Do you mean Correction and simulation of the intensity compensation algorithm used in curvature wavefront sensing? *

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Received 2014 June 24; accepted 2014 September 4

Abstract The wave-front measuring range and recovery precision of the curvature sensor can be improved by the intensity compensation algorithm. However, in a focal system with a fast f-number focus system, especially in a telescope with a large field of view, the accuracy of this algorithm cannot meet the requirements. The theoretical analysis of the correction ability provided by the intensity compensation algorithm in a focal system with a fast f-number-focus system is firstly introduced and afterwards the mathematical formulas used in this algorithm are deduced. The corrected result is then verified through simulation. The method used by such a simulation can be described as follows. First, the curvature signal of a focal system with a fast f-number-focus system is simulated by Monte Carlo ray tracing; then the wave-front result is recovered calculated by the inner loop of the FFT wave-front recovery algorithm and the outer loop of the intensity compensation algorithm. Upon the comparison of comparing the intensity compensation algorithm with the corrected intensity compensation algorithm, we reveal that the recovered precision of the curvature sensor can be greatly improved by the corrected intensity compensation algorithm.

Key words: active optics — curvature sensor — wave-front recovery — intensity compensation Please select keywords from RAA’s keywords list.

1 INTRODUCTION

In the imaging process of telescopes, many factors will lead to the decline of image quality, such as optical design, optical fabrication error, gravity deformation, and thermal deformation. The concept of active optics has changed the way of designing...
telescopes are designed (Su & Cui 1999). People can take the initiative to correct the gravity deformation, temperature deformation, support error, and even the mirror machining error, but active optics make the mean improve image quality better (Su & Cui 2004). In Active Optics, real time sensing of wave-front error is very critical. There are two main kinds of Active Optical wave-front sensors used in active optics: the Hartmann wave-front sensor and wave-front curvature sensor. Do you mean a “Shack-Hartmann” wavefront sensor instead of a “Hartmann” wavefront sensor? Please check this point. The Hartmann wave-front sensor has been used in many large telescopes, such as the Chinese LAMOST project (Cui et al. 2004, Zhang 2008), and its. The measuring precision achievable with the active optics used in the LAMOST project is extremely high (Stepp 1994). Note: according to an online search of previously published articles, our editorial staff found that LAMOST uses a “Shack-Hartmann” wavefront sensor instead of a “Hartmann” wavefront sensor. Do you agree with this change? But However, in a telescope with a large field of view, the field of view used for sensing is limited by the Hartmann wave-front sensor, and almost equal to zero. However, a wave-front curvature sensor is based on the measurement of the wave-front’s curvature. Compared with the Hartmann sensor, it has the advantages of simplicity, high throughput, and avoidance of calibration difficulties, etc (Roddi&Roddi 1993). Do you mean Researchers working with the LSST (Manuel et al. 2010) and VISTA (Patterson & Sutherland 2003) have proposed to use this method to measure the wave-front in these large telescopes?

Do you mean There are two common methods to recover the wave-front by incorporating curvature sensors: the Fast Fourier Transform (FFT) Algorithm and the Gerchberg-Saxton Algorithm. Both algorithms require that the equivalent defocus distance is small and the wave-front aberration is not very large. Do you mean Thus, Roddi & Roddi (1993) proposed an intensity compensation algorithm based on an ideal system. By compensating the wave-front error of that generates the defocused images, the wave-front measurement range and recovered precision of the curvature sensor can be greatly improved. Roddi & Roddi used their algorithm in real telescopes like the ESO NTT and obtained very good results (Roddi & Roddi 1993), but those tests were applied on the relative relatively slow Cassegrain focus and where distortion errors in the pupil grid could be ignored. In telescopes with a fast f-number-focus, the reference point is much different from that of an ideal system, and distortion errors in the pupil grid will lead to lower recovered precision, thus this algorithm cannot be used in those systems directly. In addition, in the situation of off-axis-off-axis situation, the angle between the CCD and the defocused surface can also cause the coordinate distortion. In this paper, we theoretically deduce the equations for corrected intensity compensation formula in a telescope with a fast f-number-focus telescope, and simulate the situation of on-axis and off-axis with a flat-field in telescopes with a flat image plane. Note: “flat-field” means a correction to a CCD image due to the unequal sensitivity of pixels. Our editorial staff believes this is not your intended meaning. If our interpretation is not correct, please explain your meaning more carefully. The results show that the corrected intensity performed by the compensation algorithm is more reasonable and better for wave-front recovery in telescope with a fast f-number-focus telescopes.

