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Scattering Law Analysis Based on Hapke and Lommel-Seeliger models for Asteroidal Taxonomy

Xiang-Jie Huang[1], Xiao-Ping Lu[1,2,4*], Jian-Yang Li[5], Bao Mei[1,2], Chih-Hao Hsia[1,2], Hai-Bin Zhao[3,4]

[1] Macau University of Science and Technology, Taipa, Macau; *xplu@must.edu.mo*
[2] Lunar and Planetary Science Laboratory, Macau University of Science and Technology - Partner Laboratory of Key Laboratory of Lunar and Deep Space Exploration, Chinese Academy of Sciences, Taipa 999078, Macau
[3] Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China
[4] Key Laboratory of Planetary Sciences, Chinese Academy of Sciences, Nanjing 210008, China
[5] Planetary Science Institute, 1700 E. Ft. Lowell Road, Suite 106, Tucson, AZ 85719, USA

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Abstract In deriving the physical properties of asteroids from their photometric data, the scattering law plays an important role, although the shape variations of the asteroids result in the main variations in lightcurves. By following the physical behaviours of light reflections, Hapke (2012) deduced complex functions to represent the scattering process, however, it is very hard to accurately simulate the surface scattering law in reality. For simplicity, another numerical scattering models are presented for efficiently calculating the physical properties of asteroids, such as Lommel-Seeliger (LS) model. In this article, we compare two models and find that the LS model with four parameters can numerically fit well the Hapke model with five parameters. Furthermore, the generated synthetic lightcurves by the Cellinoid shape model also show that the LS model can perform as well as the Hapke model in the inverse process. Finally, by applying the Principal Component Analysis (PCA) technique to the parameters of LS model, we present an efficient method to classify the C and S type asteroids, instead of the conventional method by the parameters of Hapke model.

Key words: Scattering law; Hapke model; Lommel-Seeliger model; Asteroid; Taxonomy;

1 INTRODUCTION

As the primitive matters of early solar system, the asteroid can reveal the evolution mechanism of planets and provide the formation information of our solar system (DeMeo & Carry 2014). From ground-based observations, the physical properties of asteroids can be derived from their photometric data such as rotational periods, pole orientations, and the overall shapes (Lu et al. 2013, 2014; Lu & Ip 2015; Lu et al. 2016).

In deriving the physical properties of asteroids from their photometry data, the scattering law plays the important role, although the shape variations of the asteroids result in the main variations in lightcurves (Karttunen 1989; Karttunen & Bowell 1989). As shown in Figure 1, the scattering geometry can be defined in terms of the incidence angle i between the light source and local surface normal,

Table 1 Physical meanings of five parameters in Hapke Model

Parameter	Symbol	Meaning
Single Scattering Albedo	ω	Fraction of total incident energy that is scattered by a single particle towards all directions
Asymmetry factor	g	Spatial energy distribution in a single particle scattering phase function
Opposition surge amplitude	B_{so}	Amplitude of opposition effect(SHOE only)
Opposition surge width	h_s	Width of opposition effect (SHOE only)
Roughness parameter	$\bar{\theta}$	Average deviation of local normal with respect to average

the emergence angle e between the observer and the local surface normal, and the phase angle α between the light source and observer, as seen from the object, in addition to the azimuthal angle ϕ between the planes of incidence and emergence. By following the physical behaviours of light reflections, Hapke (1981, 1984, 1986) and Hapke & Wells (1981) deduced complex functions to represent the scattering process. And their evolved models has been applied extensively to spacecraft and telescopic observations for a lot of objects throughout the Solar System to derive their physical and surface characteristics (Li et al. 2004, 2006, 2007, 2010, 2013).

However, Hapke model is very complicated in the application of numerical inverse process to search the physical properties of asteroids. There are many equivalent methods to attempt to numerically replace it such as the joint linear exponential model made by Muinonen et al. (2009), Minnaert model made by Minnaert (1941), and Lommel-Seeliger (LS) model constructed by Seeliger (1884). Especially, the LS model is very efficient in numerical calculations of synthetic brightness of asteroids. There are recently many applications to search the asteroidal physical parameters by LS model (Cellino et al. 2015; Muinonen & Lumme 2015).

