High Brightness Temperatures in IDV Sources

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Abstract High brightness temperatures are a characteristic feature of IntraDay Variability (IDV) of extragalactic radio sources. Recent studies of the polarization properties of some IDV sources (e.g., 1150+812, PKS 0405−385 and 0716+714) have shown that these sources harbor several compact IDV components with angular sizes of \(\sim 10–30\)\(\mu\)as and very high polarizations (of up to \(\sim 50\%–70\%\)). These results indicate the possibility of the existence of uniform magnetic fields in the IDV components. We investigate the incoherent synchrotron and self-Compton radiation of an anisotropic distribution of relativistic electrons which spin around the magnetic field lines at small pitch angles. The brightness temperature limit caused by second-order Compton losses is discussed and compared to the brightness temperatures derived from energy equipartition arguments. It is found that anisotropic distributions of electrons moving in ordered magnetic fields can raise the equipartition and Compton brightness temperatures by a factor of up to \(\sim 3–5\). This would remove some of the difficulties in the interpretation of extremely high intrinsic brightness temperatures of \(> 10^{12}\) K (or apparent brightness temperatures of \(\sim 10^{14}\) K with a Doppler factor of \(\sim 30\)).

Key words: radio continuum: galaxies – Quasars: individual: PKS 0405−385

1 INTRODUCTION
Since the discovery of IntraDay Variability (IDV) in extragalactic radio sources (Witzel et al.1986; Heeschen et al. 1987), the high apparent brightness temperatures (10\(^{17}\)–10\(^{19}\)K) involved have been becoming an important issue in the interpretation of the physical origin of IDV. These high brightness temperatures are derived on the assumption that the IDV components have linear sizes determined by the distance travelled by light during the characteristic time of variability (\(l \leq c \Delta t\), \(c\) – speed of light, \(\Delta t\) – timescale of variability). If the derived high apparent brightness temperatures are caused by relativistic Doppler amplification, Doppler factors of \(\sim 100\) would be required, much higher than those derived from the VLBI observations (e.g., Kellermann 2002).

The discovery of extremely rapid variability on timescales of hours (and even less than an hour) stretches the problem to an extremity: the inferred apparent brightness temperatures reach \(\sim 10^{21}\) K (Kedziora-Chudczer et al. 1997; Dennet-Thorpe & de Bruyn 2000). These extremely high brightness temperatures exceed the well-known Compton limit of \(\sim 10^{12}\) K (Kellermann & Pauliny-Toth 1969, 1968) by a factor of \(10^9\), requiring Doppler factors of \(\sim 10^3\) if these rapid variations are regarded as intrinsic to the source.

However, the angular sizes of IDV components derived from light-travel-time effects are in the range of a few \(\mu\)as to a few tens of \(\mu\)as. With sizes of order of the Fresnel scale set by the interstellar medium interstellar scintillation is unavoidable. Therefore, the mechanism of refractive interstellar scintillation has been proposed to explain the intraday variability phenomenon and the extremely high apparent brightness temperatures.

All observed IDV sources are either classified as Quasar or as BL Lac object. The observed ‘blazar activity’ and the small source sizes obtained from VLBI observations and flux density variability suggest
that the IDV phenomenon is a superposition of at least two mechanisms (e.g., Qian 1994a, 1994b): intrinsic variability in the source and refractive interstellar scintillation caused by the interstellar medium.

### 1.1 Source Intrinsic Mechanisms

Source intrinsic mechanism has been proposed to interpret the apparent brightness temperatures of up to \( \sim 10^{15} \) K. This interpretation was mainly motivated by the observed correlation between the optical and radio intraday variations observed in the BL Lacertae object 0716+714 (Qian et al. 1996; Wagner et al. 1996; Wagner & Witzel 1995, also see Qian et al. 2000 for 0235+176). A similar correlation with a time lag of a few days between optical and radio bands was also observed in 0954+658 (Wagner et al. 1993).

Spada et al. (1999) showed that a pile-up effect of radio photons can occur in a conical jet with oblique shocks. This effect could explain brightness temperatures of \( 3 \times 10^{17} \) K with a relativistic bulk Lorentz factor \( \Gamma \) of only 10, assuming a Doppler factor \( \delta = \Gamma \). This model shows that in certain cases and with a special jet geometry, a non-spherical relativistic aberration correction could result in an amplification of the brightness temperature by a factor equivalent to \( \sim \delta^5 \).

In a similar manner, Qian et al. (1991) proposed a shock-in-jet model in which a shock propagates through a turbulent magnetized plasma (similar to the shock model of Marscher et al. 1992, also see Blandford & Königl 1979; Königl 1981). This allowed one to interpret the observed IntraDay variations of QSO 0917+624 in terms of interaction between shock and existing inhomogeneities in the jet without the assumption of extreme Doppler factors. In this model four effects were taken into account: (1) pile-up of photons caused by the relativistic shock propagating across the small scale inhomogeneities; (2) the time-shortening caused by the light-travel time effect (similar to the interpretation of superluminal motion); (3) a slab-like very thin shock with a projected size much larger than the size derived from the variability timescale (\( \delta c \Delta t_{\text{obs}} \)), i.e., the IDV timescales being related to the passing time of the thin shock passing through the small-scale inhomogeneities (see also Marscher & Gear 1985); (4) a time variable polarization of the shock (both degree and angle of polarization) as it propagates along the jet. This model may be useful to consistently explain the entire range of intraday phenomena observed in some blazars, including the high brightness temperatures of \( \sim 10^{18} \) – \( 10^{19} \) K, the variations in the total and polarized flux density, their correlation and anti-correlation, and the variability of the angle of polarization, especially the continuous swing of \( \sim 180^\circ \), which could not be explained in the scintillation model of Rickett et al. (1995) (also see Qian & Zhang 2004).

