium isotopes) are very important and dominate for supernova explosions (e.g., Aufderheide et al. 1990, 1994; Dean et al. 1998; Heger et al. 2001;). In the process of presupernova evolution, chromium isotopes are a very important and crucial radionuclide. Aufderheide et al. (1994) detailed investigated the EC and beta decay for these nuclei in presupernova evolution. They found that the EC rates of these chromium isotopes can be of significant astrophysical importance by controlling the electronic abundance. Heger et al. (2001) also discussed weak-interaction rates for some iron group nuclei by employing shell model calculations in presupernova evolution. They found that electron capture rates on iron group nuclei would be crucial for decreasing the electronic abundance (Y_e) in stellar matter.

On the other hand, in the process of presupernova evolution of massive stars, the Gamow-Teller transitions of isotopes of chromium play a consequential role. Some studies shown that β-decay and electron capture rates on chromium isotopes significantly affect the time rate of change of lepton fraction ($\dot{Y}_e$). For example, Nabi et al. (2016) detailed the Gamow-Teller strength distributions, $\dot{Y}_e$, and neutrino energy loss rates for chromium isotopes due to weak interactions in stellar matter.

However, their works did not discuss the problem that electron screening (SES) would strongly effect on EC. What role does the EC play in stellar evolution? How does SES influence on EC reaction at high density and temperature? In order to calculate accurately the EC rates and screening correction for supernova explosion and numerical simulation, in this paper we will detailed discuss this problem.

Based on the linear response theory model (LRTM) and Random Phase Approximation (RPA), we study the strong screening EC rates of chromium isotopes in astrophysical environments by using the Shell-Model Monte Carlo (SMMC) method. In the next Section, we discuss the methods used for EC in stellar interiors in the case with and without SES. Section 3 will present some numerical results and discussions. Conclusions follow in Section 4.

2 THE EC RATES IN THE PROCESS OF STELLAR CORE COLLAPSE

2.1 The EC rates in the case without SES

For nucleus (Z, A), we calculate the stellar EC rates, which is given by a sum over the initial parent states i and the final daughter states f at temperature T and it is written by (e.g., Fuller et al. 1980, 1982)

$$\lambda_k = \sum_i \frac{(2J_i+1)e^{\frac{-E_i}{kT}}}{G(Z,A,T)} \sum_f \lambda_{if} \tag{1}$$

here J_i is the spin and E_i is excitation energies of the parent states, the nuclear partition function $G(Z, A, T)$ has been discussed by Aufderheide et al. (1990, 1994). λ_{if} is named as the rates from one of the initial states to all possible final states.

Based on the theory of RPA, the EC rates is closely related to cross section σ_{ec}, and we can written by (e.g., see detailed discussions in Dean et al. 1998; Caurier et al. 1999; Juodagalvis et al. 2010)

$$\lambda_{if} = \frac{1}{\pi^2\hbar^3} \sum_{if} \int_{\varepsilon_0}^{\infty} p_e^2 \sigma_{ec}(\sigma_e, \sigma_i, \sigma_f) f(\sigma_e, U_F, T) d\varepsilon_e \tag{2}$$

where $\varepsilon_0 = \max(Q_{if}, 1)$. The incoming electron momentum is $p_e = \sqrt{\varepsilon_e - 1}$, and ε_e is the electron energy and the electron chemical potential is given by U_F, T is the electron temperature. The energies and the moments are in units of $m_e c^2$ and $m_e c$ (m_e is the electron mass and c is the light speed), respectively.