The parameters in Hapke model have the corresponding physical meanings such as the albedo, which illustrates the ability of reflecting light on surface and others described in Table 1. Meanwhile, LS model can be applied efficiently to search the physical parameters for asteroids (Muinonen et al. 2015; Wang et al. 2016). In this article, we take an attempt to combine two models, by presenting an equivalent method to connect the physical meanings of Hapke model to the parameters of LS model. Following the method, the parameters of LS model can be efficiently derived in fitting the photometric data and subsequently they can be analyzed by the Principal Component Analysis (PCA) technique to obtain a classification for the asteroids (Abdi & Williams 2010). Ultimately, to better identify the asteroid types, the Least Absolute Shrinkage and Selection Operator (LASSO) technique is introduced to efficiently classify C and S type asteroids based on LS model parameters (Tibshirani 1996; Mei & Ling 2009; Musoro et al. 2014).

The paper will be arranged as follows. First of all, in Section 2, Hapke model and LS model will be concisely presented. In Section 3, the numerical simulations are conducted to confirm that LS model can perform well as replacing Hapke model in the inversion application. Furthermore, equivalent to Hapke model, the parameters of LS are analyzed by PCA method for classifying the C and S type asteroids. Subsequently, the LASSO method is employed for the taxonomy of C and S type asteroids from the LS model parameters. In the end, we sum up the primary work of this article and discuss the future work in Section 4.

2 SCATTERING MODELS

2.1 Hapke model

The Hapke model is based on a semi-physical model that uses the analytical solutions of radiative transfer on an asteroidal surface with simple assumptions, coupled with the empirical models that describe the scattering behaviour of the particulate surface (Hapke 2012).

There are at least five parameters in Hapke model, including the Single Scattering Albedo (ω), photometric roughness ($\bar{\theta}$), the amplitude and width of opposition effect (B_{so} and h_s for shadow-hiding opposition effect (SHOE), and B_{co} and h_c for coherent backscattering opposition effect (CBOE)), as well

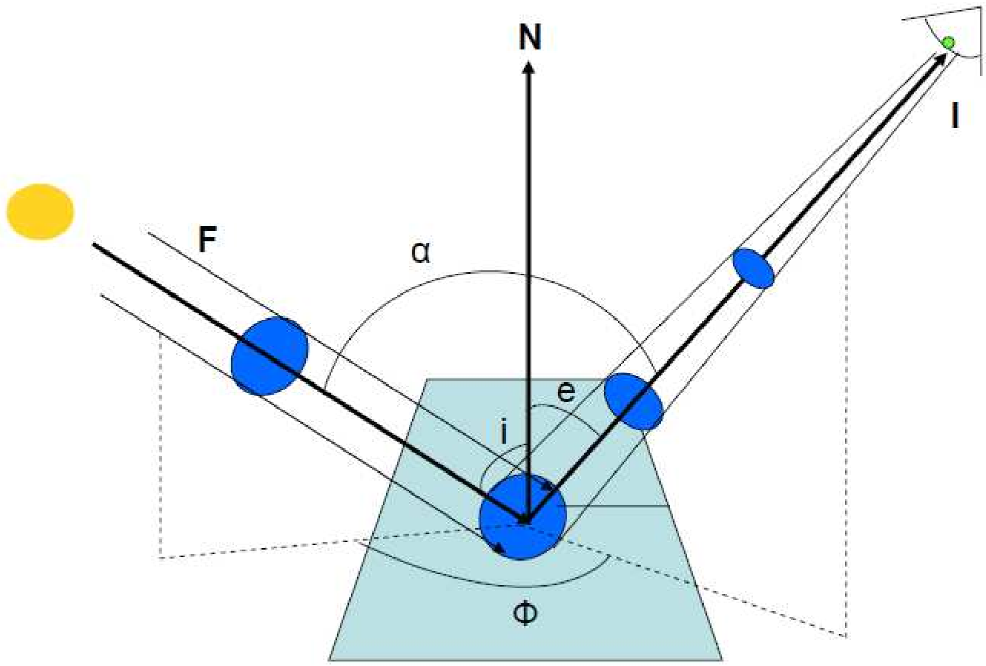

Fig. 1 Schematic diagram for bidirectional reflectance with the solar phase angle (α), incidence angle (i) and emergence angle (e).