Please provide a plan for the article in the form Section 2 gives... Section 3 describes... etc.

2 FFT WAVE-FRONT RECOVERY ALGORITHM AND INTENSITY COMPENSATION ALGORITHM

Curvature wave-front sensing was first proposed by Roddi in (1989). Roddi has successfully been used in adaptive optics successfully. Do you mean The principle of how a curvature sensor is operated is shown in Figure 1E. I1 and I2 are the defocused images. We can obtain the curvature signal by subtracting I2 from I1 and then perform normalization. Apply the expression? Note: our editorial staff does not understand why you call this equation a “normalization” since normalization generally means a sum is applied to make the values
equal 1. If our interpretation is not correct, please explain your meaning more clearly.

\[ S(r) = \frac{-1}{\Delta z} \left( I_2(r) - I_1(r) \right) \approx \nabla^2 W - \delta_c \frac{\partial W}{\partial n}. \]  

(1)

With Here, \( \Delta z = \frac{f}{f-l} \), \( f \) is the focal length of the optical system, \( l \) is the defocus distance, and \( \delta_c \) is the delta function at the aperture edge. The main task of wave-front recovery is to solve this Poisson equation with a Neumann boundary condition. If we constrain \( \partial W / \partial n \) on the aperture edge, \( \delta_c \) can be absorbed into Laplacian (Claver et al. 2012):

\[ S = \nabla^2 W. \]  

(2)

Do you mean Make the We apply a Fourier transform to both sides of formula Equation (2) and yielding \( \)

\[ FT_{\mu,\nu}[\nabla^2 W(x, y)] = -4\pi^2 \left( \mu^2 + v^2 \right) \, FT_{\mu,\nu}[W(x, y)]. \]  

(3)

Thus

\[ W = IFT_{x,y} \left\{ \frac{FT_{\mu,\nu}(S)}{-4\pi^2 (\mu^2 + v^2)} \right\}. \]  

(4)

The method mentioned above is only a first order approximation, valid for small \( \Delta z \), i.e., highly defocused images. The solution of the FFT algorithm should be used as a first order solution. Roddier discovered that the intensity compensation algorithm, which compensates the residual aberration of the defocused images, can improve the precision of wave-front recovery (Roddier & Roddier 1993). As shown in Figure 2, \( R \) is the radius of the telescope’s pupil, and the normalized coordinate of the pupil is \( x = U / R, y = V / R \). The normalized coordinate in the defocused surface is \( x' = u f / l R, y' = v f / l R \).

Do you mean Assuming no aberration, the ray from \( M \) will converge toward \( F \) and cross the defocused surface at point \( N' \). Because of the aberration \( W(x, y) \), the ray from \( M \) will converge to \( N' \). The coordinate of \( N' \) is given by

\[ \begin{cases} x' = x + C \partial W(x, y) / \partial x \\ y' = y + C \partial W(x, y) / \partial y \end{cases}, \]  

(5)
Do you mean the principle of how the intensity compensation algorithm operates? Please increase the resolution of the plots. Please use a uniform font size for all the legends.

With \[ C = -\frac{f(f-1)}{F^2} \]. Do you mean To compensate the intensity of \( N' \) to \( N \), the formula should be?

\[
I(x, y) = I'(x', y') \left\{ 1 + C \left( \frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} \right) + C^2 \left[ \frac{\partial^2 W}{\partial x^2} \frac{\partial^2 W}{\partial y^2} - \left( \frac{\partial^2 W}{\partial xy} \right)^2 \right] \right\}.
\] (6)