The electron chemical potential is obtained by

$$n_e = \frac{\rho}{\mu_e} = \frac{8\pi}{(2\pi)^3} \int_0^{\infty} p_e^2 (G_{-e} - G_{+e}) dp_e \tag{3}$$

here μ_e, ρ are the average molecular weight and the density in $\mathrm{g/cm^3}$, respectively. $\lambda_e = \frac{h}{m_e c}$ is the Compton wavelength, $G_{-e} = [1 + \exp(\frac{\varepsilon_e - U_F - 1}{kT})]^{-1}$ and $G_{+e} = [1 + \exp(\frac{\varepsilon_e + U_F + 1}{kT})]^{-1}$ are the electron and positron distribution functions respectively, k is the Boltzmann constant. The phase space factor is defined as

$$f(\varepsilon_e, U_F, T) = [1 + \exp(\frac{\varepsilon_e - U_F}{kT})]^{-1} \tag{4}$$

According to the energy conservation, the electron, proton and neutron energies are related to the neutrino energy, and Q-value for the capture reaction (Cooperstein et al. 1984)

$$Q_{if} = \varepsilon_e - \varepsilon_\nu = \varepsilon_n - \varepsilon_\nu = \varepsilon_f^n - \varepsilon_i^p \tag{5}$$

and we have

$$\varepsilon_f^n - \varepsilon_i^p = \varepsilon_{if}^* + \hat{\mu} + \Delta_{np} \tag{6}$$

where ε_ν is neutrino energy, ε_i^p is the energy of an initial proton single particle state, ε_f^n is the energy of a neutron single particle state. $\hat{\mu} = \mu_n - \mu_p$ and $\Delta_{np} = M_n c^2 - M_p c^2 = 1.293$MeV are the chemical potentials and mass difference between neutron and proton in the nucleus, respectively. $Q_{00} = M_f c^2 - M_i c^2 = \hat{\mu} + \Delta_{np}$, and the masses of the parent nucleus and the daughter nucleus are corresponding to M_i and M_f; ε_{if}^* is the excitation energies for daughter nucleus at zero temperature state.

The total cross section in the process of EC reaction is given by (e.g., Dean et al. 1998; Caurier et al. 1999; Juodagalvis et al. 2010)

$$\begin{aligned}\sigma_{ec} = \sigma_{ec}(\varepsilon_e) &= \sum_{if} \frac{(2J_i+1)\exp(-\beta\varepsilon_i)}{Z_A}\sigma_{fi}(\varepsilon_e) = \sum_{if} \frac{(2J_i+1)\exp(-\beta\varepsilon_i)}{Z_A}\sigma_{fi}(\varepsilon_e) \\ &= 6g_{wk}^2 \int d\xi(\varepsilon_e - \xi)^2 \frac{G_A^2}{12\pi} S_{\mathrm{GT}^+}(\xi) F(Z,\varepsilon_e)\end{aligned} \tag{7}$$

where $g_{wk} = 1.1661 \times 10^{-5}\mathrm{GeV}^{-2}$ is the weak coupling constant and $G_A = 1.25$. $F(Z,\varepsilon_e)$ is the factor of Coulomb wave correction.

The total amount of Gamow-teller(GT) strength is S_{GT^+} which is by summing over a complete set from an initial state to final states. The response function $R_A(\tau)$ of an operator $\hat{A}$ at an imaginary-time τ is calculated by using the method of SMMC. Thus, $R_A(\tau)$ is given by (e.g., Dean et al. 1998; Juodagalvis et al. 2010)

$$R_A(\tau) = \frac{\sum_{if}(2J_i+1)e^{-\beta\varepsilon_i}e^{-\tau(\varepsilon_f-\varepsilon_i)}|\langle f|\hat{A}|i\rangle|^2}{\sum_i(2J_i+1)e^{-\beta\varepsilon_i}} \tag{8}$$

The strength distribution is is related to $R_A(\tau)$ by a Laplace Transform $R_A(\tau) = \int_{-\infty}^{\infty} S_A(\varepsilon)e^{-\tau\varepsilon}d\varepsilon$ and given by (e.g., Dean et al. 1998; Caurier et al. 1999; Juodagalvis et al. 2010)

$$S_{GT^+}(\varepsilon) = S_A(\varepsilon) = \frac{\sum_{if}\delta(\varepsilon - \varepsilon_f + \varepsilon_i)(2J_i+1)e^{-\beta\varepsilon_i}|\langle f|\hat{A}|i\rangle|^2}{\sum_i(2J_i+1)e^{-\beta\varepsilon_i}} \tag{9}$$

here ε is the energy transfer within the parent nucleus, and the $S_{\mathrm{GT}^+}(\varepsilon)$ is in units of MeV^{-1} and $\beta = \frac{1}{T_N}$, and T_N is the nuclear temperature.