as one or more parameters to describe single-particle phase function. The model exploited throughout this article is a five-parameters version, which only considers the SHOE, and adopts a one-term Henyey-Greenstein (HG) function (Buratti & Veverka 1983; Li et al. 2015) involving one asymmetry parameter g to describe the single-particle phase function. The five parameters are summarized in Table 1 and the Radiance Factor (RADF), the ratio of the bidirectional reflectance of a surface to that of a perfectly diffuse surface illuminated at $i = 0°$ (Hapke 2012), is shown in Eq. (1), where $B(\alpha)$ is the shadow hiding opposition surge function, $p(\alpha)$ is the average particle single-scattering phase function, $H(x)$ is Ambartsumian-Chandrasekhar function for multiple scattering, and S is the Hapke roughness shadowing function (Hapke 2012).

$$RADF(i,e,\alpha) = \frac{\omega}{4}\frac{\mu_0}{\mu_0+\mu}\left\{[1+B(\alpha)]\,p(\alpha)+H(\mu_0)H(\mu)-1\right\}S(i,e,\alpha,\bar{\theta}), \tag{1}$$

$$B(\alpha) = \frac{B_{so}}{1+\frac{1}{h_s}\tan\frac{\alpha}{2}}, \tag{2}$$

$$p(\alpha) = \frac{1-g^2}{(1+2g\cos\alpha+g^2)^{\frac{3}{2}}}, \tag{3}$$

$$H(x) = \frac{1+2x}{1+2\sqrt{1-\omega x}}, \tag{4}$$

Table 2 The equivalent parameters of LS model

	Our study	Takir et al. (2015)
A_{ls}	0.0286	0.030
β	-4.20×10^{-2}	-4.36×10^{-2}
γ	2.63×10^{-4}	2.69×10^{-4}
δ	-1.09×10^{-6}	-9.90×10^{-7}

2.2 LS model

The LS model is a widely-used scattering model (Besse et al. 2013), developed by Seeliger (1884) and improved by Hapke (2012). The RADF (Takir et al. 2015) of LS model with four free parameters is shown in Eq. (5), where $f(\alpha) = e^{\beta\alpha+\gamma\alpha^2+\delta\alpha^3}$ with three polynomial coefficients β, γ, and δ, describes the variation in surface reflectance with respect to phase angle α (in degrees) and $A_{ls} = \frac{\omega}{4\pi}$ is the Lommel-Seeliger albedo with the average particle single scattering albedo, ω.

Here in both of Eq. (1) and Eq. (5), the μ_0 and μ are defined as the projections of the illuminating direction and emitting direction on the normal direction of the surface, respectively.

$$RADF(i, e, \alpha) = A_{ls} \frac{\mu_0}{\mu_0 + \mu} f(\alpha), \tag{5}$$

3 SCATTERING LAW ANALYSIS FOR HAPKE AND LS MODELS

Both of Hapke and LS models are shown in the previous section. Intuitively, the LS model has a much simpler form than Hapke model. In this section firstly the equivalent parameters of LS model are numerically fitted to the given parameters of Hapke model. And by applying the two equivalent parameters to generate the synthetic lightcurves based on a Cellinoid shape model, their similar morphologies of synthetic lightcurves confirm that LS model can replace Hapke model in the more efficient way. Then the PCA technique is applied to analyze the equivalent parameters of LS model, derived from the Hapke parameters of C and S type asteroids. Then the result shows that the different type asteroids can be identified from the parameters of LS model.

3.1 Equivalent Parameters

To better simulate the real circumstances, the derived parameters of difference scattering models for (101955) Bennu by Takir et al. (2015), are exploited for comparison of the equivalent parameters.

The phase curves of (101955) Bennu for Hapke model with $\omega = 0.031, B_{so} = 3.9, h_s = 0.11, g = -0.32$, and $\bar{\theta} = 20^\circ$ and fitted LS model are shown in Figure 2. From Figure 2, it is clear that LS model can fit Hapke model very well and the opposition effect in phase curve is also fitted well. The fit SSE(Sum of Squared Errors), defined as

$$SSE = \sum_i \left(R^i_{Hapke} - R^i_{LS}\right)^2 \tag{6}$$

is 0.0001312, where R^i_{Hapke} and R^i_{LS} denote the reflectances of the Hapke and LS model for the corresponding phase angle i.

Additionally, the fitted equivalent parameters of LS model are listed in Table 2. Meanwhile, the parameters of LS model derived by Takir et al. (2015) is also listed for comparison and confirmation.

