Correction and simulation of the intensity compensation algorithm in curvature wavefront sensing *

Zhi-Xu Wu¹,²,³, Hua Bai¹,² and Xiang-Qun Cui¹,²

¹ National Astronomical Observatories/Nanjing Institute of Astronomical Optical & Technology, Nanjing, China; zxwu@niaot.ac.cn
² Key Laboratory of Astronomical Optics & Technology, Nanjing Institute of Astronomical Optics & Technology, Chinese Academy of Sciences, Nanjing 210042, China
³ University of Chinese Academy of Sciences College, Beijing, 100049

Received 2014 June 24; accepted 2014 September 4

Abstract The wave-front measuring range and recovery precision of the curvature sensor can be improved by Intensity compensation Algorithm. However, in the fast f-number focus system, especially in a telescope with large field of view, the accuracy of this algorithm can not meet the requirements. The theoretical analysis of the correction of Intensity compensation algorithm in the fast f-number focus system is firstly introduced and afterwards the mathematical formulas of this algorithm are deduced. The correction result is then verified through simulation. The method of such simulation is that: First, the curvature signal of the fast f-number focus system is simulated by Monte Carlo ray tracing; then the wave-front result is recovered by the inner loop of the FFT wave-front recovery algorithm and the outer loop of Intensity compensation Algorithm. Upon the comparison of the Intensity compensation Algorithm of the ideal system and the corrected Intensity compensation Algorithm, we reveal that the curvature sensor recovery precision can be greatly improved by the corrected Intensity compensation Algorithm.

Key words: active optics; curvature sensor; wave-front recovery; intensity compensation

1 INTRODUCTION

In the imaging process of telescopes, many factors will lead to the decline of image quality, such as optical design, optical fabrication error, gravity deformation, thermal deformation. The concept of Active Optics has changed the way of designing telescopes (Su & Cui 1999). People can take the initiative to correct the gravity deformation, temperature deformation, support error, and even the mirror machining error, and active optics make the image quality become better (Su & Cui 2004). In Active Optics, real time sensing of wave-front error is very critical. There are two main kinds of Active Optical wave-front sensors: Hartmann wave-front sensor and the wave-front curvature sensor. Hartmann wave-front sensor had been used in many large telescopes, such as Chinese LAMOST (Cui et al. 2004, Zhang 2008), and its measuring precision is extremely high (Stepp 1994). But in the telescope with large field of view, the field of view used for sensing is limited by Hartmann wave-front sensor, almost equal to zero. Wave-front curvature sensor is based on the measurement of the wave-front curvature. Compared with the

* Supported by the National Natural Science Foundation of China.
Hartmann sensor, it has the advantages of simplicity, high throughput, and avoidance of calibration difficulties, etc (Roddier & Roddier 1993). LSST (Manuel et al. 2010) and VISTA (Patterson & Sutherland 2003) is proposed to use this method to measure the wave-front.

There are two common methods to recover wave-front by curvature sensors: FFT Algorithm and G-S Algorithm. Both algorithms require that the equivalent defocus distance is small and the wave-front aberration is not very large. Thus, Roddier proposed Intensity compensation Algorithm based on the ideal system in 1993. By compensating the wave-front error of the defocus images, the wave-front measure range and recover precision of the curvature sensor can be improved greatly. Roddier used his algorithm to real telescopes like ESO NTT and got very good results (Roddier & Roddier 1993), but those testings were applied on the relative slow Cassegrain focus and the pupil grid distortion errors can be ignored. In the fast f-number focus telescopes, the reference point is much different to that of the ideal system, and the pupil grid distortion errors will lead to lower recovery precision, thus this algorithm can not be used in those systems directly. In addition, in the situation of off-axis, the angle between the CCD and defocus surface can also cause the coordinate distortion. In this paper, we theoretically deduce the corrected Intensity compensation formula in a fast f-number focus telescopes, and simulate the situation of on-axis and off-axis with a flat-field telescope. The results show that the corrected Intensity compensation algorithm is more reasonable for wave-front recovery in the fast f-number focus telescopes.

2 FFT WAVE FRONT RECOVERY ALGORITHM AND INTENSITY COMPENSATION ALGORITHM

Fig. 1 The principle of the curvature sensor.