The models of Qian et al. (1991) and Spada et al. (1999) both use non-standard shock-in-jet geometries, and try to explain the excessive brightness temperatures using established values for the Lorentz- and Doppler-factors (\( \Gamma \leq 25 \) and \( \delta \leq 50 \) and, in addition, a relativistic transformation which yields magnification mechanisms more effective than that for simple spherical geometries (S \( \propto \delta^{-3} \)). In this regard, Qian et al. (1996, 1996a) have also proposed a model of shock propagating in an oscillatory jet to interpret the intraday variations in some sources, especially the correlation between the radio and optical IDV observed in 0716+714 (further discussion will be given in a forthcoming paper by Qian et al.).

In addition to the aforementioned models, we note that high apparent brightness temperatures may also indicate an intrinsic violation of the \( 10^{12} \) K inverse Compton limit. Slysh (1992) argued that in a nonstationary case an intrinsic brightness temperature of \( T_b \), \( \sim 10^{14} \) – \( 10^{15} \) K could be reached for limited time periods of days to weeks.

### 1.2 Refractive Interstellar Scintillation

The light travel time argument indicates angular source sizes in the range of a few \( \mu \)as to a few tens of \( \mu \)as (taking into account of Doppler corrections with \( \delta \simeq 10 \)–30). Therefore it is nearly unavoidable that intraday and intra-hour variable sources should scintillate. As the observed variability time scales are too long for diffusive scintillation, refractive interstellar scintillation has been proposed to explain the total intensity and polarization IDV (c.f. Qian 1994a, b; Rickett et al. 1995; Qian et al. 2001; Qian et al. 2005; Qian et al. 2006). In particular, for the very high brightness temperatures (\( \sim 10^{19} \)–\( 10^{21} \) K), derived in the ultra-rapid sources (PKS 0405–385: Kedziora-Chudzcer et al. 1997; J1819+3845: Dennett-Thorpe & de Bruyn 2000) the scintillation models provide a comprehensible interpretation. In a few sources the refractive scintillation mechanism has been verified by the detection of annual modulation of IDV timescale caused by the orbital motion of the Earth (0917+624: Qian & Zhang 2001a; Rickett et al. 2001; J1819+3845: Dennett-Thorpe...

Generally, in the refractive scintillation explanation, the angular sizes of the IDV components can be much larger than those determined from intrinsic mechanisms and Doppler factors of 10–30 are sufficient to explain the intraday variations without exceeding the inverse Compton limit of the brightness temperature. However, in a few IDV sources the situation is still in question. For example, for PKS 0405–385, the scintillation interpretation seriously depends on the assumption regarding the distance to the scattering screen and its velocity relative to the observer.

(1) Kedziora-Chudczer et al. (1997) assumed a distance to the scattering screen of \( \sim 500 \text{ pc} \) and a relative velocity of 50 km s\(^{-1}\). They estimated the size of the scintillating component to be \( \sim 5 \mu \text{as} \) and the corresponding brightness temperature to be \( T_b \sim 5 \times 10^{14} \text{ K} \). A similar high brightness temperature was recently derived for J1819+3845 based on observed diffractive interstellar scintillation at \( \lambda 21 \text{ cm} \) (Macquart & de Bruyn 2005). For both sources a large Doppler factor of \( \sim 10^{2} - 10^{3} \) is required in order to reduce these brightness temperatures to the equipartition value or to the inverse Compton limit (\( \sim 10^{11} - 10^{12} \text{ K} \)). However, with a Doppler factor of this order, a source intrinsic interpretation of the IDV would again become possible and variability brightness temperatures of up to \( 10^{21} \text{ K} \) could be reached. In view of the known jet speeds from VLBI, such high Doppler factors clearly pose a problem.

(2) Recently, Rickett et al. (2002, 2002a) re-analyzed the flux density and polarization variability of PKS 0405–385 observed in 1996 and suggested that the preferred distance to the scattering screen is only 25 pc (reduced by a factor of 20 compared to previous estimates). With an assumed lower velocity of 30 km s\(^{-1}\), the source size increases to \( \sim 30 \mu \text{as} \). The corresponding brightness temperature is then reduced to \( 2 \times 10^{13} \text{ K} \), from which a Doppler factor of \( \delta \sim 75 \) is derived (on taking brightness temperature limit to be \( 10^{11.5} \text{ K} \)). This value of Doppler factor is still much larger than those measured by VLBI observations. It is clear that the interpretation of intra-hour variations in terms of refractive scintillation stretches the parameters for the scattering medium and the Doppler factor to extreme.

1.3 Motivation of This Work

From the above we see the interpretation of IDV is not unambiguous and that there are still unsolved problems, in particular about the hard limits to the intrinsic brightness temperatures and Doppler factors. It is still unclear if the usual Compton limit of \( T_b \) can be exceeded and if so, by how much, and what the maximum Doppler factor can be. In the following we therefore will discuss some possibilities that may reduce some of the difficulties in the interpretation of IDV. Here we mainly like to follow and apply the theory proposed by Burbidge et al. (1974): the inherent synchrotron and self–Compton emission of anisotropic distributions of relativistic electrons in ordered magnetic fields.