Furthermore, the parameters of Hapke model and the fitted equivalent parameters of LS model listed in Table 2 are applied to generate the synthetic lightcurves based on a Cellinoid shape model presented by Lu et al. (2017), which are shown in Figure 3. Both of the synthetic lightcurves have the similar morphology with the same positions of maximum and minimum. Although the deviations between the maximum and minimum of the two synthetic lightcurves exist, the equivalent parameters of LS model

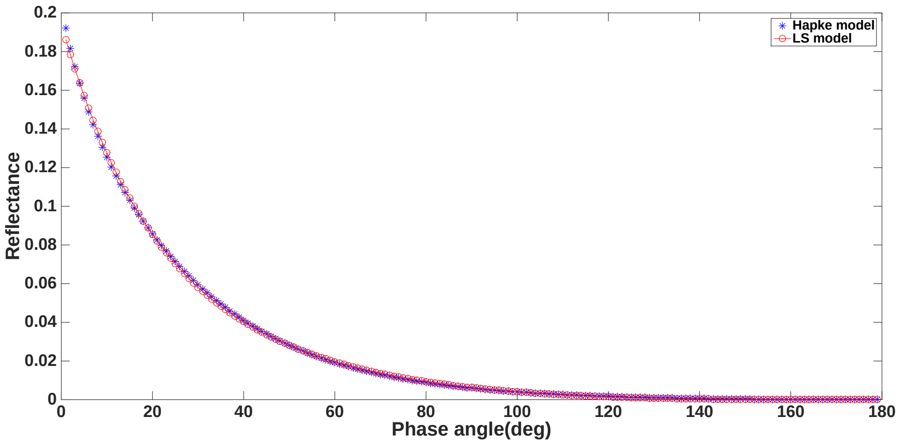

Fig. 2 The phase curves of (101955) Bennu for Hapke (shown as blue '*') and LS (shown as red 'o-') models, respectively.

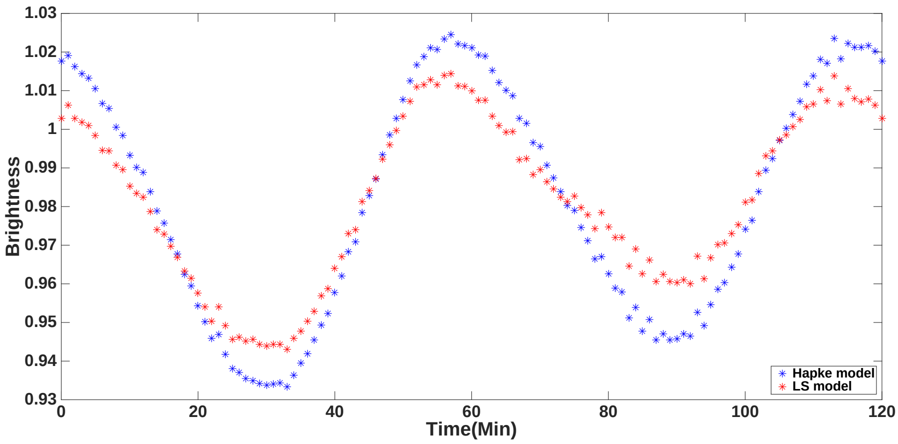

Fig. 3 The synthetic lightcurves based on Cellinoid shape with scattering parameters of Hapke (Blue '*') and LS (Red '*') models.

can fit the Hapke model well as the relative brightness is commonly applied in real application. The slightly larger amplitude of LS model than Hapke model will not affect the results of inverse process (Lu et al. 2017).

3.2 Asteroidal Taxonomy Based on LS model

As the parameters in Hapke model can reveal the surface information of asteroids, Helfenstein & Veverka (1989) introduced the taxonomy of C and S type asteroids according to the five parameters. The mean parameters are listed in Table 3, where the equivalent parameters of LS model are also listed. The corresponding phase curves for C and S type asteroids with these mean