Curvature wave-front sensing was first proposed by Roddier in 1989 (Roddier 1989), and had been used in adaptive optics successfully. The principles of curvature sensor is showed in Figure 1. I1 and I2 are the defocus images, we can get the curvature signal by subtracting I2 from I1 and then perform normalization:

\[ S(r) = \frac{-1}{\Delta z} \frac{I_2(r) - I_1(r)}{I_2(r) + I_1(r)} \approx \nabla^2 W - \delta_c \frac{\partial W}{\partial n}. \]  

(1)

With \( \Delta z = \frac{f(l-f)}{l} \), \( f \) is the focal length of the optical system, \( l \) is the defocus distance, \( \delta_c \) is the delta function at the aperture edge. The main task of wave-front recovery is to solve this Poisson equation with Neumann boundary condition. If we constrain \( \frac{\partial W}{\partial n} \) on the aperture edge, \( \delta_c \) can be absorbed into Laplacian (Claver et al. 2012):

\[ S = \nabla^2 W. \]

(2)
Make the Fourier transform at both side of formula (2) and
\[ FT_{\mu, \nu} \{ \nabla^2 W(x, y) \} = -4\pi^2 (\mu^2 + \nu^2) FT_{\mu, \nu} \{ W(x, y) \}. \] (3)

Thus:
\[ W = \text{IFT}_{x,y} \left\{ \frac{FT_{\mu, \nu}(S)}{-4\pi^2 (\mu^2 + \nu^2)} \right\}. \] (4)

The method mentioned above is only a first order approximation, valid for small \( \Delta z \), i.e., highly de-focused images. The solution of FFT algorithm should be used as the first order solution. Roddier discovered that the intensity compensation algorithm which compensates the residual aberration of the defocus images can improve the precision of wave-front recovery \((\text{Roddier} \ & \ \text{Roddier} \ 1993)\). As shown in Figure 2, \( R \) is the radius of the telescope’s pupil, and the normalized coordinate of the pupil is \( x = U/R, y = V/R \). The normalized coordinate of the defocus surface is \( x' = uf/lR, y' = vf/lR \).

**Fig. 2** The principle of the Intensity compensation Algorithm.

Assuming no aberration, the ray from \( M \) will converge toward \( F \) and cross the defocus surface to point \( N \). Because of the aberration \( W(x, y) \), the ray from \( M \) will converge to \( N' \). The coordinate of \( N' \) is given by:
\[ \begin{align*}
  x' &= x + C \frac{\partial W(x, y)}{\partial x} \\
  y' &= y + C \frac{\partial W(x, y)}{\partial y}
\end{align*} \] (5)

With \( C = -\frac{f(f-1)}{4l^2} \). To compensate the Intensity of \( N' \) to \( N \), the formula should be:
\[ I(x, y) = I'(x', y') \left\{ 1 + C \left( \frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} \right) + C^2 \left[ \frac{\partial^2 W}{\partial x^2} \frac{\partial^2 W}{\partial y^2} - \left( \frac{\partial^2 W}{\partial x \partial y} \right)^2 \right] \right\}. \] (6)

### 3 THE CORRECTION OF INTENSITY COMPENSATION IN THE FAST F-NUMBER FOCUS TELESCOPES

#### 3.1 The Analysis of Correction of Intensity Compensation

In the fast f-number focus telescopes, the wave-front of the exit pupil in the un-aberrationed system is spherical centered at the point of image. As shown in Figure 3(a), the ray tracing of this spherical wave-front is equivalent to the ideal surface \( S \), the diameter of \( S \) is obviously larger than the exit pupil, thus the defocus images is different from the ideal system. Further more, FFT algorithm must take into consideration of the edge information \( \delta_s \frac{\partial W}{\partial n} \) of curvature signal. The projection of the equivalent surface \( S \) will cause the curvature signal stretch beyond the bounds of pupil edge. Those information will be
Fig. 3 (a) The correction of the on-Axis Coordinates; (b) The correction of the off-Axis Coordinates.

masked off by the pupil. Therefore, the coordinates in reference of the spherical wave front need to be corrected to the coordinates in reference of surface $S$, the corrected formula is:

$$\begin{align*}
x' &= D_1(x, y) \cdot x + C \partial W(x, y) / \partial x \\
y' &= D_2(x, y) \cdot y + C \partial W(x, y) / \partial y
\end{align*}$$

(7)

where $D_1(x, y), D_2(x, y)$ is the coordinates correction factor, which is equals to 1 for paraxial, $C$ is constant. Because $\frac{GE}{DC} = \frac{GO}{DO}$, and $GO = f$, $DO = \sqrt{CO^2 - DC^2} = \sqrt{f^2 - (x^2 + y^2) \cdot R^2}$, thus:

$$GE = \frac{f \cdot R}{\sqrt{f^2 - (x^2 + y^2) \cdot R^2}} \cdot \sqrt{(x^2 + y^2)}.$$

(8)

The component of $GE$ on x-axis is:

$$X = \frac{1}{R} \cdot GE \cdot \cos \theta = \frac{1}{R} GE \cdot \frac{x}{\sqrt{(x^2 + y^2)}} = \frac{f}{\sqrt{f^2 - (x^2 + y^2) \cdot R^2}} \cdot x.$$

(9)