To structure the problem, we can divide this issue mainly into three questions:

(1) Are the observed variability time scales related to refractive scintillation or to source intrinsic variations, the latter affected by relativistic bulk motion? In other words, are the angular sizes of the IDV components determined by \( \delta \varepsilon \Delta t / D_l \) (\( D_l \) is the luminosity distance) or by the angular sizes derived from scintillation interpretations?

(2) Is the brightness temperature upper limit controlled by the second-order Compton scattering or by energy equipartition between electrons and magnetic field (here we define the Compton limit as \( E_{\text{esc}}(T_b) = 1 \); and the equipartition limit as \( E_{\text{em}}(T_b) = 1 \); see below)? This leads to a significant difference between these two limits by a factor of \( \sim 10 \) (the equipartition limit is \( \sim 10^{11} \text{ K} \) and the Compton limit is \( \sim 10^{12} \text{ K} \), Readhead 1994, also see Scheuer & Williams 1968; Scott & Readhead 1977);

(3) Is there any possibility to increase the value of the limiting brightness temperature, either the inverse Compton limit (\( 10^{12} \text{ K} \)) or the equipartition limit (\( 10^{11} \text{ K} \)) by some factor? For example, an increase of the limiting \( T_b \) by a factor of \( \sim 3–5 \) would already reduce the need for excessive Doppler factors in a large number of IDV sources.

We point out that recent studies of the polarization properties of some IDV sources (e.g., 1150+812: Qian et al. 2006; PKS 0405–385: Rickett et al. 2002 and 0716+714: Bach et al. 2006) have shown that the
polarization of the compact IDV components may be very high (up to \(\sim 50\% - 70\%\)). This would imply the existence of rather uniform magnetic fields. In this paper we therefore would like to concentrate on the third problem and investigate the influence on the brightness temperature limit by an anisotropic distribution of the electron velocity in ordered magnetic field. We will argue that anisotropy could raise the Compton- and equipartition-limits by a factor of 3–5, depending on the pitch angles with which the electrons spin around the magnetic field lines.

2 THEORY AND FORMULISM

2.1 Introduction

The well known \(\sim 10^{12}\) K inverse-Compton limit for the intrinsic brightness temperature proposed by Kellermann & Pauliny-Toth (1969) is based on a number of assumptions:

1. the radiation mechanism is incoherent synchrotron process;
2. the source has a spherical geometry;
3. an isotropic distribution of the electron velocity;
4. a disordered (randomly distributed) magnetic field;
5. both first-order and second-order Compton scattering occur in the Thomson regime.

If one (or more) of these conditions is not satisfied, the inverse-Compton limit would become invalid. For example, in the case of electron synchrotron emission (proton synchrotron emission will be not discussed in this paper), several possibilities have been proposed for excessive intrinsic brightness temperatures.

1. Anisotropic geometry
   Protheroe (2003) proposed that in the case of a long narrow optically thin synchrotron emitting region (i.e., a filament) the synchrotron self-Compton emission could be largely reduced, when it is viewed along its axis. In this case, the average energy density in the emission region can be many orders of magnitude lower than that calculated from the observed intensity, if one assumed a spherical emission region.

2. Coherent mechanism
   Coherent emission mechanisms have been proposed to explain high brightness temperatures (for example, Melrose 1999; Benford 1992; Benford & Tzach 2000) by collective plasma processes. However, the main problem with these mechanisms is their probably too narrow emission bandwidth, which is difficult to reconcile with the observed broad spectra of extragalactic radio sources (including IDV sources). Synchrotron masers (Zheleznyakov et al. 2000) also belong to this kind of mechanism.

3. Non-stationary processes
   Slysh (1992) proposed that continuous acceleration of electrons might be able to balance the inverse-Compton radiative losses and allow the source having brightness temperatures exceeding the inverse-Compton limit for limited periods of time. However, the model only considers the first-order Compton radiative losses and first-order Fermi acceleration in shocks, and does not take into account the second-order inverse-Compton scattering. To our knowledge, there is no acceleration mechanism which can balance the second-order inverse-Compton losses (see below). It seems that the only possibility for this is continuous injection of relativistic electrons from a reservoir which is not emitting itself and which channels the electrons into the emitting region.

4. Anisotropic distribution of electrons
   For us, the most promising mechanism which can be applied to sustain excessive brightness temperatures may be an anisotropic electron distribution in an ordered magnetic field. In this case the inverse-Compton scattering would be greatly reduced due to the small interaction angle between the electrons and photons. This possibility is of particular interest to IDV sources, because recent studies of the polarization structure of some IDV sources indicate the existence of very high polarization. Rickett et al. (2002) have derived a model to interpret the polarization variations of PKS 0405–385 and found the degree of polarization of the IDV components to be extremely high (\(\sim 70\%\)). Moreover, Qian et al. (2006) have interpreted the rapid polarization angle swing of \(\sim 180^\circ\) observed in 1150+812 in terms of scintillation by interstellar clouds and found the IDV component to have a degree of polarization...
of > 50%. These high polarizations may imply that the magnetic fields in some IDV components are ordered and anisotropic distribution of electrons could be formed.