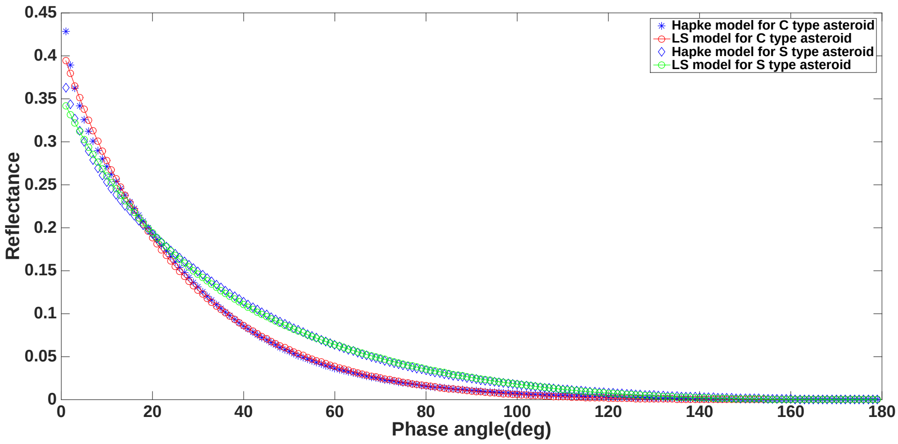

Fig. 4 C and S type asteroids phase curves derived from Hapke and LS models, respectively.

Table 3 Mean parameters of Hapke and LS models for C and S type asteroids.

		Mean of C type		Mean of S type
Hapke model	ω	0.037	ω	0.23
	B_{so}	1.03	B_{so}	1.6
	h_s	0.025	h_s	0.08
	g	-0.47	g	-0.27
	$\bar{\theta}$	20°	$\bar{\theta}$	20°
LS model	A_{ls}	0.0218	A_{ls}	0.0938
	β	-3.77×10^{-2}	β	-2.99×10^{-2}
	γ	1.51×10^{-4}	γ	2.12×10^{-4}
	δ	-7.68×10^{-7}	δ	-9.12×10^{-7}

parameters of Hapke and LS models are shown in Figure 4. Apparently, for both type asteroids, the LS model can fit the Hapke model well in simulating the phase curves.

In order to find the taxonomic relations in LS model for the two type asteroids, we build a test sets containing 1200 sample points for each type, which are selected respectively from the C and S type asteroids with the corresponding five Hapke parameters. Subsequently, the equivalent parameters of LS model for two type asteroids are derived.

Then the PCA technique (Abdi & Williams 2010) is applied to determine the correlation of the four variables in LS model and the results are presented in Table 4. The cumulative probability of previous variables of Z_1 and Z_2 is more than 95% and their dependence relations to the four variables of LS model are formulated in Eq. (7) and Eq. (8), respectively. Eventually, the distributions of the two principal components for C and S type asteroids are shown in Figure 5 with a clear cluster feature. The regression lines for C and S type asteroids are shown in the same figure and their formulas are given in Eq. (9) and Eq. (10), respectively.

$$Z_1 = 0.2461A_{ls} - 0.5197\beta + 0.5921\gamma - 0.5647\delta, \tag{7}$$

$$Z_2 = 0.9366A_{ls} - 0.0773\beta - 0.1798\gamma + 0.2907\delta, \tag{8}$$

Table 4 PCA results.

Principal Component	Z_1	Z_2	Z_3	Z_4
Percent Variance(%)	70.74	25.24	3.59	0.43
Cumulative Percent(%)	70.74	95.98	99.57	100

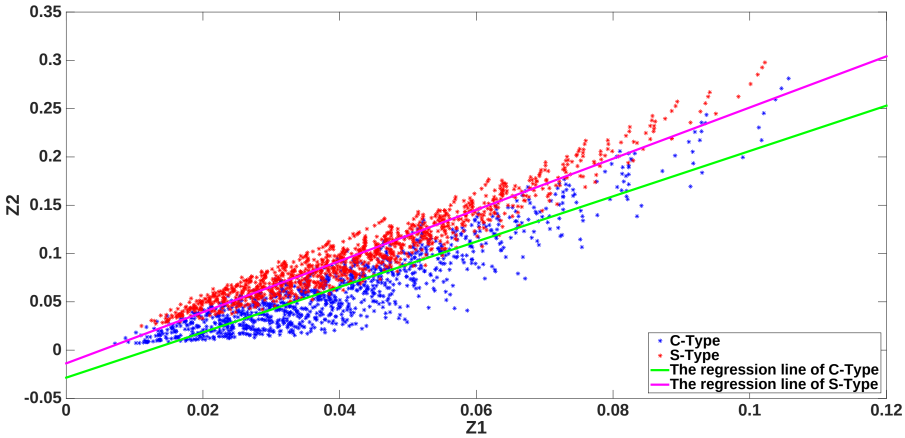

Fig. 5 Cluster distribution map of C (shown as Blue '*') and S (shown as Red '*') type asteroids. The regression line of C type is represented as green line and the one of S type is shown as pink line.

$$Z_2 = 2.3467 Z_1 - 0.0285, \quad (9)$$

$$Z_2 = 2.6490 Z_1 - 0.0137, \quad (10)$$

The PCA results show that the parameters of LS model for C and S type asteroids appear clustering, which can be applied in asteroid taxonomy. Following this, we try to find an efficient way to classify the C and S type asteroids based on the LS model parameters in the subsequent subsection.