Thus the correction factor on x-axis is:

$$D_1(x, y) = \frac{f}{\sqrt{f^2 - (x^2 + y^2) \cdot R^2}}.$$

(10)

Similarly, we can get the correction factor on y-axis:

$$D_2(x, y) = \frac{f}{\sqrt{f^2 - (x^2 + y^2) \cdot R^2}}.$$

(11)

In the situation of off-axis, as shown in Figure 3(b), CCD is perpendicular to the optical axis, resulting in an angle between the CCD and the curvature signal surface. Firstly we must compensate the coordinate distortion induced by this angle, and then use the on-axis compensation formula to compensate the coordinate. Assuming the field of view is $(\alpha, \beta)$, in the meridional plane we have:

$$\frac{AB}{AC} = \frac{\sin \angle ACB}{\sin \angle ABC} = \frac{\sin \angle ACB}{\sin(180^\circ - \angle ACB - \beta)}.$$

(12)
since \( \tan \angle AOC = \frac{yR}{f \cos \beta} \), then:

\[
AC = \frac{AB}{\cos \beta - \sin \beta \cdot \cos \beta \frac{yR}{f}}.
\] (13)

Similarly, we can get the coordinate compensate formula in the sagittal plane:

\[
AC = \frac{AB}{\cos \alpha - \sin \alpha \cdot \cos \alpha \frac{yR}{f}}.
\] (14)

Fig. 4 (a) the on-axis coordinates correction factor; (b) the vertical samples of correction factor; (c) the off-Axis coordinates correction factor; (d) the vertical samples of correction factor.

3.2 Compare the Coordinates Before and After Correction

The compensation of the distortion of the coordinate on axis is a procedure of non-linear scaling, as shown in Figure 4(a) and Figure 4(b), the correction factor is the smallest in the center of the aperture, and the scale of the ration is 1. In the situation of off axis, we can see from Figure 4(c) and Figure 4(d) that aside from the non-linear scaling of the coordinate, there is also a tilt factor on the coordinate.

4 RESULTS OF THE SIMULATION

In order to testify the Intensity compensate algorithm, we used a flat-field Schmidt telescope (as shown in Fig. 5) to simulate the curvature signal. The system parameters are: \( f = 100 \text{ mm} \), aperture diameter
$D = 75$ mm, defocus length $l = 1$ mm, wavelength $\lambda = 550$ nm, simulate field of view are $(0^\circ, 0^\circ)$ and $(0^\circ, 0.5^\circ)$.

\[ \text{Fig. 5} \quad \text{(a) Flat-field Schmidt Telescope; (b) The defocus surface.} \]

Even though the ray tracing ignored the diffraction effect of the aperture edge, the curvature signal simulate by ray tracing has the sufficient accuracy to recover the wave-front. The size of CCD that we simulate at defocus surface is 1 mm, and the pixels are $200 \times 200$. The entry pupil is sampled by Monte Carlo method, perform ray trace for each sample point. We can get the defocus images by tracing 100 thousands rays.

\[ \text{Fig. 6} \quad \text{the block diagram of wave-front recovery algorithm.} \]

The wave-front recovery process is shown in Figure 6: Firstly, we simulate the defocus images by Monte Carlo ray tracing and compensate the off-axis coordinate distortion. Then we create the curvature
Correction and Simulation of the Intensity Compensation Algorithm

Fig. 7 the changes of curvature signal during the compensation, the numbers on the upper left corner is the iteration numbers.

The curvature signal variation in on-axis situation is shown in Figure 7. Z4 is being compensated in the first iteration. Z4-Z6 of Zernike coefficients are being compensated from the second to fifth iterations. Z4-Z13 of Zernike coefficients are being compensated from the eighth to twelfth iterations. Z4-Z22 of Zernike coefficients are being compensated from the twelfth to twentieth iterations. We can see the last result that the curvature information of the curvature signal is almost zero, and the information of the aperture edge is also zero.

The on-axis and off-axis wave-front is showed in Figure 8. The on-axis aberration parameters are: PV value is 6.653 \( \lambda \), RMS is 1.137 \( \lambda \); and the off-axis aberration parameters are: PV value is 6.663 \( \lambda \), RMS is 1.14 \( \lambda \) (\( \lambda = 550 \text{ nm} \)). The coefficients of the wave-front zernike terms are shown in the first row of Tables 1 and 2. Because of vignetting, the top information of the off-axis wave-front is lost. Although the vignetting will introduces fitting errors of zernike coefficients, but the errors are too small to effect the accuracy of wave-front recovery.