Before we discuss this possibility and derive examples of higher brightness temperature limits, we should mention that Woltjer (1966) and Burbidge et al. (1974) have discussed the case of a source with an ordered magnetic field and an anisotropic distribution of electrons and photons. They argued that self-Compton scattering can be greatly reduced if the interaction angle between the electrons and photons becomes very small. Reynolds (1982) has also discussed the synchrotron self-Compton radiation under the condition of an anisotropic distributions and confirmed the conclusions by Burbidge et al. (1974). In this paper, we will follow the theory proposed by Jones et al. (1974a, b) and Burbidge et al. (1974). These authors simplified the calculations by introducing an averaged (or characteristic) interaction angle between electrons and photons. They considered the incoherent synchrotron and self-Compton radiation for an anisotropic distributions of electrons in ordered fields, taking into account also the relativistic bulk motion (relativistic Doppler effects). For the IDV sources considered here, we assume relativistic flows of high energy particles that move with a large bulk Lorentz factor and draw the magnetic field lines along the flow direction. The relativistic electrons spin around the field lines with a small pitch angle. In this case the synchrotron self-Compton radiation will be greatly reduced due to the small interaction angle between the synchrotron photons and electrons. The situation is in contrast to the theory described by Kellermann & Pauliny-Toth (1969) and Readhead (1994), who assumed isotropic distribution of electrons and disordered magnetic fields.

In the following we will apply the formalism of Burbidge et al. (1974) and Jones et al. (1974a, b) to discuss the synchrotron-self-Compton emission of a spherical source with radial fields in which relativistic electrons flow radially outwards at small pitch angles\(^1\). In order to emphasize the dependence of the source properties on brightness temperature we will express all the relevant formulae in terms of the observed (or apparent) brightness temperature. Intrinsic parameters (in the rest frame of the components) will be designated by the subscript \(*\). We use cgs units, and express temperatures in units of \(m_e c^2/k(=10^{9.77})\), \(m_e\) – rest mass of electron, \(k\) – Boltzmann constant, \(c\) – speed of light). We only discuss synchrotron self-Compton emission of the electrons.

### 2.2 Formulae

1. The observed (apparent) brightness temperature \(T_n\) at frequency \(\nu_n\).

\[
T_n(K) = \frac{2S_\nu}{m_e\nu_n^2\Omega_s},
\]

where \(S_\nu\) is the spectral flux density at the fiducial frequency \(\nu_n\). \((S_\nu, \nu_\nu)\) represents the intersection point of the power-law extrapolations of the thin and thick parts of the synchrotron spectrum. The solid angle is \(\Omega_s = \pi\theta_\nu^2\), \(\theta_\nu\) is the observed half angular size.

2. The intrinsic electron Lorentz factor,

\[
\gamma_{n,*} = \frac{T_n}{i_{\alpha_0}O(\tau_n)} \frac{1}{\delta(1+z)}
\]

where \(i_{\alpha_0}\) is a constant depending on the spectral index \(\alpha\) (Jones et al. 1974a). \(O(\tau_n)\) is the optical depth at the frequency \(\nu_n\), \(\delta\) is the Doppler factor and \(z\) is the redshift of the source.

3. The magnetic energy density,

\[
u_{m,*} = (6.85\times10^6)(i_{\alpha_0})^4O(\tau_n)^4 \left(\frac{\delta}{1+z}\right)^2 \left(\frac{T_n}{\sin\theta}\right)^{-4},
\]

where \(\theta\) is the pitch angle with which the electrons spin around the magnetic field lines.

\(^1\) In the case of fields parallel to jet flow, which may be more appropriate to IDV components, the results are similar to the case of radial fields.
(4) The synchrotron radiation energy density,
\[ u_{\text{syn}} = \frac{1}{1 - \alpha} \left[ \frac{\nu_n}{\nu_h} \right]^{1 - \alpha} - 1 \frac{\sin^2 \theta}{(\lambda_n)^3} \left( \frac{1 + z}{\delta} \right)^4 T_n, \]
where \( \nu_h \) is the high-frequency cutoff of the synchrotron spectrum.

(5) The electron energy density,
\[ u_{\text{es}} = 1.1 \times 10^6 (1 + \frac{1}{3} \cos^2 \theta) \sin^2 \theta \frac{O(\tau_n)^3 k_{\alpha 0}}{\lambda_n \nu_{\rho 0} O(\tau_n)^2 D_1 \theta_s \delta_s} \]
where \( D_1 \) is the luminosity distance, \( \delta_s \) is introduced to allow for relativistic expansion, \( k_{\alpha 0} \) is another constant depending on the spectral index (see Jones et al. 1974a), \( \nu_I \) the frequency corresponding to the low-energy cutoff of the electron energy distribution.

(6) The ratio of first-order Compton emission power to synchrotron radiation power, \( E_{\text{ssc}} \approx \frac{3 u_{\text{syn}}}{4 u_{\text{m}}}, \)
\[ \frac{(1 - \cos \phi)^2}{(2 \sin \theta)} (\phi \text{ is the average interaction angle, } \approx \theta) \text{ and thus } \]
\[ E_{\text{ SSC}} = 2 r_e \sin^2 \theta (1 - \cos \theta)^2 \frac{O(\tau_n)^2 D_1 \theta_s \delta_s}{\lambda_n \nu_{\rho 0} O(\tau_n)^2} \times \]
\[ \frac{\left[ \frac{\nu_h}{\nu_n} \right]^{1 - \alpha} - 1 \left( \frac{1 + z}{\delta} \right)^6}{1 - \alpha} (T_n)^5, \]
where \( r_e = 2.82 \times 10^{-13} \text{ cm} \) is the radius of the electron.