3.1 Do you mean The analysis of correction of intensity compensation?

In telescopes with a fast f-number focus telescopes, the wave-front of the exit pupil in an unaberrated system is spherically centered at the focal point of the image. As shown in Figure 3(a), the ray tracing of this spherical wave-front is equivalent to the ideal surface \( S \); the diameter of \( S \) is obviously larger than the exit pupil, thus the defocused images is different.
Correction and Simulation of the Intensity Compensation Algorithm

from that generated in an ideal system. Furthermore, the FFT algorithm must take into consideration the edge information of the curvature signal. The projection of the equivalent surface will cause the curvature signal to stretch beyond the bounds of the pupil edge. Those information will be masked off by the pupil. Therefore, the coordinates in reference to the spherical wave-front need to be corrected to the coordinates in reference to the surface $S$. The corrected formula for this correction is

$$\begin{cases} x' = D_1(x, y) \cdot x + C \frac{\partial W(x, y)}{\partial x} \\ y' = D_2(x, y) \cdot y + C \frac{\partial W(x, y)}{\partial y} \end{cases}$$ \tag{7}$$

where $D_1(x, y)$ and $D_2(x, y)$ are the coordinates correction factors, which are equal to 1 for the paraxial case, and $C$ is constant. Because $GE = \frac{GO}{DO}$, and

$$GO = f, \quad DO = \sqrt{CO^2 - DC^2} = \sqrt{f^2 - (x^2 + y^2) \cdot R^2},$$

thus

$$GE = \frac{f \cdot R}{\sqrt{f^2 - (x^2 + y^2) \cdot R^2}} \cdot \sqrt{(x^2 + y^2)}.$$ \tag{8}$$

The component of $GE$ on the $x$-axis is

$$X = \frac{1}{R} \cdot GE \cdot \cos \theta = \frac{1}{R} \cdot GE \cdot \frac{x}{\sqrt{x^2 + y^2}} = \frac{f}{\sqrt{f^2 - (x^2 + y^2) \cdot R^2}} \cdot x.$$ \tag{9}$$

Thus the correction factor on the $x$-axis is

$$D_1(x, y) = \frac{f}{\sqrt{f^2 - (x^2 + y^2) \cdot R^2}}.$$ \tag{10}$$

Similarly, we can derive the correction factor on the $y$-axis to be

$$D_2(x, y) = \frac{f}{\sqrt{f^2 - (x^2 + y^2) \cdot R^2}}.$$ \tag{11}$$

Do you mean In the situation of off-axis case, as shown in Figure 3(b), the CCD is perpendicular to the optical axis, resulting in an angle between the CCD and the curvature signal from the surface. Firstly we must compensate the coordinate distortion induced by this angle, and then use the equation for the on-axis compensation formula to compensate the coordinates. Assuming the field of view is $(\alpha, \beta)$, in the meridional plane we have

$$\frac{AB}{AC} = \frac{\sin \angle ACB}{\sin \angle ABC} = \frac{\sin \angle ACB}{\sin(180^\circ - \angle ACB - \beta)}.$$ \tag{12}$$

Since $\tan \angle AOC = \frac{y \cdot R}{f \cos \beta}$, then

$$AC = \frac{AB}{\cos \beta - \sin \beta \cdot \cos \beta \frac{y \cdot R}{f}}.$$ \tag{13}$$

Similarly, we can obtain the coordinate compensation equation in the sagittal plane

$$AC = \frac{AB}{\cos \alpha - \sin \alpha \cdot \cos \alpha \frac{y \cdot R}{f}}.$$ \tag{14}$$
Fig. 4 (a) Correction factor for on-axis coordinates correction factor; (b) Do you mean vertical examples of the correction factor; (c) the correction factor for off-axis coordinates correction factor; (d) Do you mean vertical examples of the correction factor? Please thicken the curves in Fig. 4(b) and (d).

3.2 Comparing the coordinates before and after correction

Do you mean The Compensation for the distortion of the coordinate on the axis is a procedure that involves non-linear scaling. As shown in Figure 4(a) and Figure 4(b), the correction factor is the smallest in the center of the aperture, and the scale of the ration is 1. In the situation of the off-axis case, we can see from Figure 4(c) and Figure 4(d) that aside from the non-linear scaling of the coordinate, there is also a tilt factor that is applied to the coordinate.

4 RESULTS OF THE SIMULATION

Do you mean In order to testify the intensity compensation algorithm, we used a flat field simulated observations using a Schmidt telescope with a flat image plane (as show in Fig. 5) to simulate and applied the curvature signal. Note: please see the previous note about “flat-field”. The system parameters we used are: $f = 100$ mm, aperture diameter $D = 75$ mm, defocus length $l = 1$ mm, wavelength $\lambda = 550$ nm, and the simulated fields of view are ($0^\circ, 0^\circ$) and ($0^\circ, 0.5^\circ$).
Do you mean Flat-field Schmidt Telescope with a flat image plane? (b) the defocused surface. The green lines in the Fig. 5(b) are not clear in the green background. Please use distinct color lines to replace them.