For degenerate relativistic electron gas, the EC rates in the case without SES are given by (e.g., Dean et al. 1998; Caurier et al. 1999; Juodagalvis et al. 2010)

$$\lambda_{ec}^0 = \frac{\ln 2}{6163}\int_0^\infty d\xi S_{\mathrm{GT}^+}\frac{c^3}{(m_e c^2)^5}\int_{p_0}^\infty dp_e p_e^2(-\xi+\varepsilon_e)^2 F(Z,\varepsilon_e) f(\varepsilon_e, U_F, T) \tag{10}$$

The p_0 is defined as

$$p_0 = \begin{cases} \sqrt{Q_{if}^2 - 1} & (Q_{if} < -1) \\ 0 & (\text{otherwise}). \end{cases} \tag{11}$$

2.2 The EC rates in the case with SES

In 2002, based on the linear response theory model (LRTM) for relativistic degenerate electrons Itoh et al.(2002) discussed the effect of the screening potential on EC. The electron is strongly degenerate in our considerable regime of the density-temperature. The condition is expressed as

$$T \ll T_F = 5.930 \times 10^9\{[1 + 1.018(\frac{Z}{A})^{2/3}(10\rho_7)^{2/3}]^{1/2} - 1\}, \tag{12}$$

here T_{F} and ρ_7 are the electron Fermi temperature and the density (in units of $10^7\mathrm{g/cm}^3$).

For relativistically degenerate electron liquid, Jancovici et al. (1962) studied the static longitudinal dielectric function. Taking into account the effect of strong screening, the electron potential energy is written by

$$V(r) = -\frac{Ze^2(2k_{\rm F})}{2k_{\rm F}r}\frac{2}{\pi}\int_0^\infty \frac{\sin[(2k_{\rm F}r)]q}{q\epsilon(q,0)}dq, \qquad (13)$$

where $\epsilon(q,0)$ is Jancovicis static longitudinal dielectric function and $k_{\rm F}$ is the electron Fermi wave-number.

The screening potential for relativistic degenerate electrons by linear response theory is written by (Itoh et al. 2002)

$$D = 7.525\times 10^{-3}Z(\frac{10z\rho_7}{A})^{\frac{1}{3}}J(r_s,R) \ \ ({\rm MeV}) \qquad (14)$$

Itoh et al.(2002) detailed discussed the parameters $J(r_s,R)$, r_s and R. The Eq. (14) is fulfilled in the pre-supernova environment and is satisfied for $10^{-5}\le r_s \le 10^{-1}, \ \ 0\le R\le 50$.

The screening energy is sufficiently high enough such that we can not neglect its influence at high density when electrons are strongly screened. The electron screening will make electron energy decrease from ε to $\varepsilon^{'}=\varepsilon - D$ in the process of EC. Meanwhile the screening relatively increases threshold energy from ε_0 to $\varepsilon_s=\varepsilon_0+D$ for electron capture. So the EC rates in SES is given by (e.g., Juodagalvis et al. 2010; Liu. 2014)

$$\lambda_{ec}^s = \frac{\ln 2}{6163}\int_0^\infty d\xi S_{GT^+}\frac{c^3}{(m_ec^2)^5}$$
$$\int_{\varepsilon_s}^\infty d\varepsilon^{'}\varepsilon'(\varepsilon^{'2}-1)^{\frac{1}{2}}(-\xi+\varepsilon^{'})^2F(Z,\varepsilon^{'})f(\varepsilon_e,U_F,T) \qquad (15)$$

The nuclear binding energy will increase due to interactions with the dense electron gas in the plasma. The effective nuclear Q-value ($\rm Q_{if}$), will change at high density due to the influence of the charge dependence of this binding. When we take account into the effect of SES, the electron capture Q-value will increase by (Fuller et al(1982))

$$\Delta Q \approx 2.940\times 10^{-5}Z^{2/3}(\rho Y_e)^{1/3} \quad {\rm MeV}. \qquad (16)$$