The results of the simulation are shown in Tables 1 and 2. The third row in the tables is the zernike coefficients recovered by FFT wave-front recovery algorithm. For large aberrations (about 1 wavelength), the RMS of the recovered on-axis wave-front is 114 nm, and the RMS of recovered off-axis wave-front is 128 nm. The results don’t meet the requirements of wave-front sensing. The results show in the fourth row are the zernike coefficients recovered by FFT wave-front recovery algorithm with ideal intensity compensation algorithm. Even though more curvature signals are used in FFT wave-front recovery algorithm, the accuracy still can not meet the requirements because of the coordinate distortion. The last row in Tables 1 and 2 are the results recovered by inner iteration of FFT algorithm and outer iteration of corrected intensity compensation algorithm. The accuracy is highly improved because the
Fig. 8 (a) the on-axis wave-front s; (b) the off-axis wave-front.

distortion of coordinate is removed by the coordinate correction. The RMS on-axis decline from 52.9 nm to 12.4 nm, and the RMS off-axis decline from 82.5 nm to 13.7 nm. We reveal by the simulation results that the corrected intensity compensation algorithm is more reasonable for the wave-front recovery of the fast f-number focus system.

Table 1 the result of on-axis wave-front recovery

<table>
<thead>
<tr>
<th>Zernike Terms</th>
<th>$Z_4$</th>
<th>$Z_5$</th>
<th>$Z_6$</th>
<th>$Z_7$</th>
<th>$Z_8$</th>
<th>$Z_9$</th>
<th>$Z_{10}$</th>
<th>$Z_{11}$</th>
<th>$Z_{12}$</th>
<th>RMS (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zernike Coefficients ($\lambda$)</td>
<td>$-0.720$</td>
<td>$-0.602$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0.502$</td>
<td>$0$</td>
<td>$-0.401$</td>
<td>$-0.017$</td>
<td>$0$</td>
<td>$114$</td>
</tr>
<tr>
<td>FFT</td>
<td>$-0.591$</td>
<td>$-0.051$</td>
<td>$-0.013$</td>
<td>$0.025$</td>
<td>$0.528$</td>
<td>$0.023$</td>
<td>$-0.243$</td>
<td>$0.006$</td>
<td>$-0.011$</td>
<td>$52.9$</td>
</tr>
<tr>
<td>on-Axis</td>
<td>$-0.725$</td>
<td>$-0.609$</td>
<td>$0.018$</td>
<td>$0.013$</td>
<td>$0.500$</td>
<td>$0.008$</td>
<td>$-0.403$</td>
<td>$-0.018$</td>
<td>$0$</td>
<td>$12.4$</td>
</tr>
</tbody>
</table>

Table 2 the result of off-axis wave-front recovery

<table>
<thead>
<tr>
<th>Zernike Terms</th>
<th>$Z_4$</th>
<th>$Z_5$</th>
<th>$Z_6$</th>
<th>$Z_7$</th>
<th>$Z_8$</th>
<th>$Z_9$</th>
<th>$Z_{10}$</th>
<th>$Z_{11}$</th>
<th>$Z_{12}$</th>
<th>RMS (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zernike Coefficients ($\lambda$)</td>
<td>$-0.729$</td>
<td>$0.002$</td>
<td>$0.004$</td>
<td>$0.013$</td>
<td>$0.502$</td>
<td>$0$</td>
<td>$-0.401$</td>
<td>$-0.017$</td>
<td>$0$</td>
<td>$128$</td>
</tr>
<tr>
<td>FFT</td>
<td>$-0.533$</td>
<td>$-0.062$</td>
<td>$0.032$</td>
<td>$0.018$</td>
<td>$0.512$</td>
<td>$0.018$</td>
<td>$-0.427$</td>
<td>$0.001$</td>
<td>$-0.002$</td>
<td>$82.5$</td>
</tr>
<tr>
<td>Paraxial</td>
<td>$-0.654$</td>
<td>$-0.633$</td>
<td>$-0.030$</td>
<td>$0.032$</td>
<td>$0.390$</td>
<td>$0$</td>
<td>$-0.430$</td>
<td>$-0.028$</td>
<td>$0.014$</td>
<td>$13.7$</td>
</tr>
<tr>
<td>off-Axis</td>
<td>$-0.727$</td>
<td>$-0.614$</td>
<td>$0.023$</td>
<td>$0.017$</td>
<td>$0.488$</td>
<td>$0.008$</td>
<td>$-0.390$</td>
<td>$-0.016$</td>
<td>$0$</td>
<td>$12.4$</td>
</tr>
</tbody>
</table>

5 CONCLUSIONS

In this paper, We analyze the corrected intensity compensation algorithm of the fast f-number focus system, and apply this algorithm to recovery the wavefront of a flat-field Schmidt telescope. By comparing simulation results of ideal intensity compensation algorithm and corrected intensity compensation algorithm, we confirm that the corrected intensity compensation algorithm is more reasonable for the fast f-number focus system.

Acknowledgements This work was supported by the National Natural Science Foundation of China (11103048).
References

Su, D., & Cui, X. 1999, Progress in Astronomy, 17, 1