Even though the ray tracing ignored the effect of diffraction from the edge of the aperture, the curvature signal simulated by ray tracing has sufficient accuracy to recover the wave-front. The size of the CCD that we simulate at the defocused surface is 1 mm, and the pixels are arranged in a 200 × 200 grid. The entry pupil is sampled by the Monte Carlo method, which performs a ray trace for each sample point. We can simulate the defocused images by tracing 100 thousand 10^5 rays.

The wave-front recovery process is shown in Figure 6: Firstly, we simulate the defocused images by Monte Carlo ray tracing and compensate the off-axis coordinate distortion.
Fig. 7 The changes in the curvature signal during the compensation. The numbers in the upper left corner are the iteration numbers.

Fig. 8 (a) The on-axis wave-front; (b) the off-axis wave-front.

Then, we create the curvature signal to recover the raw wave-front by the FFT wave-front recovery algorithm. Secondly, by inputting the raw wave-front to the corrected intensity compensation algorithm, we obtain the compensated defocused images. Thirdly, we estimate the wave-front by the FFT algorithm and repeat these steps until the accuracy of the estimated wave-front meets the requirements. In order to prevent the oscillations caused by intensity compensation, a compensating factor needs to be multiplied (in this paper, the compensating factor is 0.6). Furthermore, we compensate the high order aberrations after compensating the lower order aberrations and the maximum order is 22.

Do you mean Variation in the curvature signal variation in the on-axis situation is shown in Figure 7. Z4 is being compensated in the first iteration. Z4–Z13 of the Zernike coefficients are being compensated from the second to the fifth iterations. Z4–Z22 of the Zernike coefficients are being compensated from the eighth to the twelfth iterations. Z4–Z22 of the Zernike coefficients are being
compensated from the twelfth to the twentieth iterations. We can see from the last result that the curvature information of the aperture edge is also zero.

The on-axis and off-axis wave-front is shown in Figure 8. The on-axis aberration parameters are: the PV value is 6.653 λ, and the RMS is 1.137 λ; the off-axis aberration parameters are: the PV value is 6.663 λ, and the RMS is 1.14 λ (λ = 550 nm). The coefficients of the wave-front Zernike terms are shown in the first row of Tables 1 and 2. Do you mean the curvature signal is almost zero, and the information of the aperture edge is also zero.

The results of the simulation are shown in Tables 1 and 2. The third row in the tables is the Zernike coefficients recovered by the FFT wave-front recovery algorithm. For large aberrations (about 1 wavelength), the RMS of the recovered on-axis wave-front is 114 nm, and the RMS of the recovered off-axis wave-front is 128 nm. The results do not meet the requirements of wave-front sensing. The results shown in the fourth row are the Zernike coefficients recovered by the FFT wave-front recovery algorithm with the ideal intensity compensation algorithm. Even though more curvature signals are used in the FFT wave-front recovery algorithm, the accuracy still cannot meet the requirements because of the coordinate distortion. The last rows in Tables 1 and 2 are the results recovered by the inner iteration of the FFT algorithm and the outer iteration of the corrected intensity compensation algorithm. The accuracy is highly improved because the distortion in the coordinate is removed by the coordinate correction. The RMS on-axis and the RMS off-axis decline from 52.9 nm to 12.4 nm, and the RMS off-axis RMS declines from 82.5 nm to 13.7 nm. We reveal that the simulation results indicate that the corrected intensity compensation algorithm is more reasonable for the wave-front recovery of the in a telescope with a fast f-number focus system.

5 CONCLUSIONS

Do you mean that we analyze the corrected intensity compensation algorithm of the focal system with a fast f-number focus system, and apply this algorithm to recover the wavefront of a flat field Schmidt telescope with a flat image plane? By comparing simulation results of the ideal intensity compensation algorithm and with the corrected intensity compensation algorithm, we confirm that the corrected intensity compensation algorithm is more reasonable for the focal system with a fast f-number focus system.
Acknowledgements  This work was supported by the National Natural Science Foundation of China (Grand No. 11103048).

References


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