Therefore, The Q-value of EC increases from Q_{if} to $Q'_{if}=Q_{if}+\Delta Q$. The ε_s is defined as

$$\varepsilon_s = \begin{cases} Q'_{if}+D & (Q'_{if}<-m_ec^2) \\ m_ec^2+D & ({\rm otherwise}). \end{cases} \qquad (17)$$

We define the screening enhancement factor C to enable a comparison of the results as follows

$$C = \frac{\lambda_{ec}^s}{\lambda_{ec}^0} \qquad (18)$$

3 NUMERICAL CALCULATIONS OF EC RATES AND DISCUSSION

The influences of SES on EC rates for these chromium isotopes at some typical astrophysics condition are shown in Figure 1. Note that the no SES and SES rates correspond to solid and dotted line. We detailed the EC process according to SMMC method, especially for the contribution for EC due to the GT transition. For a given temperature, the EC rates increases by more than six orders of magnitude as the density increases. Based on proton-neutron quasiparticle RPA model, Nabi & Klapdor-Kleingrothaus also detailed investigated the EC rates in the case without SES. Their results also shown that the density strongly influence on the EC rates for a given temperature. For examples, the EC rates for ^{61}Cr increases from $6.3096\times10^{-23}{\rm s^{-1}}$ to $3.71535\times10^{2}{\rm s^{-1}}$ when the density changes from $10^7{\rm g/cm^3}$ to $10^{11}{\rm g/cm^3}$