(7) The ratio of electron energy density to magnetic energy density, \( E_{\text{em}} = \frac{u_{\text{em}}}{u_{\text{m}}}, \)
\[ E_{\text{em}} = \frac{1}{2 \pi} \frac{\lambda_n \sin \theta (1 + \frac{\cos \theta}{\sin \theta})^2}{3 O(\tau_n)^2 D_1 \theta_s \delta_s} \times \]
\[ \frac{1}{2 \alpha - 1} \left[ \frac{\nu_n}{\nu_h} \right]^{\alpha - 0.5} - \left( \frac{\nu_n}{\nu_h} \right)^{\alpha - 0.5} \left( 1 + \frac{z}{\delta} \right)^2 \left( \frac{1 + z}{\delta} \right)^7 (T_n)^8. \]

(8) The ratio of the synchrotron radiation energy density to the electron density, \( E_{\text{syn,es}} = \frac{u_{\text{syn}}}{u_{\text{es}}}, \)
\[ E_{\text{syn,es}} = \frac{16 \pi}{3} \frac{i_{\alpha 0} O(\tau_n)^3 k_{\alpha 0} D_1 \theta_s \delta_s}{\lambda_n \sin \theta (1 + \frac{\cos \theta}{\sin \theta})} \times \]
\[ \frac{2 \alpha - 1}{1 - \alpha} \left( \frac{\nu_n}{\nu_h} \right)^{\alpha - 0.5} - \left( \frac{\nu_n}{\nu_h} \right)^{\alpha - 0.5} \frac{r_e}{(1 + z)^2} \frac{\delta}{1 + z} (T_n)^3. \]

### 2.3 Equipartition- and Compton- Brightness Temperatures

In the following we define two brightness temperatures: equipartition– and Compton– brightness temperatures \( T_{\text{eq}} \) and \( T_{\text{sc}} \).

(1) Equipartition brightness temperature \( T_{\text{eq}} \) is defined by \( E_{\text{em}}(T_{\text{eq}}) = 1 \). At this apparent temperature the emitting component has equipartition between the electron and magnetic energy density in the emitting region.

(2) Compton brightness temperature \( T_{\text{sc}} \) is defined by \( E_{\text{ssc}}(T_{\text{sc}}) = 1 \). When the apparent brightness temperature exceeds \( T_{\text{sc}} \), the second-order inverse-Compton scattering will dominate the radiative losses and a Compton catastrophe will occur.

Assuming the second-order inverse-Compton scattering takes place also in the Thomson regime, we have \(^2\)
\[ \frac{P_{2,ic}}{P_{1,ic}} = \frac{P_{1,ic}}{P_{\text{syn}}} = E_{\text{ssc}}, \]
(see Jones 1979). Here \( P_{1,ic} \) and \( P_{2,ic} \) are the power of the first- and second-order self-Compton emissions, \( P_{\text{syn}} \) is the power of the incoherent synchrotron emission.

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\(^2\) In fact, part of the second-order Compton scattering occurs in Klein-Nishina regime, which reduces the second-order Compton emission. See Tavecchio et al. (1998).
2.4 Lifetimes of Electrons due to Synchrotron Self-Compton losses

We define the life timescale of electrons due to radiative losses as \( t_{\gamma} = \frac{1}{2} \frac{E}{\alpha \dot{E}} \) (Readhead 1994).

- The life timescale due to synchrotron radiative loss \( t_{\gamma,\text{syn}} \) is

\[
t_{\gamma,\text{syn}} = 1.47 \frac{\lambda_n^2}{\theta_{\text{obs}}^2} \frac{O(\tau_n)}{O(\tau)} \frac{1 + \frac{z}{\delta}}{(T_n)^3}.
\]

- The life timescale due to synchrotron plus first- and second-order Compton radiative losses, \( t_{\gamma,\text{syn+1c+2c}} \), is

\[
t_{\gamma,\text{syn+1c+2c}} = t_{\gamma,\text{syn}} \left[ 1 + F_{\text{osc}}^* + F_{\text{osc}^2}^* \right]^{-1}.
\]

3 RESULTS AND DISCUSSION

To demonstrate the consequences of our calculation, we may consider the following example assuming incoherent synchrotron self-Compton emission of an anisotropic distribution of relativistic electrons in an ordered magnetic field, as described above.

We choose some of the parameters with reference to the IDV source PKS 0405–385. Adopting a cold-dark-matter model (ΛCDM) with Hubble constant \( H_0 = 71 \, \text{km s}^{-1} \text{Mpc}^{-1} \), \( \Omega_m = 0.3 \) and \( \Omega_\Lambda = 0.7 \) (Hogg 1999; Peacock et al. 2001; Pen 1999; Spergel et al. 2003), we obtain for a source at redshift of \( z = 1 \) a luminosity distance of \( D_L = 6.5 \, \text{Gpc} \). We assume a source size \( \theta_s = 15 \, \mu\text{as} \), a Doppler factor\(^3\) \( \delta = 30 \), \( \delta_s = 1 \) (without consideration of relativistic expansion) and spectral index \( \alpha = 0.75 \), thus we obtain \( i_{\alpha\gamma} = 0.212 \) and \( k_{\alpha\gamma} = 3.34 \) (Jones et al. 1974a). We further assume \( \nu_1 = 0.1 \, \text{GHz} \), \( \nu_n = 10 \, \text{GHz} \) (\( \lambda_n = 3 \, \text{cm} \)), \( \nu_h = 1000 \, \text{GHz} \) and \( \Omega(\tau_n) = 1 \) (the low-frequency turnover of the synchrotron spectrum is assumed to be due to self-absorption).