3.3 Identify C and S types Based on LS model

As previously described, the principal components of four parameters in LS model show a clear clustering. Here we present an efficient technique to identify the taxonomy of an asteroid from its LS parameters. The Least Absolute Shrinkage and Selection Operator (LASSO) (Tibshirani 1996; Mei & Ling 2009; Musoro et al. 2014) in machine learning is commonly applied to search the best-fit solution with sparse non-zero variables.

The asteroid taxonomy from the LS parameters can be expressed in the LASSO format as a linear system of equations,

$$\mathbf{y} = \begin{bmatrix} \mathbf{T_c} & \mathbf{T_s} \end{bmatrix} \begin{bmatrix} \mathbf{d_c} \\ \mathbf{d_s} \end{bmatrix} = \mathbf{Td} \quad (11)$$

where the coefficient matrix $\mathbf{T} = [\mathbf{T_c}, \mathbf{T_s}]$ consists of the known LS parameters for C and S type asteroids, respectively. In the context of machine learning, the coefficient matrix $\mathbf{T}$ is the training set.

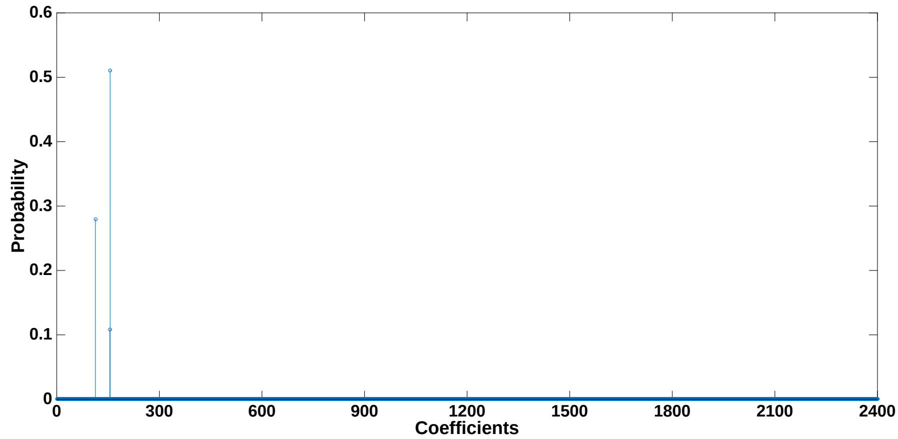

Fig. 6 The successfully derived indicators for C type asteroids by LASSO method

For any given LS parameters $\mathbf{y}$, by solving the linear system (11) the vector $\mathbf{d} = [\mathbf{d_c}, \mathbf{d_s}]^{\mathbf{T}}$ will indicate the possible asteroid type.

As LASSO method minimizes the following optimization problem with the ℓ_1 regularization term,

$$\mathbf{d}^* = \arg\min_{\mathbf{d}} \frac{\mathbf{1}}{\mathbf{2}} \|\mathbf{y} - \mathbf{Td}\|_{\mathbf{2}}^{\mathbf{2}} + \lambda \|\mathbf{d}\|_{\mathbf{1}} \tag{12}$$

the probability distribution can be derived to confirm that the asteroid with this group of LS parameters can be classified to C or S type. As a result of the employed ℓ_1 regularization, which encourages small components of $\mathbf{d}$ to become exactly zero, the derived result $\mathbf{d}^*$ presents sparse solutions with several non zero components, indicating the asteroid type.

First, the training set including total 2400 sample points is built by merging the LS model parameters for known C and S type asteroids with respective 1200 sample points, where the LS model parameters are calculated equivalently from Hapke model parameters for known C and S type asteroids. Then we also generate a test set containing 200 points for known C and S type asteroids. Finally, by applying Gradient Projection method (Figueiredo et al. 2007) to solve the problem in Eq. (12), the taxonomy for the test set is derived.