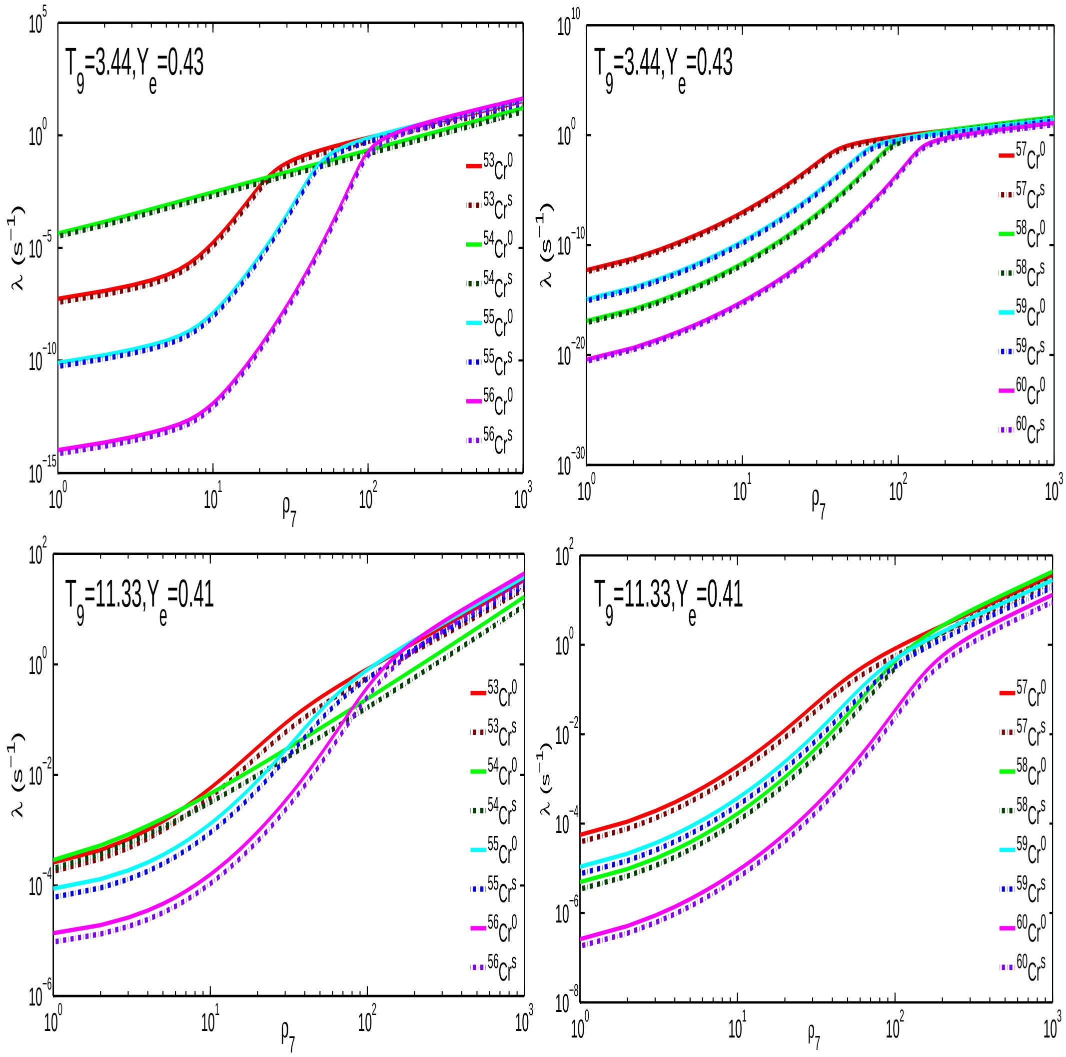

Fig. 1 The no SES and SES rates corresponding to solid and dotted line for chromium isotopes as a function of the density ρ_7 at the temperature of $T_9 = 3.44, Y_e = 0.43$ and $T_9 = 11.33, Y_e = 0.41$.

at $T_9 = 3$ (see the detailed discussions in Nabi & Klapdor-Kleingrothaus. 1999). under the same conditions, the FFN rates for ^{60}Cr increases from $8.3946 \times 10^{-26}\text{s}^{-1}$ to $1.2388 \times 10^{3}\text{s}^{-1}$ (see Fuller et al. 1982). These studies show that the stellar weak rates play a key role in the dynamics of the core collapse calculations and stellar numerical simulation.

According to our calculations, the GT transition EC reaction may not be dominant process at lower temperature. On the other hand, the higher the temperature, the larger the electron energy, the larger the density, the higher the electron Fermi energy becomes. Therefore, a lot of electrons join in EC reaction and the GT transition would be very active and have dominated contribution to total EC rates. Figure 1 shows the screening rates and no screening rates, which corresponding to solid and dotted line as a function of density. We find that the screening rates are commonly lower than no screening rates.

The Gamow-Teller strength distributions play a significant role in supernova evolution. But the GT^{+} transitions is addressed only qualitatively in pre-supernova simulations because of the insufficient