In Figures 1–10 and Tables 1–6 (all the tables are published in this Journal’s web page), we summarize the quantities given in Equations (2)–(8) and (10)–(11) and for different electron pitch angles. All calculations were made for a discrete sequence of pitch angles, spanning the interval \( \theta = 0.72^\circ – 57.3^\circ \). We note that \( \theta = 57.3^\circ \) is close to the case of an isotropic distribution, and \( \theta < 57^\circ \) can be regarded as representing the extreme anisotropic case. In Tables 1–3 the parameters are calculated for the source with equipartition brightness temperature \( T_{\text{eq}} \), and in Tables 4–6 for a source having the Compton brightness temperature \( T_{\text{sc}} \). It should be noted that \( T_{\text{eq}} \) and \( T_{\text{sc}} \) represent the apparent brightness temperatures, and \( T_{\text{eq}}/\delta (\equiv T_{\text{eq}}^*) \) and \( T_{\text{sc}}/\delta (\equiv T_{\text{sc}}^*) \) represent the intrinsic brightness temperatures, after correction of relativistic boosting.

The figures and tables show that an anisotropic distribution of relativistic electrons in an ordered magnetic field reduces the self-Compton emission and thus the radiative losses of relativistic electrons. The equipartition– and Compton–brightness temperatures therefore can be increased. We summarize the results as follows.

3.1 Relations: \( T_{\text{eq}} \theta \) and \( T_{\text{sc}} \theta \)

As shown in Figure 1 (Tables 1 and 4), with the pitch angle decreases (or as the anisotropy increases), the equipartition- and Compton-brightness temperatures \( T_{\text{eq}} \) and \( T_{\text{sc}} \) increase. For example, at pitch angle \( \theta = 5.73^\circ \), \( T_{\text{eq}} = 8.8 \times 10^{12} \, \text{K} \) and \( T_{\text{sc}} = 2.1 \times 10^{14} \, \text{K} \), compared to \( T_{\text{eq}} = 2.4 \times 10^{12} \, \text{K} \) and \( T_{\text{sc}} = 1.5 \times 10^{13} \, \text{K} \) at pitch angle \( \theta = 57.3^\circ \), which is close to the case of an isotropic distribution of electrons (note the intrinsic brightness temperatures can be obtained from the apparent brightness temperatures by dividing by the Doppler factor \( \delta = 30 \)). The Compton brightness temperature is usually larger than the equipartition brightness temperature by a factor of 5–50. Because the Doppler factor is moderate, it is more favourable for IDV sources to adopt the Compton brightness temperature as the upper limit to the intrinsic brightness temperature. However, the extreme departure from the equipartition between the electron energy and field energy is still a problem. Plausibly, we would assume that the equipartition is a consequence of some isotropization process, which occurs along the jet and does not hold for very compact sources (with

\(^3\) In the case of an anisotropic distribution of electrons, the viewing angle \( \theta_{\text{obs}} \) is approximately equal to \( \theta/\delta \) and is very small, thus the bulk Lorentz factor \( \Gamma \approx \delta/2 = 15 \), which is a moderate value observed by VLBI in superluminal sources.
Fig. 1 Dependence of apparent brightness temperature $T_{\text{eq}}$ (K) and $T_{\text{sc}}$ (K) on pitch angle. It is shown that $T_{\text{eq}}$ and $T_{\text{sc}}$ increase with decreasing pitch angle or with increasing anisotropy.

Fig. 2 Dependence of field strength $B_\star$ (Gauss) on pitch angle. It is shown that in the case of $T_n = T_{\text{sc}}$ the field strength rapidly decreases with decreasing pitch angle. In the case of equipartition ($T_n = T_{\text{eq}}$), the field strength decreases much less.

Fig. 3 Dependence of field energy density $u_{m\star}$ (erg cm$^{-3}$) on pitch angle. In the case of $T_n = T_{\text{sc}}$, $u_{m\star}$ rapidly decreases with decreasing pitch angle.

Fig. 4 Dependence of electron energy density $u_{e\star}$ (erg cm$^{-3}$) on pitch angle. It is shown that in the case of Compton brightness temperature $T_{\text{sc}}$, the electron energy density rapidly increases with decreasing pitch angle. This is due to the rapid departure from equipartition and the rapid decreasing field strength. In contrast, in the case of equipartition brightness temperature $T_{\text{eq}}$ the electron energy density varies much less.

angular sizes of a few tens of $\mu$as), i.e., if the component is in the innermost region of the jet where the electrons are accelerated along the magnetic fields.

3.2 Relation: $\frac{T_{\text{sc}}}{T_{\text{eq}}} - \theta$

With increasing anisotropy (or decreasing pitch angle), the ratio $\frac{T_{\text{sc}}}{T_{\text{eq}}}$ increases. This leads to an increasing departure from the equipartition state, if the source is at its Compton brightness temperatures (Table 6). For example, $E_{\text{em}\star} = 9.7 \times 10^{10}$ at $\theta = 5.73^\circ$ compared to $E_{\text{em}\star} = 1.9 \times 10^{10}$ at $\theta = 57.3^\circ$. This is in agreement with the arguments of Readhead (1994) that the radiation at the Compton limit requires significant departure from equipartition between the electron energy density and the magnetic energy density. The main reason for this phenomenon is that $E_{\text{em}\star}$ is propotional to the 8th power of the brightness temperature: $E_{\text{em}\star} \propto T_b^8$
Dependence of electron Lorentz factor $\gamma_{\text{n}*}$ on pitch angle. It is shown that in the case of Compton limit $T_{\text{sc}}$, not only the electron energy density, but also the electron Lorentz factor rapidly increase with decreasing pitch angle. This is due to the rapid decrease of the magnetic field strength while the observed radiation frequency remains constant.

Dependence of the ratio $E_{\text{em}*}$ between electron and magnetic energy density on pitch angle. It is shown that in the case of $T_n = T_{\text{eq}}$, this ratio rapidly increases with decreasing pitch angle, showing its increasing departure from equipartition.