As shown in Figure 6 and Figure 7, the LASSO method can successfully derive the asteroid type. However, there are also some exception situations as shown in Figure 8. The probabilities for either of C and S type are too close to discriminate them successfully. In all, there are 9 and 13 indeterminate cases for C and S type asteroids, respectively. Compared to the total test set with 200 sample points, the LASSO technique can identify the C and S type asteroids from LS model parameters in an approximate probability of 90%. Furthermore, as shown in Figure 6 and 7, this method can also identify the similarity of two asteroids based on their LS model parameters.

4 CONCLUSIONS

In this article, we analyze the two scattering models, Hapke and LS models. Considering the Hapke model is deduced with physical meanings and LS model is efficient in numerical calculation, we numerically compare the two scattering models and confirm that LS model can fit Hapke model well in phase curves. Additionally, for the shape determination of asteroids, the LS model can generate the similar lightcurves with the same morphology to the Hapke model. Finally, as the parameters in Hapke model are commonly exploited to classify asteroids. We primarily investigate two type asteroids, C and S type.

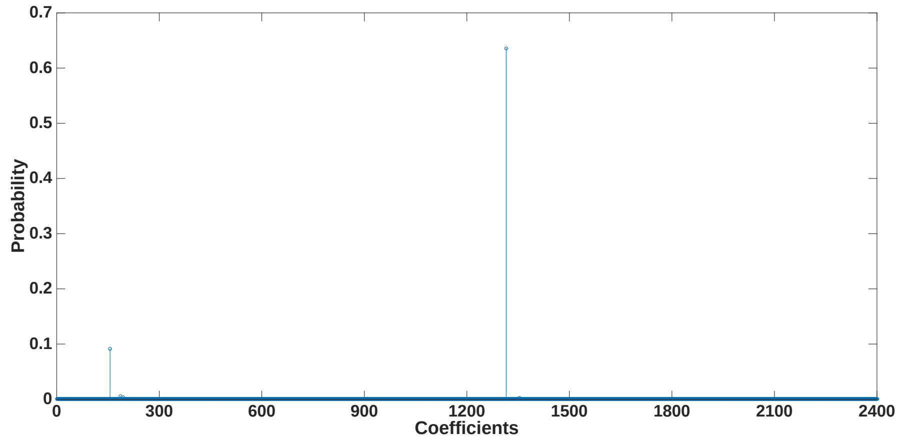

Fig. 7 The successfully derived indicators for S type asteroids by LASSO method

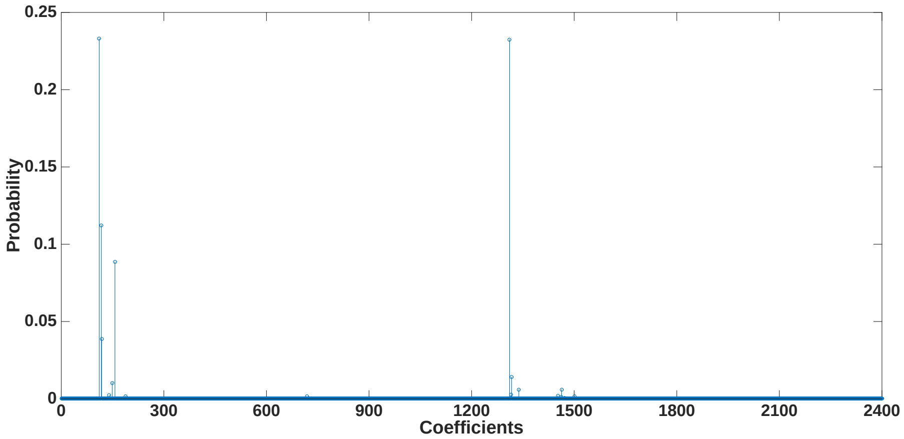

Fig. 8 The indeterminate indicators for C and S type asteroids by LASSO method

By calculating the equivalent parameters of LS model, we apply the PCA technique to show the clear cluster features of former two principal components of LS model for C and S type asteroids. Moreover, we also introduce the method based on LASSO to classify an asteroid from LS scattering parameters to C or S type. This is very useful in real application. The shape, pole, rotational period, as well as the four LS scattering parameters can be derived from photometric data by the inverse process such as Kaasalainen inversion method and Cellinoid inverse process. Then by applying the technique presented in this article, the asteroid can be classified to C or S types in a fast way.

As now we only test the C and S type asteroids, we want to find more taxonomy of asteroids and try to apply our technique to classify them in the future. And we also expect that a new taxonomy based on LS model can be given out in the future. This will be very useful for asteroids, especially for the cases of inverse process.

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