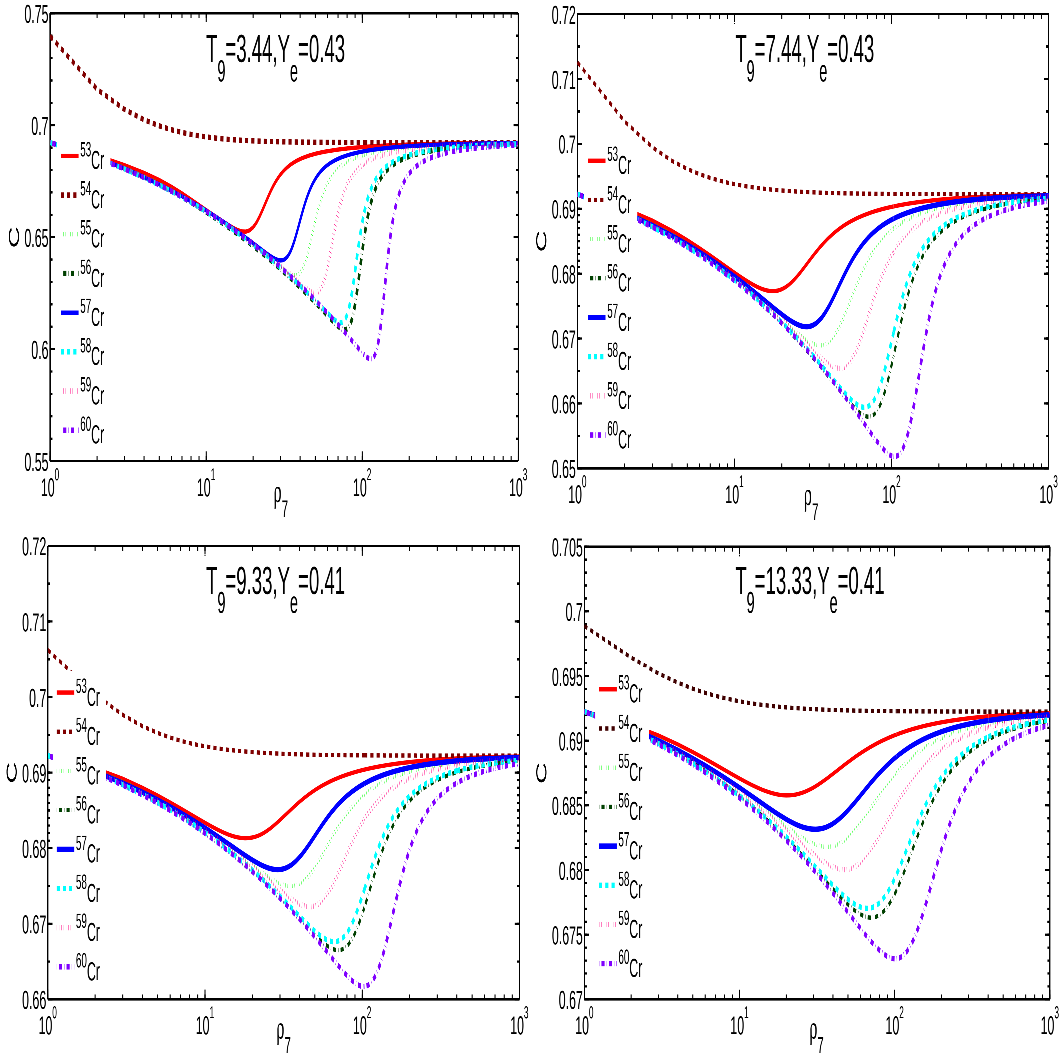

Fig. 2 The SES enhancement factor C for chromium isotopes as a function of the density ρ_7 at the temperature of $T_9 = 3.44, 7.44, Y_e = 0.43$ and $T_9 = 9.33, 11.33, Y_e = 0.41$.

of experimental information. The general rule is that the energy for the daughter ground state is parameterized phenomenologically by assuming the GT^+ strength resides in a single resonance. Charge exchange reactions (n, p) and (p, n) would, if obtainable supply us with plenty of experimental information. However, any available experimental GT+ strength distributions for these nuclei can not obtained except for theoretical calculations. Table 1 present some information about the comparison of our results by SMMC for total strength, centroid and width of calculated GT strength distributions with those of NKK (Nabi et al. 2016) for EC of $^{53-60}$Cr. Our results of GT strength distributions calculated are higher than those of NKK.

Based the pn-QRPA theory, NKK analyzed nuclear excitation energy distribution by taking into consideration the particle emission processes. They calculated stronger Gamow-Teller strength distribution from these excited states compared to those assumed using Brinks hypothesis. However, in their works, they only discussed the low angular momentum states. By using the method of SMMC, GT in-

Table 1 Comparison of our results by SMMC for total strength, centroid and width of calculated GT strength distributions with those of NKK (Nabi et al. 2016)for EC of $^{53-60}$Cr.

	$\sum B(GT)_+$		E_+(MeV)		$Width_+$ (MeV)	
Nuclide	NKK	SMMC	NKK	SMMC	NKK	SMMC
^{53}Cr	0.51	0.5625	6.21	6.334	2.72	2.813
^{54}Cr	1.95	2.2340	2.88	2.912	3.32	3.406
^{55}Cr	0.39	0.4130	4.06	4.126	3.47	3.675
^{56}Cr	1.31	1.3326	1.77	1.791	2.14	2.366
^{57}Cr	0.25	0.2740	5.21	5.267	2.84	2.972
^{58}Cr	0.82	0.8411	1.57	1.605	2.49	2.560
^{59}Cr	0.24	0.2520	1.26	1.302	2.24	2.272
^{60}Cr	0.39	0.4012	3.03	3.201	4.99	5.017

tensity distribution is detailed discussed and actually an average value of the distribution is adopted in our paper.