Dependence of ratio $E_{\text{ssc}*}$ between self-Compton radiation and synchrotron radiation on pitch angle. It is shown that for the equipartition case this ratio is always much less than 1, i.e., self-Compton radiation is not important relative to synchrotron radiation (no Compton catastrophe occurs).

Dependence of synchrotron radiation energy density $u_{\text{syn}*}$ on pitch angle. It is shown that for both the Compton and the equipartition cases, the synchrotron radiation energy density decreases similarly with the decreasing pitch angle.

(see Eq. (7)). If a source has an apparent brightness temperature of $2T_{\text{sc}}$ or $3T_{\text{sc}}$ the departure from the equipartition will be very extreme.

3.3 Relation: $\gamma_{\text{n}*}\theta$

The Lorentz factor $\gamma_{\text{n}*}$ of the electrons emitting at the frequency $\nu_n$ increases with decreasing $\theta$ (Tables 1 and 4). When $T_n = T_{\text{eq}}$, $\gamma_{\text{n}*} < 2 \times 10^3$, but when $T_n = T_{\text{sc}}$ and $\theta < 5.73^\circ$ (extreme anisotropy) $\gamma_{\text{n}*}$ reaches $\sim 10^4 - 10^5$. These extremely high energy electrons would possibly Compton-scatter the external isotropic photons (e.g., the optical photons surrounding the component and the cosmic IR background) to produce high energy radiation in hard X-rays and $\gamma$-rays. Moreover, in these extreme anisotropic cases,

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4 In this paper, we assume the angular size of the IDV component is constant ($15 \mu\text{as}$), and does not depend on the brightness temperature. Thus the values of $u_{\text{e}*}$ (and correspondingly $E_{\text{em}*}$) calculated for the higher brightness temperatures could be underestimated and the values of $E_{\text{syn}*}$ could be overestimated. The differences, however, are less than a factor of 10.
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**Fig. 9** Dependence of the ratio $E_{\text{syn+}}$ of synchrotron radiation energy density to electron energy density on pitch angle.

**Fig. 10** Doppler factor $\delta = 30$: Half-life timescales $t_{1/2}$ of electrons emitting at peak frequency $\nu_m$ due to (synchrotron + first-order inverse-Compton scattering + second-order inverse-Compton scattering) radiative losses. The two solid lines are for pitch angles $\theta = 57.3^\circ$ and $0.72^\circ$. The dashed lines are for $\theta = 28.7^\circ, 22.9^\circ, 17.2^\circ, 11.5^\circ, 5.73^\circ, 2.87^\circ$ and $1.43^\circ$. $\theta = 57.3^\circ$ approximately resembles the isotropic case discussed in Readhead (1994).

$E_{\text{ens}}$ reaches $10^{11} - 10^{15}$ (Table 6), which is an extreme departure from equipartition. Therefore, we would not suggest that such an extreme anisotropic case occurs in IDV sources. Most plausible distributions would invoke $\theta > \sim 0.1$ radians.

### 3.4 Relation: $t_{1/2,*} - T_n$ (Synchrotron Loss)

In Figure 10 we plot the half-life timescale $t_{1/2,*}$ for radiative losses from synchrotron, first- and second-order inverse-Compton scattering, as function of apparent brightness temperature (c.f. Eq. (11)). The curve for $\theta = 57.3^\circ$, corresponding to nearly isotropic distribution of relativistic electrons, is similar to that given by Readhead (1994). These curves show that with increasing anisotropy, the synchrotron state and the peak of the curves (where $E_{\text{ssc*}} \simeq 1$) move towards higher brightness temperatures. Correspondingly, the half-life timescale increases and is much longer than in the isotropic case. At equipartition brightness temperatures (for all values of $\theta$) the synchrotron radiative losses dominate and $E_{\text{ssc*}} \ll 1$ (see Fig. 7), the life timescale of electrons increases with decreasing pitch angle (increasing brightness temperature). This is because the magnetic field strength is inversely proportional to the 2nd power of the brightness temperature (see Eq. (3), Fig. 2 and Table 1). Thus the effect of an anisotropic distribution is to extend the synchrotron state to higher
brightness temperatures, i.e., anisotropic distributions can raise the equipartition and Compton limits to higher temperatures.

3.5 Relation: $t_{1/2,b} - T_n$ (Compton Loss)

As shown in Figure 10, beyond the peak of the curve (or beyond the Compton brightness temperature $T_n$) the second-order inverse-Compton scattering dominates the radiative losses and the life timescale of electrons rapidly decreases with the increasing brightness temperatures: $t_{\frac{1}{2}} \propto T_n^{-7}$. This part of the curves is the region of the well-known “Compton catastrophe”. It can be seen that for a given apparent brightness temperature, anisotropy can greatly increase the half-life timescale. For example, for $\theta = 57.3^\circ$ (nearly isotropic case) the half-life timescale is $\sim 10^{-9}$ years for a source with an apparent brightness temperature of $10^{15}$ K ($T_{bn} = 3.3 \times 10^{13}$ K). In comparison, for $\theta = 5.73^\circ$ the corresponding timescale is order of $\sim 10^4$ years. This explains why in the isotropic case the intrinsic brightness temperatures could not exceed the Compton limit of $\sim 10^{12}$ K, as already suggested by Kellermann & Pauliny-Toth (1969). The main reason is that any re-acceleration processes in the source (e.g., by a relativistic shock) could not compensate the rapid ‘Compton cooling’ of the high energetic electrons, because the acceleration timescale is too long compared to the radiative loss timescales. This is in contrast to the conclusion obtained by Slysh (1992), in which only the first-order Compton loss was considered, and the second-order inverse-Compton scattering was not taken into account for the evolution of the energy distribution of the electrons. This poses a problem for the study of a non-stationary process, as radiative losses depend on the brightness temperature of the source in that the radiative loss moves from being dominated by synchrotron plus first-order Compton scattering to being dominated by second-order Compton scattering when the brightness temperature exceeds the Compton limit (i.e., $E_{\text{sec}} > 1$ or $T_{bn} \sim 10^{12}$ K) (also see Ghisellini et al. 1998).