The screening factors C is plotted as a function of ρ_7 in figure 2. Due to SES, the rates decrease by about 40.43%. The lower the temperature, the larger the effect of SES on EC rates is. This is due to the fact that the SES mainly decreased the number of higher energy electrons, which can actively join in the EC reaction. Moreover, the SES can also make the EC threshold energy increases greatly. As a matter of fact, SES will strongly weaken the progress of EC reactions. One can also find that the screening factor almost tends to the same value at higher density and it is not dependent on the temperature and density. The reason is that at higher density the electron energy is mainly determined by its Fermi energy, which is strongly decided by density.

Table 2 shows the numerical calculations about the minimum values of screening factor $C_{\rm min}$ in detail. One finds that the EC rates decrease greatly due to SES. For instance, from Table 2 of the factor $C_{\rm min}$, the rates decrease about $34.75\%, 30.77\%, 36.92\%, 39.07\%, 35.98\%, 38.81\%, 37.50\%, 40.43\%$ for $^{53-60}$Cr at $T_9 = 3.44, Y_e = 0.43$, respectively. This is due to the fact that the SES mainly decreased the number of higher energy electrons, which can actively join in the EC reactions. On the other hand, the screening of nuclear electric charges with a high electron density means a short screening length, which results in a lower enhancement factor from Coulomb wave correction. However, even a relatively short electric charge screening length will not have much effect on the overall rate due to the weak interaction being effectively a contact potential. A bigger effect is that electrons are bound in the plasma.

Synthesizes the above analysis, the effects of the charge screening on the nuclear physics (e.g., EC and beta decay) come at least from following factors. First, the screening potential will change the electron Coulomb wave function in nuclear reactions. Second, the electron screening potential decreases the energy of incident electrons joining the capture reactions. Third, the electron screening increases the energy of atomic nuclei (i.e., increases the single particle energy) in nuclear reactions. Finally, the electron screening effectively decreases the number of the higher-energy electrons, whose energy is more than the threshold of the capture reaction. Therefore, screening relatively increases the threshold needed for capture reactions and decreases the capture rates.

4 CONCLUSION REMARKS

In this paper, based on the theory of RPA and LRTM, by using the method of SMMC, we investigated the EC rates in SES. The EC rates increase greatly by more than six orders of magnitude as the density increases. On the other hand, by taking into account the influence of SES on the energy of incident electrons and threshold energy of electron capture, the EC rates decrease by $\sim 40.43\%$.

Electron captures play an important role in the dynamics process of the collapsing core of a massive star. It is a main parameter for supernova explosion and stellar collapse. The SES strongly influences the EC and may influences the cooling rate and evolutionary timescale of stellar evolution. Thus, the con-

Table 2 The minimums value of strong screening factor C for some typical astronomical condition when $1 \leq \rho_7 \leq 10^3$.

	$T_9 = 3.44, Y_e = 0.43$		$T_9 = 7.44, Y_e = 0.43$		$T_9 = 9.33, Y_e = 0.41$		$T_9 = 13.33, Y_e = 0.41$	
Nuclide	ρ_7	$C_{\min}$	ρ_7	$C_{\min}$	ρ_7	$C_{\min}$	ρ_7	$C_{\min}$
^{53}Cr	18	0.6525	19	0.6774	19	0.6813	20	0.6858
^{54}Cr	62	0.6923	65	0.6924	66	0.6924	67	0.6924
^{55}Cr	38	0.6308	37	0.6690	36	0.6750	37	0.6818
^{56}Cr	81	0.6093	72	0.6580	71	0.6665	71	0.6763
^{57}Cr	32	0.6402	30	0.6719	31	0.6772	33	0.6832
^{58}Cr	74	0.6119	69	0.6594	67	0.6676	67	0.6770
^{59}Cr	50	0.6250	47	0.6654	49	0.6723	48	0.6800
^{60}Cr	115	0.5957	106	0.6518	104	0.6617	99	0.6731

clusions we obtained may have a significant influence on the further research of supernova explosions and numerical simulations.

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