In contrast with the isotropic case, for $\theta = 5.73^\circ$ (an extreme anisotropic case) the half-life timescales are of the order of $10^7$ and $10^4$ years for the source at $T_{bn} = 10^{13}$ K ($T_{bn} = 3.3 \times 10^{12}$ K) and $10^{15}$ K ($T_{bn} = 3.3 \times 10^{13}$ K), respectively. Thus we can see that in the anisotropic cases the apparent brightness temperatures could reach $\sim 10^{14}$–$10^{15}$ K or the intrinsic brightness temperatures could reach $\sim 10^{13}$ K. In addition, in these anisotropic cases the life timescales would be mainly determined by the isotropization processes and much shorter than the radiative-loss timescales.

3.6 Circular Polarization

In the case of ordered field and anisotropic distribution of electrons, synchrotron radiation would have significant circular polarization. Under the assumption of unidirectional field and for the electron-proton plasma, the circular polarization can be estimated as

$$p_c \approx \frac{\cot \theta}{\gamma_{ns}} \left(\frac{T}{10^6}\right) \left(\frac{T_n}{10^9}\right) \left(\frac{\theta}{5.73}\right)^{3/4} \left(\frac{10^{13} K}{T_n}\right),$$

where $\gamma_{ns} \approx 5.34 \left(\frac{T_n}{10^9}\right)$ (see Eq. (2)). The results are given in Table 7 for apparent brightness temperatures of $10^{13}$–$10^{15}$ K, which are typical for some extreme IDV sources that have been interpreted in terms of interstellar scintillation (Kedziora-Chudzzer et al. 2000; Dennett-Thorpe & de Bruyn 2002). It can be seen that the calculated circular polarization are $0.1\%$–$2\%$ for $\theta$ in the range of $5.7^\circ$–$12^\circ$. These values seem too high to be comparable with the available observations. This might imply that the effects caused by reversal of field direction and pair plasma should be taken into account for IDV sources (Wardle & Homan 2001). In addition, Kedziora-Chudzzer (2000) showed that in PKS 1519–273 the observed degrees of circular polarization are $-2.6\%$, $-3.8\%$ and $-2.4\%$ at 8.6, 4.8 and 2.5 GHz, respectively, for an apparent brightness temperature of $2 \times 10^{14}$ K. From Table 7 it can be seen that the modelled circular polarizations are much less than those observed in PKS 1519–273. This implies that anisotropic distributions of electrons in ordered fields could not explain the circular polarization in PKS 1519–273. This is consistent with the fact that the observed circular polarization disagrees with the $\nu^{-1/2}$ (or $\gamma^{-1}$) dependence expected from the synchrotron theory. Thus the circular polarization in PKS 1519–273 is probably produced by propagation effects: transformation from linear polarization by Faraday rotation or by birefringent scintillation due to propagation through a scintillating screen. Since the variations in the circular polarization are observed to be correlated with the variations in the intensity, the latter mechanism may dominate (Kedziora-Chudzzer et al. 2000; Macquart & Melrose 2000; Macquart et al. 2000).
4 SUMMARY

In this paper we have argued that in IDV sources compact regions of high linear polarization emission may exist (Rickett et al. 2002; Qian et al. 2006). This would indicate the presence of a highly ordered magnetic field on angular scales of 10–30 µas. The bulk motion of relativistic electrons may draw out the magnetic field and the electrons may spin around the field lines with small pitch angles, when they are accelerated by some mechanism along the field. In the case of an anisotropic distribution of the relativistic electrons in an ordered magnetic field, the inverse-Compton scattering in both Thomson and Klein-Nishina regimes will be largely reduced. This will lead to an increase of the inverse Compton limit of brightness temperature, which then could exceed the well-known limit of $10^{11}–10^{12}$ K. A higher value of the intrinsic brightness temperature limit would reduce the difficulties in the interpretation of the physical origin of IDV, as it would lead to less extreme requirements for the Lorentz– and Doppler– factors in these sources (see Qian et al.1991; Qian 1994a; Rickett et al.2002; Spada et al. 1999; Kedziora-Chudczer et al.1997; Macquart & de Bruyn 2005). Moreover, the isotropization of electron distributions can be regarded as an alternative possibility for the disappearance of IDV sources of excessive brightness temperatures.

From this study it is difficult to decide which brightness temperatur limit, equipartition limit or Compton limit, applies, because the mechanisms which produce equipartition are not known and the mechanisms which can prevent large departure from equipartition are not known either. For interpretation of the extremely high brightness temperatures observed in a few sources (like PKS 0405–385 and J1819+3845), however, the combination of refractive scintillation, equipartition, anisotropy and relativistic bulk motion may be a plausible mechanism. Adopting equipartition the brightness temperature limit seems physically more preferable, because the total energy budget (particle energy plus field energy) is at minimum and little Compton losses or no Compton catastrophe could occur. X-ray emission from IDV components may be used to distinguish the brightness temperature limits obtained from equipartition and from the compton catastrophe (see Fig. 7).

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References

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